Funding Value Adjustments

LEIF ANDERSEN, DARRELL DUFFIE, and YANG SONG*

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Abstract

We demonstrate that the funding value adjustments (FVAs) of major dealers are debt-overhang costs to their shareholders. In order to maximize shareholder value, dealer quotations therefore adjust for FVAs. Our case examples include interest-rate swap FVAs and violations of covered interest parity. Contrary to current valuation practice, FVAs are not themselves components of the market values of the positions being financed. Current dealer practice does, however, align incentives between trading desks and shareholders. We also establish a pecking order for preferred asset financing strategies and provide a new interpretation of the standard debit value adjustment (DVA).

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*Andersen is with Bank of America Merrill Lynch. Duffie is with Stanford University Graduate School of Business and NBER. Song is with Stanford University Graduate School of Business. We are grateful for comments from the referees, the associate editor, and the editor, as well as Claudio Albanese, Shalom Benaim, Damiano Brigo, Stéphane Crépey, Yuanchu Dang, Rupert Brotherton-Ratcliffe, Yann Coatanlem, Stéphane Crépey, Youssef Elouerkhaoui, Marco Francischello, Jon Gregory, Lincoln Hannah, Burton Hollifield, John Hull, David Lando, Wujiang Lou, Alexander Marini, Martin Oehmke, Andrea Pallavicini, Stephen Ryan, Steven Shreve, Taylor Spears, and Hongjun Yan.

We calculate the debt-overhang costs to dealer-bank shareholders associated with financing new balance-sheet positions. For debt-financed swap positions, we show that these shareholder costs are equal to the funding value adjustments (FVAs) that dealers have been making to the reported market values of their swap books. Contrary to this dealer practice, however, FVAs are not actually components of the market values of the positions being financed. Instead, they are debt-overhang transfers from shareholders to creditors.

We show that dealer bid and ask quotes, if aligned with shareholder interests, must incorporate the debt-overhang costs represented by FVAs. That is, in order to maximize their equity value, dealers must quote so as to extract enough trading profit from their counterparties to overcome the FVA-associated costs to their shareholders. This wedge represents a significant friction in over-the-counter markets.

The following simple example illustrates the meaning of an FVA. A dealer purchases $100 face value of one-year T-Bills, and commits to hold them to maturity. Risk-free interest rates are assumed to be zero. The dealer purchases the T-bills at their mid-market value, $100. The purchase is funded by issuing unsecured debt, which could be motivated by a desire to increase the dealer's regulatory measure of High Quality Liquid Assets (HQLA). The dealer has an unsecured one-year credit spread of 50 basis points. At the end of the year, the T-bills will pay $100 and the dealer will repay $100.50 on its financing. The dealer's shareholders will therefore suffer a net loss in one year, after financing costs, of $0.50. This loss will be borne by the dealer's shareholders only if the dealer survives. Assuming the dealer's one-year risk-neutral survival probability p^* is 0.99, the shareholder equity value is thus reduced by $p^* \times 0.50 = 0.495$. This cost to shareholders is the FVA for this trade. The FVA is a transfer in value to legacy creditors, who now have access to an additional safe asset in the event of default.

If the dealer were to apply FVA-based valuation practice to the T-bills following the same method currently used for swaps,[1] the dealer would assign the T-bills a market value equal to the mid-market value of $100 less a funding value adjustment of $0.495, for a net market value of only $99.505. By assumption, however, the T-bills have an actual market value of $100, implying an inconsistency.

Were it not for the HQLA requirement in this example, the dealer would not conduct this trade at the given pricing terms. The dealer's shareholders benefit from this trade only if the T-bills can be purchased at a price below $99.505. More generally, in order to align its market-making function with shareholder interests, a dealer's price quotation practice must reflect funding value adjustments. Thus, even though the current FVA practice of dealers is not correct from the perspective of market valuation, it does achieve this alignment of

[1] In current practice, dealers do not typically apply FVAs to their bond positions.

Table I Funding value adjustments of major dealers (millions). Source: supplementary notes of quarterly or annual financial disclosures. The $1.5 billion 2013 FVA of JP Morgan includes an FVA of about $1.1 billion for derivatives and about $400 million for structured notes.

	Amount	Date Disclosed
Bank of America Merrill Lynch	$497	Q4 2014
Morgan Stanley	$468	Q4 2014
Citi	$474	Q4 2014
HSBC	$263	Q4 2014
Royal Bank of Canada	C$105	Q4 2014
UBS	Fr267	Q3 2014
Crédit Suisse	Fr279	Q3 2014
BNP Paribas	€166	Q2 2014
Crédit Agricole	€167	Q2 2014
J.P. Morgan Chase	$1,500	Q4 2013
Nomura	$98	Q1 2014
ANZ	AUD61	Q4 2013
Bank of Ireland	€36	Q4 2013
Deutsche Bank	€364	Q4 2012
Royal Bank of Scotland	$475	Q4 2012
Barclays	£101	Q4 2012
Lloyds Banking Group	€143	Q4 2012
Goldman Sachs	Unknown	Q4 2011

incentives. Being forced to mark down the value of the T-Bills by the FVA implies that traders will not be credited with a trading profit unless they can purchase the T-Bills at a price that is below the true market value by at least the FVA. As we will discuss, there are other ways to obtain this shareholder alignment that do not involve valuation inconsistencies.

Funding costs have long been informally considered an input to dealer trading decisions. Beginning in 2011, major dealer banks started to formally show FVAs on their balance sheets, as described by Cameron (2014b) and Becker (2015), and as shown in Table I. Details on how these adjustments have been made are discussed by Albanese, Andersen, and Iabichino (2015).

The move by dealers to formally introduce funding value adjustments probably has several causes. First, beginning in 2008, severe deviations of dealers' borrowing rates from risk-free rates resulted in funding costs that were so large that excluding them from financial statements might have been considered imprudent. (Indeed, we provide assumptions under which large FVAs should be made, although not to the asset side of the balance sheet.) Second, the finance departments of many dealers now feel confident that funding cost adjustments are observable in market transaction terms. (Our model explains why this should be the case.) Third, despite the absence of published financial accounting standards that support FVA practice, large accounting firms have signaled a willingness to accept FVA disclosures in dealers' financial statements. See, for example, Ernst and Young (2012) and

KPMG (2013).

Current practice also implies that FVAs generate tax savings for dealers, because their taxable incomes are lowered whenever swap values are lowered by FVAs. As we show however, in economic terms, FVAs do not actually involve a reduction in income.

Missing from the controversy over FVA, to this point, has been a model that is consistent with underpinning theories of asset pricing and corporate finance and that accounts for the impact of funding strategies on the market valuation of claims on a dealer's assets, most importantly equity and debt. We provide such a model, along with a number of implications for dealer quotations, trading desk incentives, and preferred financing strategies.

We show, by theory and calibrated numerical examples, that FVAs are also an important determinant of dealer bid-ask spreads. Because the financing of collateral or cash upfront payments can cause a change in capital structure that is costly to dealer shareholders, dealers maximize shareholder value by using quoting strategies that overcome this cost to their shareholders with a sufficient widening of bid-ask spreads.

As an empirical example, Wang, Wu, Yan, and Zhong (2016) estimate the impact of the 2009 "big-bang" introduction of upfront payments for credit default swaps on CDS bid-ask spreads. They write: "Intuitively, the upfront payment is an impediment to trading, and so reduces the market liquidity, leading to higher bid-ask spreads." Our model justifies this intuition. Wang et al. (2016) indeed find that big-bang upfronts widened bid-ask spreads significantly.[2]

As another example, we consider the post-crisis violations of covered interest parity (CIP) documented by Du, Tepper, and Verdelhan (2018) and Rime, Schrimpf, and Syrstad (2017). For a dealer to benefit its shareholders by arbitraging a CIP violation, our FVA calculations imply that the magnitude of the CIP basis must roughly exceed the dealer's credit spread.

More generally, our results are part of a growing body of work, including for example Adrian, Etula, and Muir (2014) and Brunnermeier and Pedersen (2009), that examines the impact of dealer capital structure on asset price behavior. Because over-the-counter (OTC) markets rely heavily on intermediation by dealers, FVAs can play a significant role in the liquidity of OTC products whose intermediation requires substantial amounts of dealer funding.

The rest of this paper is organized as follows. Section I outlines prior research on FVAs. Section II introduces a basic two-period model of the marginal effects of investments and investment financing decisions on the market valuation of the firm's debt and equity. Section

[2] They find that "for a CDS contract with a spread level of 300 basis points, at the average level of the Libor-OIS spread in our sample, 32 basis points, the upfront payment introduced by the CDS Big Bang increases the bid-ask spread by 1.5 basis points. This is a sizeable effect as the bid-ask spread in our sample has a mean of 9.6 basis points and median of 5.3 basis points."

III applies and extends these basic results to swap valuation, the impact of swap valuation on a dealer's equity and debt, and swap rate quotation. Here, we provide a new theoretical foundation for funding value adjustment, showing how it applies to a dealer's equity with a compensating partial adjustment to debt valuation, but with no impact on fair swap valuation. We treat swaps with and without upfront payments, as well as the impact of initial and variation margin. Section IV illustrates how FVA significantly reduces the incentive of most banks to exploit violations of covered interest parity. In Section V, we illustrate the magnitudes and directional responses of FVAs and DVAs that may be anticipated in practical settings of plain-vanilla interest-rate swaps, based on a reduced-form analogue of a structural multi-period version of the model. Section VI summarizes our key results and discusses some of its broader implications. Proofs and other extensions are found in appendices.

I. Prior Research

While including an FVA as a component of the market value of swaps has seemed natural to many practitioners, the practice has been controversial. Concerns about the validity of FVA methodology have been raised, for instance by Hull and White (2012, 2016), Cameron (2013, 2014a), Becker and Sherif (2015), and Sherif (2016b). Some have pointed to questionable asset-liability valuation asymmetries induced by FVAs, a seeming absence of accounting for the DVA effects of the associated debt issuance, and an incongruity in the way that FVA for derivatives liabilities overlap with already-reported DVA for derivatives. These issues have been discussed by Hull and White (2012, 2014, 2016), Albanese and Andersen (2014), and Albanese et al. (2015), among others. In addition, there appears to be significant variation across dealers in the manner in which dealers compute their FVA metrics, particularly with respect to measurement of the relevant unsecured borrowing rates. Recently, the Office of the Comptroller of the Currency, a U.S. banking regulator, announced the formation of a working group to examine industry practices for FVA determination. (See Sherif (2015b).)

To our knowledge, of prior related work on FVA,[3] only Burgard and Kjaer (2011) and Castagna (2013, 2014) specifically incorporate the incremental cash flows of a swap into a model of the balance sheet of a dealer. Using a reduced-form model of the event of the dealer's default, but explicitly capturing the impact of swaps on the dealer's default recovery, Burgard and Kjaer (2011) show that adding an appropriately hedged derivative has no impact on the dealer's funding costs.[4] They do not use their balance-sheet model to

[3]There is a large body of applied derivatives valuation research that addresses FVA and related concepts. Key examples include Pallavicini, Perini, and Brigo (2012), Pallavicini, Perini, and Brigo (2011) and Elouerkhaoui (2016).

[4]See, for example, their equations (20)-(25).

isolate the nature of FVA as a cost to shareholders. Indeed, contrary to our results, their approach allows swap market values to be affected by dealer funding costs.[5] In a narrower setting, Castagna (2013, 2014) calculates a marginal funding-cost impact on shareholders that is similar in spirit to our own. In the end, however, Castagna (2014) concludes that the market valuation of derivatives should include the FVA component, which is opposite to our result. The similar approach but different conclusion of Castagna arises from his implicit assumption that the valuation of a financial instrument is the value of only that component of its cash flows that is ultimately assigned to equity shareholders.[6]

II. Shareholder Financing Costs

This section characterizes the effect on a firm's shareholders and creditors of financing an investment, or a package of financial transactions. We focus here on debt financing, which is the basis for FVA. Appendix A provides the analogous explicit calculations for the impact on shareholder value of equity financing and of financing with existing balance sheet cash, as well as a pecking order of preferred financing methods. These results recapitulate relatively standard concepts of asset pricing and corporate finance in a novel form that is useful for explaining the role of FVA and for solving valuation and dealer price-quotation problems.

A. Representation of Market Valuations

Our most basic setting is a market at time 0 for claims to uncertain cash flows at time 1. For simplicity, we assume that the set of possible states of the world at time 1 is finite. All of our results apply in the general case of infinitely many states of the world under standard technical continuity conditions.[7] The proofs of our results, given in Appendix B, cover both

[5] Burgard and Kjaer (2011) also construct dealer strategies that can "shield the balance sheet" from funding costs, thus eliminating or reducing inconsistencies that arise in current practice when the same swap cash flows are not valued symmetrically by their two counterparties due to funding value adjustments.

[6] For example, Castagna (2014) states, at page 14, that "The results just shown confirm also that the practice of including the funding valuation adjustment (FVA) in the valuation (i.e.: internal pricing) process of a contract is fully justified: this thesis was supported in Castagna [7] (arguing against the opposite view in Hull&White [12] and [14]) but not proved analytically."

[7] For general one-period models with the potential for infinitely many states or infinitely many traded instruments, we can fix an arbitrary probability space $(\Omega, \mathcal{F}, P)$. In addition to the given assumptions, sufficient additional regularity is obtained by assuming that the set $\mathcal{L}$ of payoffs to which a valuation is assigned is a linear subspace of the set $L^1(P)$ of random variables with finite expectation having the property that $\mathcal{L} - L^1(P)_+$ is closed in $L^1(P)$. The existence of a bounded stochastic discount factor ν then follows from Yan's Separation Theorem. See, for example, Schachermayer (1992). Dalang, Morton, and Willinger (1990) extends this representation result in the obvious way, without need for finite-state or continuity assumptions, to settings with a finite number of intermediate trading periods and with a finite number of primitive traded financial instruments.

finite-state and infinite-state cases. Without loss of generality, each state has some given strictly positive probability. All investors in our model have the same information.

In order to characterize the market valuation[8] of financial instruments that may appear on the balance sheet of a dealer, we fix the set $\mathcal{L}$ of payoffs at time 1 to which a fair value at time zero is assigned by some given "fair-market-value" function $V : \mathcal{L} \to \mathbb{R}$. We impose only minimal coherency assumptions on market-value assignments, namely that $V(\,\cdot\,)$ is linear[9] and increasing in payoffs. That is, *(i)* the value of a portfolio of different cash flows is the sum of the values of the elements of the portfolio, and *(ii)* if payoff X is greater than or equal to payoff Y in every state of the world, and if $X > Y$ in some states of the world, then $V(X) > V(Y)$.

Under these two coherency assumptions, Stiemke's Lemma implies that there is stochastic discount factor, that is, a strictly positive random variable ν with the property that the value of any payoff Y is $V(Y) = E(\nu Y)$. We take one of the payoffs to be that of a risk-free bond. The associated risk-free discount is $\delta = E(\nu)$, implying a risk-free gross rate of return of $R = \delta^{-1}$. It follows that fair valuations, henceforth called "valuations" or simply "values," can be assigned according to "risk-neutral" expectation. That is, we can define risk-neutral expectation E^* by letting $E^*(Y) = E(\nu Y)R$, so that the value of any payoff Y can be represented as $V(Y) = E(\nu Y) = \delta E^*(Y)$. The associated risk-neutral probability measure P^* is defined by $P^*(B) = E^*(1_B)$ for any event B, with indicator 1_B. Because ν is not necessarily uniquely determined, the risk-neutral probability measure P^* is not necessarily unique.

Although this seems familiar from the standard setup of an arbitrage-free asset pricing model, we do not actually assume the absence of arbitrage in the usual sense. We have merely given a representation of how market valuations are assigned by $V(\,\cdot\,)$. In an over-the-counter market, market valuations need not coincide with the prices at which instruments are actually traded by specific dealers. At or about the same point in time, the same asset can be traded at different prices, reflecting the distinct bids and offers of different

[8] Some of the controversy about FVA arises in part from tension over how to measure market values. For example, international accounting standard IFRS 13 refers to the use of "exit prices," meaning roughly the price that the firm would receive when selling (if a net asset) or transferring (if a net liability) a derivatives portfolio to a new counterparty in an orderly transaction. This approach to fair market valuation raises some additional consistency issues that we do not address. Both U.S. accounting standards (in particular FASB 157 and 159) and international accounting standards (IFRS 13) require that traded OTC derivatives be disclosed at their fair value, rather than by ordinary accrual (or cost) accounting. We merely take fair market valuation as a given concept subject only to the two coherency axioms stated above (linearity in payoffs and increasing in payoffs), which are rather compelling for any approach to measuring fair market value.

[9] That is, $\mathcal{L}$ is a linear space and for any two payoffs X and Y and any scalars a and b, the value of the portfolio payoff $aX + bY$ is $V(aX + bY) = aV(X) + bV(Y)$.

dealers. The associated price dispersion,[10] which would not occur in a competitive all-to-all market, reflects search costs, differences in dealer-client relationships, difference in netting opportunities, and differences in dealer capital structures. As we will show, a dealer should refuse to trade some types of financial instruments unless it can buy them at prices strictly below their market values, or sell them at prices strictly above market values, to extents that vary across dealers based on differences in the structures of their balance sheets. The ability of dealers to execute trades at prices that reflect non-zero bid-ask spreads arises from the imperfect nature of financial markets, particularly over-the-counter markets, in which search costs and other frictions frequently give dealers a trading advantage over non-dealers. As we shall explain, bid-ask spreads are needed to cover more than a dealer's overhead and trading expenses (which we ignore here). We will show the amounts by which a dealer, based on its own balance sheet, may need to widen its a bid-ask spread so as to overcome a variant of debt overhang, representing the cost to the dealer's shareholders of financing the cash needed to enter new positions.

B. *The Marginal Valuation of Corporate Assets, Liabilities, and other Claims*

We consider a firm whose assets and liabilities have payoffs at time 1 (before additional trades are considered) given by random variables A and L, respectively. The firm defaults in the event $D = \{A < L\}$. At default, liquidation or reorganization may lead to distress costs. The asset value remaining after default, net of distress costs, is κA, for some recovery parameter $\kappa \in (0, 1]$. The market values of the firm's equity and debt are therefore $\delta E^*[(A - L)^+]$ and $\delta E^*(\kappa A 1_D + L 1_{D^c})$, respectively, where $D^c = \{A \geq L\}$ is the event of no default.

We now consider a potential new investment by the firm, such as a swap, whose payoff Y may be positive in some states and negative in other states. The positive part $Y^+ = \max(Y, 0)$ is an asset of the firm. The negative part $Y^- = \max(-Y, 0)$ is a contingent liability. The positive part Y^+ is measured net of any losses due to counterparty default. Our convention is that the liability component Y^- is the contractual amount due, before considering the firm's potential for default. If the contingent liability Y^- is fully secured,[11] then it has a value to the firm of $-\delta E^*(Y^-)$, so that the total value of the financial instrument is $\delta E^*(Y)$.

If the contingent liability Y^- is not fully secured, we must specify how the associated counterparty recovers on its claim in case the firm defaults. We assume throughout that

[10] For example evidence of price dispersion in OTC markets, see Jankowitscha, Nashikkara, and Marti (2011) and Green, Hollifield, and Schürhoff (2007).

[11] The liability is secured, for example, if $A > Y^-$ and the liability is collateralized or otherwise takes priority over other liabilities.

the firm's unsecured liabilities are pari passu with each other, so that the various claimants' default recoveries are pro rata with their claim sizes. In practice, the unsecured portions of a firm's swap contingent liabilities are normally pari passu with its unsecured senior debt claims. If the firm acquires the candidate new claim to Y, the liability component Y^- is the firm's only other unsecured default claim, so that the value to the firm of this claim is $\delta E^*(\mathcal{C})$, where $\mathcal{C}$ is net actual cash flow to the firm, given by

$$\mathcal{C} = 1_{\{A+Y \geq L\}} Y + 1_{\{A+Y<L\}} Y^+ - 1_{\{A+Y < L\}} \rho\kappa A, \tag{1}$$

where

$$\rho = \frac{Y^-}{L+Y^-}$$

is the pro-rata share of this contingent liability.

In order to later treat collateralized swap positions, we will also need to consider cases in which the contingent liabilities include both secured and unsecured components. For this purpose, we allow for the case of a financial position whose cash flows to be paid to the firm at time 1, before considering the effect of the firm's own default, have a decomposition of the form $Y = Y_1 + Y_2$, where the first contingent liability Y_1^- is secured and the second contingent liability Y_2^- is unsecured and pari passu in default with other unsecured creditor claims. In this case, the firm's valuation of the associated net time-1 cash flow is $\delta E^*(\mathcal{C})$, where $\mathcal{C}$ is the net actual cash flow at time 1, given by

$$\mathcal{C} = Y_1 + 1_{\{A+Y \geq L\}} Y_2 + 1_{\{A+Y<L\}} Y_2^+ - 1_{\{A+Y < L\}} \kappa(A+Y_1)\rho, \tag{2}$$

where

$$\rho = \frac{Y_2^-}{L+Y_2^-}$$

is the pro-rata share of the unsecured liability Y_2^-. (Here, we have assumed for simplicity that adding the given position has no impact on the proportional default recovery coefficient κ.) A necessary condition for the contingent liability Y_1^- to be secured is that $A + Y_1 \geq 0$, which we assume.

For a position that has net actual cash flows at time 0 of c_0 and at time 1 of c_1, the total valuation is of course $c_0 + \delta E^*(c_1)$. In the next subsection, we examine the preferences of the firm's shareholders for how the initial cash flow c_0 is financed, meaning transformed into time-1 cash flows by issuing new debt or new equity.

C. The Marginal Value to Shareholders of a Debt-Financed Investment

The firm contemplates entering some quantity q of an investment, such as a package of financial instruments with one or more counterparties. In this subsection, we are mainly concerned with the impact of entering this investment on the firm's shareholders. Before considering the effect of the firm's default, the per-unit payoff of the package at time 1 is given by some random variable Y, which may have a negative outcome with positive probability. The net cash-flow to the firm at time 1 for a position of size q is therefore qY. We allow that Y may be of the form $Y = Y_1 + Y_2$, where Y_1^- is secured and Y_2^- is unsecured.

As shown in Appendix B, the following calculations also apply without change to an infinite-state setting provided that, with respect to P^*, the random variables A, L, Y_1, and Y_2 have finite expectations, and provided that A and L have a continuous joint probability density, or if A has a continuous density and L is a constant.

The investment cost for q units of the new position is some given amount $U(q)$, which is not necessarily equal to the market value of the position's cash flows, allowing for the possibility of a trading profit. The marginal investment cost, $u \equiv \lim_{q\downarrow 0} U(q)/q$ is assumed to be well-defined. We allow $U(q)$ to have either sign. If $U(q)$ is positive, the initial investment cost must be financed at time 0. If $U(q)$ is negative, the firm may invest the cash received, $-U(q)$, or use it to retire debt or equity.

We assume for simplicity that the firm faces a competitive capital market for new debt and equity issuances. That is, those competing to offer equity or debt financing to the firm break even by paying the market value of any claim issued to them by the firm. This implies in particular that the yield spread paid on debt issuances is entirely driven by credit considerations. Any part of the spread originating with, say, imperfect liquidity is not treated here. At a cost in complexity, one could extend our model to incorporate a liquidity spread on debt.

We now calculate the marginal value of the investment for the firm's shareholders, assuming debt financing. Appendix A provides the analogous explicit calculations for equity and cash financing. In order to avoid singularities when calculating derivatives, we maintain throughout the assumption that $P(A = L) = 0$. In the finite-state case, this assumption holds generically in the space of all model parameters.[12]

Throughout the remainder, "marginal value" means the first derivative of the market value of the claim under consideration, per unit of the claim. Except for cases in which the

[12]With some finite number n of states, we can treat (A, L) as a vector in $\mathbb{R}_+^{2n}$. The property that $P(A = L) = 0$ holds generically, that is, for all such pairs of vectors except for a closed subset of $\mathbb{R}_+^{2n}$ of Lebesgue measure zero. For the infinite state case, $P(A = L) = 0$ holds if A and L have a joint density, or if A has a density and L is a constant, among other mild technical conditions.

size of the investment is large relative to the firm's entire balance sheet, this first-order valuation approach accurately characterizes the benefit of the investment, and provides intuitively natural and simple analytical results. Appendix B shows how the second-order valuation effect (in the sense of the Taylor series) explicitly reflects additional asset-substitution benefits to shareholders associated with increasing the riskiness of the firm's assets.[13]

For an investment of q units, let $s(q)$ be the credit spread on the new debt that must be issued to finance the cost $U(q)$ of the new position. If $U(q)$ is negative, the associated cash proceeds to the firm are used to retire debt by purchasing it on the capital market.

Because we assume that the new creditors who finance the cost $U(q)$ are pari passu with all of the other unsecured senior creditors of the firm (including the unsecured counterparty of the new position), the credit spread $s(q)$ is determined by both the legacy balance sheet and the new position. A detailed calculation of $s(q)$ is provided in Appendix B.

Although $s(q)$ depends in general on the decomposition of Y into the sum $Y_1 + Y_2$ of its secured and unsecured components, we also show in Appendix B that the limiting spread $\lim_{q\downarrow 0} s(q)$ is invariant, and given by

$$S = \frac{E^*(\phi)R}{1 - E^*(\phi)},$$

reflecting the proportional default loss to creditors of

$$\phi = \frac{L - \kappa A}{L} 1_D. \tag{3}$$

In the case that L is deterministic, S is identical to the credit spread of the firm's legacy debt.

The contractual new debt payback at time 1 is $(R + s(q))U(q)$. Shareholders receive the residual $A + qY - L - U(q)(R + s(q))$, unless this amount is negative, in which case the firm defaults and shareholders get nothing. The marginal increase in the value of the firm's equity, per unit investment, is therefore

$$G = \left. \frac{\partial E^*[\delta(A + qY - L - U(q)(R + s(q)))^+]}{\partial q} \right|_{q=0}, \tag{4}$$

provided of course that this derivative is well defined, which we will show to be the case. The appendix includes a proof of the next result, and of all results to follow.

[13]The potential for a strictly positive gain to shareholders from the purchase of risky assets, even at an investment cost that is equal to or somewhat above the fair market value $\delta E^*(Y)$, is commonly known as "asset substitution," as characterized by Jensen and Meckling (1976) and Myers (1977).

PROPOSITION 1: *THE MARGINAL VALUE TO SHAREHOLDERS OF DEBT FINANCING. The marginal gain G in equity value is well defined and given by*

$$G = p^* \pi - \delta \operatorname{cov}^*(1_D, Y) - \Phi, \tag{5}$$

where

$p^* = P^*(D^c)$ *is the risk-neutral survival probability of the bank.*

$\pi = \delta E^*(Y) - u$ *is the marginal profit on the trade for a hypothetical risk-free dealer.*

$\Phi = p^* \delta u S$ *is defined to be the funding value adjustment (FVA).*

The term $\operatorname{cov}^*(1_D, Y)$ in equation (5) is the marginal asset-substitution cost to shareholders of investing in an asset whose payoff is positively correlated with the firm's default, given that shareholders give up all payoffs to creditors in the event of default. The second-order asset-substitution benefit, associated with increasing the total variance of the firm's assets, is calculated in Appendix B.C. The last term, the funding value adjustment Φ, is the present value to shareholders of their share of the net financing costs, uS. Shareholders pay these financing costs if and only if the firm survives.

In typical practice, dealers differ from our FVA formula $\Phi = p^* \delta u S$ by replacing the marginal purchase price u of the asset with the corresponding value $\delta E^*(Y)$, a practice that we later motivate with equation (11). Our formula Φ for FVA is numerically similar to that used in practice, and represents a more consistent measure of actual funding costs. There are other small variations within industry practice with respect to the exact calculation of FVAs.[14]

Proposition 1 reflects a well known principle of corporate finance known as "debt overhang," by which even an investment whose upfront cost u is strictly below the market value of an asset may sometimes be declined by a firm because the payoffs accrue excessively to creditors rather than shareholders.[15]

Appendix A provides the explicit marginal valuations to equity shareholders associated with equity financing and with cash financing. Under a non-degeneracy condition, we show a strict pecking order. For an investment requiring funding, the case of $u > 0$, cash financing (if feasible) is strictly preferred by shareholders over debt financing, which is in turn strictly preferred over equity financing. Other financing strategies could be considered. For instance, the firm could sell non-cash assets or could arrange a combination of equity, cash, and debt

[14] Some dealers ignore counterparty default risk when computing FVA. Some dealers replace their own credit spread S in the formula for FVA with an estimated average of major-dealer credit spreads.

[15] See Myers (1977).

funding. Song (2016) extends to the case of repo financing. Dealer industry metrics are rarely based on these alternative strategies, and we shall not consider them further here.

Under a linear combination of different financing methods, our technical assumptions imply that valuation is continuously differentiable in the quantity of each of the types of financing. This implies that a linear combination of financing strategies generates the corresponding linear combination of the respective marginal shareholder values.

III. How Funding Costs Affect Swap Valuation

We now apply the basic theory of the previous section to a dealer's swap transactions. Interest rate swaps, a primary example in practice and the focus of our numerical examples in Section V, make up the majority of a typical dealer's derivatives inventory, representing tens of trillions of dollars of total notional positions for each of the largest dealers.

Our main objective here is to calculate the impact of FVA on the swap prices that a dealer would quote in order for its shareholders to break even, after considering FVA. This section also provides a novel implication of debit value adjustments (DVAs) for shareholder break-even swap rate quotation.

In this section, we consider an unsecured swap transaction. Appendix C extends to the cases of *(i)* an unsecured swap transaction packaged with an inter-dealer hedge, and *(ii)* a swap secured by initial margin. The funding value adjustment associated with initial margin is known in industry practice as a "margin value adjustment" (MVA), rather than an FVA.

Appendix D generalizes the basic one-period model of this section to a two-period (three-date) model that allows for the financing of intermediate-date coupons and variation margin payments, and also allows for default at the intermediate date.

In the one-period setting considered here, a swap is a contract promising some underlying floating payment $X > 0$ in exchange for some fixed payment K. We take K as given for now, and assume that the dealer pays fixed and receives floating, for a net contractual receivable at time 1 of $X - K$, before considering the effect of counterparty default. Results for the reverse case, in which the dealer receives fixed and pays floating, are obvious by analogy.

We assume that the dealer's survival probability is not zero. In the infinite-state case, the following calculations apply if A, L, and X have finite risk-neutral expectations and a continuous joint risk-neutral density function, or if (A, X) has a continuous joint density and L is a constant.

A. *Valuing Unsecured Swaps with Upfront Payments*

In this subsection, the swap is assumed to be fully unsecured, that is, not covered by collateral. For simplicity, we suppose that there are no pre-existing positions between the swap client and the dealer. Otherwise, the results would be complicated by the effect of netting the new swap cash flows against those of the dealer's legacy positions with the same client. This more general case is analyzed in Appendix F.

We let B denote the event of the client's default, at which the dealer recovers a fraction β, possibly random, of any remaining contractual amount due to the dealer, $(X - K)^+$. In the event that $X < K$ and the dealer defaults, the unsecured swap client recovers a pro-rata share of the dealer's estate, pari passu with the dealer's unsecured creditors.

A swap position of size q requires the dealer to make an upfront payment of $U(q)$. Given our pecking order for dealer financing preferences, a positive payment is preferably funded by excess balance-sheet cash, and a negative payment is preferably used to retire equity. In practice, however, dealers' swap trading units are typically cash-constrained and are not in a position to freely retire equity. Consistent with industry practice, we therefore assume that a positive financing requirement amount is funded by issuing debt. Likewise, any net positive cash flow to the dealer is used to retire debt.

Our resulting definition of FVA is therefore "symmetric," in the sense that cash inflows and outflows are assumed to be financed or to reduce financings, respectively, at a spread of S. For the case of cash inflows, this implicitly assumes that there is always some short-term unsecured debt to roll over whose total amount can be reduced by swap cash inflows. This is a simplifying abstraction of a practical setting in which much of the surplus funds created temporarily by derivatives trading would more likely be "parked" in short-term low-risk assets. A corresponding definition of "asymmetric funding value adjustment" (AFVA) is provided by Albanese and Andersen (2014). Asymmetric funding strategies of this and other types are captured in a straightforward, albeit more complicated, way within our modeling framework by assuming that cash inflows are financed with unsecured debt and cash outflows are financed at the risk-free rate. The basic thrust of our conclusions, however, is not changed when substituting FVA with AFVA. In the simple one-period model of this section, the AFVA is merely the positive part of the FVA.

In the absence of a dealer default, the payment flowing to the dealer at time 1, per unit notional position, is

$$Y = y(K) \equiv X - K - \gamma(X - K)^+, \tag{6}$$

where $\gamma = (1 - \beta)1_B$ is the fractional counterparty default loss.

In order to calculate the market value of the swap, we must consider the potential default of the dealer. With q units of the swap traded, we can use (1) to express the effective time-1 payoff of the swap to the dealer as

$$\mathcal{C}(q) = q(X-K) - q\gamma(X-K)^{+} + (1-\kappa\rho(q))\, q\,(X-K)^{-}\, 1_{\mathcal{D}(q)},$$

where, given debt financing, the asset-to-debt payoff ratio is

$$\rho(q) = \frac{A}{L + U(q)\,(R+s(q)) + q\,(X-K)^{-}},$$

and where

$$\mathcal{D}(q) = \{A - L + qY - U(q)(R+s(q)) < 0\}$$

is the dealer's default event after considering the new position. Our basic valuation framework of the previous section implies that the fair value of the swap payoff is $\mathcal{V}(q) = \delta E^{*}\left(\mathcal{C}(q)\right)$.

The proof of the following proposition, provided in Appendix B, shows that the marginal value $v = \partial\mathcal{V}(q)/\partial q|_{q=0}$ of the swap payoff at time 1, after financing the upfront, does not depend on the financing strategy. This invariance of the marginal value to the financing method can be thought of as a consequence of the Modigliani-Miller Theorem.[16] Nevertheless, the value $\mathcal{V}(q)$ of a non-trivial position of size $q > 0$ in general depends non-trivially on the financing method, because the incremental distress costs depend on the financing method.

PROPOSITION 2: *FAIR MARKET VALUE OF AN UNSECURED SWAP. Consider a swap position with cash flow Y defined by* (6)*. Whether the dealer finances the swap by issuing debt, issuing equity, or using existing cash on its balance sheet, the marginal value of the swap is well defined and given by*

$$v = \delta E^{*}(X-K) - \text{CVA} + \text{DVA}, \tag{7}$$

where $\text{CVA} = \delta E^{*}\left(\gamma(X-K)^{+}\right)$ *is known as the credit value adjustment and* $\text{DVA} = \delta E^{*}\left(\phi\,(X-K)^{-}\right)$ *is known as the debit value adjustment.*

The CVA and DVA adjustments have been characterized in the literature, and are now accepted in practice.[17]

If there are no default distress costs ($\kappa = 1$), we may view v as the choice of upfront

[16] See Modigliani and Miller (1958).

[17] For DVA and CVA analysis, see, for example, Sorensen and Bollier (1994), Duffie and Huang (1996), and Gregory (2015). Spears (2017) discusses the history of DVA and CVA adjustments.

payment u that would make a total claimant on the dealer's balance sheet (debt plus equity) indifferent to entering the swap transaction. Whenever trading decisions are made, however, we assume that the dealer's preferences are determined by shareholder value maximization. We therefore focus on the upfront payment v^* for the swap that would leave shareholders indifferent to the swap transaction.

PROPOSITION 3: *SHAREHOLDER BREAKEVEN VALUE OF UNSECURED SWAPS. Consider a swap position with cash flow Y defined by* (6)*. Under debt financing, the upfront payment for the swap that is breakeven for dealer shareholders, in the sense that $G = 0$, is*

$$v^* = \frac{E^*(Y)}{R+S} - \frac{\text{cov}^*(1_D, Y)}{p^*(R+S)}. \tag{8}$$

If the dealer's default indicator 1_D and the swap cash flow Y are uncorrelated under P^*, then

$$v^* = (v - \text{DVA})\,\frac{R}{R+S}. \tag{9}$$

In the simple case covered by (9), the shareholder breakeven upfront price v^* for entering the swap is an adjustment of the fair market value v that:

(i) Removes the DVA from v.

(ii) Substitutes the dealer's unsecured discount rate $R + S$ for the risk-free rate R.

The first of these adjustments does not depend on the funding strategy and reflects the lack of any shareholder benefit from paying the swap counterparty less than the contractually promised amount when the dealer defaults (because the equity holder receives nothing at default). The second adjustment is for the funding cost to shareholders, who must pay the credit spread S to the new creditors without gaining any marginal benefit from the right to default on the new debt.

If the upfront payment u is negative, then dealer shareholders benefit from a negative FVA, which is known in industry practice as a "funding benefit adjustment" (FBA).

When ignoring distress costs (by taking $\kappa = 1$), the difference between the shareholder break-even value v^* and the total value v to all dealer claimants (debt plus equity) amounts to a wealth transfer by the dealer's equity shareholders to the dealer's creditors. This wealth transfer is triggered both by the swap cash flow itself (through the DVA) and also by the financing strategy used by the dealer to fund the upfront. If the dealer has distress costs at default then the net shareholder cost $v^* - v$ of entering the swap is not entirely transferred to other stakeholders. For the general case, the net gain to the dealer's legacy creditors is calculated in Appendix B.

B. Dealer Quotation and FVA for Unsecured Swaps

Assuming that the dealer maximizes shareholder value, it would rationally not trade the swap unless the upfront payment to the dealer is at least v^*. If the dealer manages to execute the trade at this level, the firm as a whole would make a trading profit of $v - v^*$. This profit can have either sign. Although the DVA effect always lowers[18] v^* relative to v, the funding-cost component can either increase v^* relative to v (which occurs if $v < \text{DVA}$), or decrease it (whenever $v > \text{DVA}$). Loosely speaking, the funding component increases shareholder value for swaps that are predominantly liabilities (have a high fixed rate K relative to $E^*(X)$) and decreases shareholder value for swaps that are predominantly assets (have a low K relative to $E^*(X)$).

Before the introduction of FVAs, bank quotation practices adjusted appropriately for the DVA effect, but did not correctly account for the funding-cost effect. That is, before the introduction of FVA, rather than quoting v^* as suggested by the shareholder breakeven upfront payment (8), banks quoted

$$v - \text{DVA} = \delta E^*(X - K) - \text{CVA}, \tag{10}$$

which is the fair-market value of a default-free swap less the CVA, but removing the DVA adjustment that is now an accepted element of fair value accounting for swaps reflected in (7).

If the swap is executed at this conventional level $v - \text{DVA}$, then (5) implies that shareholders experience a marginal loss in value of $\delta \operatorname{cov}(1_D, Y) + \text{FVA}$, where, under these pricing terms, we have

$$\text{FVA} = p^* \delta(\delta E^*(Y))S. \tag{11}$$

As we mentioned earlier, dealers now incorporate into their quotes this variation (11) of the formula Φ for FVA given in Proposition 1. Although we show that an FVA is actually a transfer of wealth away from dealer's shareholders due to the adverse impact of funding costs, this conceptual basis for FVA is not commonly recognized within the dealer community. Whether viewed correctly as a equity value transfer or incorrectly as a reduction in the market value of the swaps, one would expect dealers to incorporate FVAs into their quotes.

In order to make this point more transparent, we Taylor-expand the expression (9) for the shareholder valuation v^* of the swap position, for a small credit spread S and for a survival

[18]This is true unless the swap is a pure asset with no DVA at all, that is, unless K is so low that $X - K > 0$.

probability p^* close to 1. We see that

$$v^* = (v - \text{DVA})\,\frac{1}{1+S/R} \approx (v - \text{DVA})\left(1 - \frac{S}{R}\right) \approx v - \text{DVA} - \text{FVA}, \qquad (12)$$

taking FVA as defined by (11). Thus, the current practice by dealers of making a downward FVA adjustment to their mark-to-market swap valuations, although not consistent from a valuation viewpoint, causes a valuation bias that leads to quotations that align the interests of the dealer's traders with those of the dealer's shareholders.

In order to trade with a dealer that quotes swaps in a manner reflecting these shareholder incentives, the client swap counterparties must be willing to "donate" the sum of the DVA and the FVA. In practice, this "donation" would be implemented through an effective widening of the dealer's bid-ask spread, manifested either in the upfront u or in the swap rate K, or both. Section V provides a numerical example illustrating the magnitudes of compensating bid-ask spreads. We argue that these magnitudes are economically significant.

It follows from our results that the most creditworthy dealers, those with the lowest credit spread S and therefore the lowest FVAs and DVAs, usually have a head start over less well capitalized dealers in finding swap clients willing to enter trades at terms that are beneficial to the dealer's shareholders. Even the best capitalized dealer, however, must attract clients that are sufficiently anxious to trade (given their own hedging or speculative motives) that they are willing to give up some value to the dealer. This concession can be buried into the bid-ask spread quoted by the dealer.

Appendix C extends the results of this section to treat hedged swaps and swaps that are secured with variation margin and, potentially, initial margin. In industry terminology, the additional funding value adjustment associated with the financing of initial margin is called a "margin value adjustment" (MVA) rather than a "funding value adjustment."

A dealer sometimes finds itself in a position to enter a swap that *lowers* its aggregate margin requirement, because the new swap hedges or offsets a legacy position with the same counterparty. In this case, the margin that is released by the trade is a source of profit to the dealer's shareholders in the form of a reduction in FVA, as shown in Appendix F. This funding benefit adjustment (FBA) gives the dealer an advantage over other dealers (even some dealers with lower credit spreads) in "winning" the trade. We do not model the associated strategic implications.

IV. FVA and Arbitrage of Covered Interest Parity Violations

Although FVAs are most prominently associated with swaps, the same trading friction can play a significant role in the attractiveness of other potential dealer trades that call for significant unsecured debt financing. In this section, we consider the opportunity for trades that exploit significant recent violations of covered interest parity.

Du, Tepper, and Verdelhan (2018) and Rime, Schrimpf, and Syrstad (2017) have shown that the interest rates at which some big banks borrow US dollars outright in wholesale funding markets have been significantly below the rates for synthetic US dollar borrowing that could be obtained via foreign exchange (FX) markets. The synthetic method is to borrow a foreign currency, euros for example, and to exchange the euros for dollars (at spot, and back again at maturity) using FX forwards or cross-currency swaps. If the credit qualities of the two dollar positions, direct and synthetic, are the same, then the associated interest rates "should" be the same absent trade frictions, a point first noted by Keynes (1923) and now known as covered interest parity (CIP). Any difference in these two rates, actual minus synthetic, is called the CIP basis.

Between 2010 and 2016, on average over major currencies, Du et al. (2018) estimate a CIP basis of about minus 24 basis points at 3 months and about minus 27 basis points at 5 years. In some currencies, especially the Yen, they show that the basis has been much wider. Rime et al. (2017) show that, once accounting for actual available transactions prices, profitable arbitrage of the CIP basis is possible for only a subset of highly capitalized banks. Neither of these studies, however, consider whether CIP arbitrage is beneficial to bank shareholders, that is, after considering the adverse impact of FVAs, among other potential frictions.

Suppose, for a simple numerical example, that a bank has a one-year risk-neutral default probability of 70 basis points and that its creditors suffer a fractional loss given default of 50%. The bank's one-year credit spread is thus $S = 35$ basis points. For illustrative simplicity, the risk-free US dollar interest rate is assumed to be zero, so that $R = \delta = 1$.

The bank will fund a CIP basis trade by borrowing \$100 in the one-year USD commercial paper (CP), thus promising a repayment of \$100.35. The bank invests the \$100 proceeds in one-year Euro CP, swapped to USD with an FX forward, such that the synthetic US dollar asset has same all-in credit quality (same risk-neutral default probability and same fractional loss given default) as that of the bank's own CP. For simplicity, the default risk of this asset is assumed to be uncorrelated with the bank's default event.

The synthetic asset, however, has an all-in US-dollar interest rate of 60 basis points. That is, absent default, the asset payoff is \$100.60, implying a CIP basis of -25 basis points. The bank has a new liability with a market value of \$100 and a new asset with a market value of approximately \$100.25, for a mark-to-market trade profit of approximately \$0.25.

However, the marginal impact of the trade on the market value of the bank's equity is negative, because the \$0.25 profit is more than offset by the FVA cost to equity of $\delta p^* uS \simeq 0.35$, for a net loss of about \$0.10. In order for a trade like this to benefit shareholders, the CIP basis would need to exceed the proportional FVA of approximately 35 basis points.[19]

Most or all of the effective CIP violations documented by Rime et al. (2017) are below the associated proportional FVAs of global banks, based on current credit spreads.

As noted by Du et al. (2018), CIP violations were extremely small before the financial crisis of 2007-2009. Consistent with this, major dealer-bank credit spreads (thus FVAs) were also extremely small before the financial crisis.

Regulatory capital requirements pose an additional friction on CIP arbitrage that can be analyzed within our modeling framework. Under the leverage-ratio rule, a bank may be required to finance a fraction α of an investment with new equity, and only $1-\alpha$ with debt. In that case, based on the marginal value to shareholders of equity financing that is computed in Equation (24) of Appendix A, the marginal funding cost of an asset purchase to bank shareholders, above that for all-debt financing, is

$$\alpha u[1 - p^*(1 + \delta S)]. \tag{13}$$

For the largest U.S. bank dealers, the supplementary leverage ratio rule implies that $\alpha = 6\%$. From (13), the additional cost to the shareholders for the CIP basis trade described in the above example is 2.1 basis points, for a total proportional funding cost to shareholders of approximately $35 + 2 = 37$ basis points.

In practice, a bank would not obtain equity funding on a trade-by-trade basis. The bank would instead arrange in advance for enough excess regulatory equity capital to accommodate its likely potential trades. We do not model the more complicated role of anticipatory funding.

[19]The value of this trade to dealer shareholders can also be computed directly in this simple example. The risk-neutral expected payoff B of the bank's euros-swapped-to-dollars CP asset is the contractual payoff of \$100.60 multiplied by the sum of (a) the risk-neutral probability 0.993 of the survival and (b) the risk-neutral probability 0.007 of default multiplied by the fraction 0.5 lost given default. The net profit to the bank's shareholders is the product of the risk-neutral survival probability of the bank and the expected trade net profit allocated to shareholders, after financing costs, conditional on the event of survival. This shareholder profit is $0.993 \times \big(B - \$100.35\big) \simeq -\$0.10$.

V. Valuation Adjustments for Long-Term Swaps

We now illustrate the numerical implications of our model for valuation adjustments of swaps. After setting up a general reduced-form swap valuation framework that parallels the structural model of the previous sections and Appendix D, we provide a numerical illustration of the magnitude of valuation adjustments for plain-vanilla interest rate swaps, showing that funding valuation adjustments are economically important in practice, and also indicating relative responses to the term structure of interest rates and to the fixed coupon rate of swaps.

For this purpose, we begin with a typical continuous-time setting that allows us to appeal to standard reduced-form models of the term structure of interest rates, default timing, and default recovery. Our reduced-form model is otherwise conceptually faithful to the solution for funding value adjustments in our structural model. In order to capture the effects of interim coupon and variation margin payments in a manner consistent with the spirit of the structural model, Appendix D generalizes the basic one-period model of Section III to a two-period model that allows for the financing of coupon and intermediate-date margin payments, and also allows for default at the intermediate date.

A. *Reduced-Form Valuation Framework*

Our continuous-time framework is based on standard technical assumptions given in Appendix E. The model begins with a default-risk-free short-rate process $r = \{r_t : t \geq 0\}$, implying that the risk-free discount at time t for risk-free cash flows at time T is $E_t^*(\delta_{t,T})$, where $\delta_{t,T} = e^{-\int_t^T r(s)\,ds}$.

Before considering the effect of incremental cash flows associated with a new position, the derivatives dealer defaults at a stopping time τ_D whose conditional mean arrival rate at time t is $\lambda_D(t)$. The fractional loss to the creditor claim associated with default at time t is $\ell_D(t)$. That is, an unsecured claim of size C on the dealer's estate at default is paid $(1 - \ell_D(\tau_D))C$, for some proportional loss process ℓ_D taking outcomes in $[0, 1]$. (This incorporates the impact of fractional default recovery, captured in the one-period model by the parameter κ.) This dealer's short-term credit spread at time t is $S_t = \lambda_D(t)\ell_D(t)$. That is, each unit of the dealer's short-term unsecured debt can be continually renewed, or "rolled over," by making continual floating-rate interest payments at the adjusting rate $r_t + S_t$, as justified in Appendix E.

Similarly, a given client swap counterparty has default risk characterized by a default time τ_C whose conditional mean arrival rate at time t is $\lambda_C(t)$, and by a proportional loss given default at time t of $\ell_C(t)$.

We will characterize various valuation adjustments for an unsecured swap between the dealer and the client. Johannes and Sundaresan (2007) and Piterbarg (2010) have modeled the important valuation distinction between unsecured and collateralized swaps. Our objective here, instead, is to calculate the swap FVA, MVA,[20] CVA, and DVA.

The swap that we will evaluate has maturity date T and contractually promises the dealer, before considering the effect of counterparty default, net payments $C_1, \ldots, C_N$ at some respective increasing sequence $\{t_1, \ldots, t_N = T\}$ of coupon exchange times. For notational simplicity, for any time $t \leq T$, we let $\eta(t) \in \{0, 1, ..., N\}$ denote the index of the associated coupon period. That is, $t \in \left(t_{\eta(t)-1}, t_{\eta(t)}\right]$, taking $t_{-1} = -\infty$.

The market value at time $t < T$ of a default-free version of the swap is, by definition,

$$V_t = E_t^* \left(\sum_{j=\eta(t)+1}^{N} \delta_{t,t_j} C_j \right).$$

By direct analogy with the structural multi-period model of Appendix D, the CVA and DVA are, respectively,

$$\text{CVA} = E^* \left(1_{\{T > \tau_C,\, \tau_D > \tau_C\}} \delta_{0,\tau_C} \ell_C V(\tau_C)^+ \right) = \int_0^T E^* \left(\delta_{0,t} \xi_t \lambda_C(t) \ell_C V_t^+ \right) \, dt, \tag{14}$$

where $\xi_t = e^{\int_0^t -[\lambda_D(s) + \lambda_C(s)] \, ds}$, and

$$\text{DVA} = E^* \left(1_{\{T > \tau_D,\, \tau_C > \tau_D\}} \delta_{0,\tau_D} \ell_D V(\tau_D)^- \right) = \int_0^T E^* \left(\delta_{0,t} \xi_t \lambda_D(t) \ell_D V_t^- \right) \, dt. \tag{15}$$

Again by analogy with the marginal valuation of the swap that we provided for our discrete-time structural model, the market value of the swap is

$$v \equiv V_0 - \text{CVA} + \text{DVA}. \tag{16}$$

To repeat, this is the total value of the swap cash flows to both equity and debt claimants. By implication of the structural model, there is no funding value adjustment assignable to this (total) swap market value.

In order to compute funding value adjustments to shareholder value, we suppose that the dealer can enter small notional positions of the swap at a per unit upfront payment of u. Just as for our structural model, we do not require that this upfront payment u is the equal to the initial value v of the swap. Because this is merely a reduced-form model as opposed

[20] As we discuss in Appendix C.C, MVA applies if the dealer hedges the unsecured client swap with an inter-dealer swap that requires the dealer to post initial margin.

to a structural model, there is no point in making a distinction here between the marginal value and the per-unit value of a position of a given non-zero size.

As for computing the FVA, we suppose that the dealer issues short-term unsecured debt to finance any pre-default swap-related payments, including the upfront payment and any interim coupon payments. Any swap-related receivables to the dealer are likewise used to retire outstanding short-term unsecured debt. The FVA of swap position can in this case be defined by direct analogy with that of the multi-period model of Appendix D by

$$\Phi(u) = E^* \left(u \int_0^{\tau} S_t \, dt - \sum_{i=0}^{\eta(\tau)-1} \delta_{0,t_i} \, C_i \int_{t_i}^{\tau} S_t \, dt \right),$$

where $\tau = \min(\tau_C, \tau_D, T)$. Given our default-time assumptions, this reduces to

$$\Phi(u) = E^* \left(u \int_0^{T} \xi_t \, S_t \, dt - \sum_{i=0}^{N-1} \delta_{0,t_i} C_i \int_{t_i}^{T} \xi_t S_t \, dt \right). \tag{17}$$

For the special case in which the unsecured swap is executed at an upfront payment equal to the default-free market value V_0, direct algebraic calculations, as in Appendix D, yield the FVA

$$\Phi(V_0) = E^* \left(\int_0^{T} \xi_t V_t \delta_t S_t \, dt \right). \tag{18}$$

If the dealer hedges the unsecured swap with a fully collateralized inter-dealer swap that requires the dealer to post variation margin and intial margin, as now required under Dodd-Frank and MiFID regulations, there is also a margin value adjustment (MVA), which can be computed by analogy with the multi-period structural model of Appendix D as

$$\Psi = E^* \left(\int_0^{T} \xi_t I_t \delta_t S_t \, dt \right), \tag{19}$$

where I_t is the initial margin at time t. By analogy with (12), the upfront payment v^* that would leave shareholders indifferent to the swap transactions is approximated as

$$v^* \approx V_0 - \text{CVA} - \Phi(V_0) - \Psi = v - \text{DVA} - \Phi(V_0) - \Psi. \tag{20}$$

B. Illustrative Numerical Example of XVAs

We now give illustrative magnitudes of FVAs, MVAs, DVAs, and CVAs based on a simple parametric term-structure model. While the term-structure models used by major dealers are generally more sophisticated than our illustrative model, we believe that the magnitudes of these "XVAs" that we calculate give realistic indications of their relative economic importance in practice, and help us understand how they vary with swap rates, credit risk, and the slope of the term structure.

For this purpose, we consider an unsecured 10-year, semi-annual-coupon, plain-vanilla interest rate swap with a notional size of $100 million. The underlying floating rate is six-month LIBOR. For our example, this floating rate is the simple six-month (money-market) interest rate associated with a hypothetical borrower whose six-month credit spread over the risk-free six-month simple interest rate is taken to be some constant ϵ. At base case, we take ϵ to be 30 basis points. For the initial term structure of risk-free interest rates, we calibrate to the risk-free discount term structure given by

$$p(0,t) = E^*(\delta_{0,t}) = e^{-(0.005+0.001t)t}, \tag{21}$$

roughly corresponding to market conditions in January 2016. That is, the continuously compounding yield curve starts at 50 basis points and slopes upward at a rate of 10 basis points per year.

Until default, the net coupon C_i paid to the dealer at the i-th coupon date t_i is the current six-month LIBOR floating rate less some fixed coupon rate K. (We will consider various fixed coupon rates.) In addition to this payer swap, we will also provide results for the corresponding receiver swap, by which the dealer receives the fixed rate K net of the six-month LIBOR floating rate. A default-free swap whose market value V_0 is zero corresponds to a fixed coupon rate of $K = 1.783\%$.

The risk-free short-rate process r, which we treat as the short rate underlying the overnight index swap (OIS) swap term structure, is determined by a one-factor Hull-White term-structure model[21] calibrated consistently with (21). That is, the short-rate process r satisfies $r_t = -d\log(p(0,t))/dt + z_t$, where $p(0,t)$ is given by (21) and z_t satisfies the stochastic differential equation

$$dz_t = (\theta_t - \alpha z_t)\,dt + \sigma\,dW_t, \quad z_0 = 0, \tag{22}$$

where α and σ are constants, W is a standard Brownian motion under the valuation measure P^*, and

$$\theta_t = \int_0^t \sigma^2 e^{2\alpha(u-t)}\,du. \tag{23}$$

We set $\alpha = 0.05\%$ and $\sigma = 0.70\%$, which approximate the implied volatility levels of long-dated Bermudan LIBOR swaptions as of January 2016.

[21] See Hull and White (1993).

Table II This table shows XVAs, in thousands of dollars, for a 10-year plain-vanilla interest-rate swap of notional size \$100 million. Each cell of the table uses the format $x_P \,|\, x_R$ to show the XVA x_P of the dealer's payer version of the swap on the left and the corresponding XVA x_R of the dealer's receiver version of the same swap on the right. Shown in parentheses are the running-spread equivalents of the associated XVAs, in basis points, meaning the adjustments to the fixed swap rate K that substitute that compensate for eliminating the upfront payments. The columns of the table correspond to the fixed coupon rate of the swap. The rows correspond, respectively, to the funding value adjustment (FVA) given by $\Phi(V_0)$ of (18), the margin value adjustment (MVA) given by Ψ of (19), the credit value adjustment (CVA) given by (14), and the debit value adjustment (DVA) given by (15).

	$K = 1.0\%$	$K = 1.783\%$	$K = 2.5\%$
FVA	428 | − 428	116 | − 116	−171 | 171
	(4.6 | − 4.6)	(1.2 | − 1.2)	(−1.8 | 1.8)
MVA	116 | 116	116 | 116	116 | 116
	(1.2 | 1.2)	(1.2 | 1.2)	(1.2 | 1.2)
CVA	942 | 85	479 | 247	236 | 577
	(10.0 | 0.9)	(5.1 | 2.6)	(2.5 | 6.1)
DVA	42 | 471	124 | 240	289 | 118
	(0.5 | 5.0)	(1.3 | 2.5)	(3.1 | 1.3)

The dealer has a constant default intensity of λ_D of 2% and a constant fractional loss given default ℓ_D of 50%. We assume that the swap counterparty has a constant default intensity λ_C of 4% and a constant fractional loss given default ℓ_C of 50%. This implies a constant short credit spread S for the dealer of 1%, and for the counterparty of 2%.

We assume that the dealer hedges the unsecured swap with a fully collateralized inter-dealer swap that requires initial margin. When calculating the MVA, the initial margin I_t is set at the level required by BCBS/IOSCO (BCBS (2013)), that is, at the 99th percentile of the 2-week change in market value V_t of the default-free version of the swap, excluding any jumps associated with coupon payments. A detailed analysis of the computation of the CVA, DVA, FVA, and MVA, based on the formulae provided in the previous section, is found in Appendix E.

Table II shows these "XVAs." For the FVA, we report $\Phi(V_0)$, meaning the FVA associated with an upfront equal to the default-free market value V_0.

C. The Magnitudes of the XVAs and Their Impacts on Dealer Quotation

To interpret the results in Table II, we focus at first on the payer swap at a fixed coupon rate of 1.783%, and consider how the shareholder value v^* of (20) differs from the market value v of (16).

The market value of the swap is obtained by subtracting the CVA net of the DVA from the value V_0 of a default-risk-free swap (which in this example is zero), for a total reduction of $\$479,000 - \$124,000 = \$355,000$, which is economically equivalent to $5.1 - 1.3 = 3.8$ basis points in running-coupon terms. Relative to this market value, the funding value adjustment $\Phi(V_0)$ for the swap represents a cost to shareholders of approximately \$116,000, which is economically equivalent in terms of its cost to shareholders to an increase of approximately 1.2 basis points in the fixed coupon rate paid by the dealer. The margin value adjustment Ψ (assuming that the dealer is actually subject to initial margin) represents an additional $\$116,000$ cost to shareholders. (The approximate numerical equivalence of FVA and MVA in this example is merely coincidental.) Finally, the DVA benefit of approximately $\$124,000$ is of no value to shareholders, so the impact of the swap trade on the value of the dealer's equity is less than the market value v of the swap by approximately $\$116,000 + \$116,000 + \$124,000 = \$356,000$ (which is economically equivalent to an impact on shareholders of $1.2 + 1.2 + 1.3 = 3.7$ basis points running).

From a quotation perspective, the "par" coupon rate K, that making the swap have a zero market value, is approximately $178.3 - 3.8 = 174.5$ basis points. However, as we just noted, entering the swap at these "fair-market" terms represents a swap-rate disadvantage to the dealer's shareholders of 3.7 basis points. That is, the dealer's swap desk, if acting on behalf of shareholders, should be willing to enter the swap only if the fixed rate paid by the dealer is no greater than 170.8 basis points.

As for the receiver version of this swap, the dealer's shareholders benefit only if they receive an upfront that is increased above the initia market value of the swap by the sum of the FVA, MVA, and DVA, which is \$240,000, or a running-spread equivalent of 2.5 basis points of notional. (In this case, the FVA is negative, but this funding benefit to shareholders is more than offset by the total of the MVA and DVA.) Equivalently, the shareholder breakeven receiver swap rate is 2.5 basis points above the fair-market rate of $178.3 + 2.6 - 2.5 \simeq 178.4$ basis points. That is, the swap desk should not enter as a receiver at a zero upfront payment unless it can receive a swap rate of at least $178.4 + 2.5 = 180.9$ basis points. If quoting both sides of the swap so as to ensure that shareholders break even, this represents a bid-ask spread of approximately $180.9 - 170.8 = 11.1$ basis points, an enormous widening of the spread relative to current unsecured dealer-to-client bid-ask spreads of under 0.2 basis points.

This example, however, is extreme relative to typical XVA impacts on dealer shareholders and on bid-ask spreads. Until recently, dealers have not been providing initial margin. Removing the MVA impacts would reduce the bid-ask spread by 2.4 basis points, leaving a bid-ask spread widening effect of 8.7 basis points, corresponding to the impact on shareholders of FVA and DVA.

Furthermore, the MVA impacts of new swaps are frequently *beneficial* to the dealer's shareholders, through netting effects relative to legacy swap positions. This netting benefit is shown in a structural version of our model found in Appendix F. In practice, according to OCC (2015), on average across the largest U.S. swaps dealers as of the end of the third quarter of 2015, netting reduced the gross positive market value of swaps by 87%. There is no available breakdown, however, of the impact of netting on dealer-to-client swaps versus inter-dealer swaps, and no breakdown of the effects of netting cash flows across counterparties and netting within counterparty positions.

If, for example, the dealer has 25% more payers than receivers, implying a reduction from gross to net notional positions of 8/9, then the average MVA effect on the total book of all swaps is a loss to shareholders of only about 1/9 of the impact of a stand-alone payer, per unit of total gross notional. In our example, the MVA effects for standalone payers and receivers are the same at all of the coupon rates that we considered. This 1.2 basis point spread compensation to dealer shareholders is then reduced by netting to about 0.13 basis points running of the gross notional, or, equivalently, a market value impact on shareholders of about $12,900 per $100 million notional.

For the case of credit default swaps, the degree to which initial dealer margin to CCPs and other dealers is reduced by netting is examined empirically by Duffie, Scheicher, and Vuillemey (2015).

As opposed to the case of FVA, the impact of netting on DVA does not net across counterparties. However, DVA impacts do net across offsetting positions with the *same* counterparty. For example, if the dealer has a DVA that is reduced through counterparty-level netting by an average factor of two, then the adverse DVA effect (relative to market value) is also reduced by a factor of two. In our example of interest-rate swaps entered at a fixed rate of 178.3 basis points, the DVA effects per $100 million notional, of $124,000 for payers and $240,000 for receivers, would then each be cut in half.

These illustrative netting effects would imply an average net bid-ask running spread effect of FVA and DVA of $2 \times 0.13 = 0.26$ basis points and $(1.3 + 2.5)/2 = 1.9$ basis points respectively, for a total average widening of the bid-ask spread necessary to compensate shareholders of about 2.2 basis points. Again, these numerical effects of netting are purely illustrative. Nevertheless, they portray the importance of netting on impacts of FVA and

DVA, and they illustrate the still large residual adverse impacts on shareholders, relative to typical inter-dealer bid-ask spreads. As we have emphasized, if the dealer aligns the incentives of its swap trading desk appropriately, shareholder costs are passed through to clients in the form of wider bid-ask spreads.

Moving back to the base case of payer swaps, as shown in Table II, FVA decreases as the fixed coupon rate K is increased. At a coupon rate K of 1.0%, this FVA impact is nearly four times bigger than for a coupon rate of 1.783%. That is, the higher is the fixed rate, the lower is the value to the dealer, resulting in lower upfront financing costs to shareholders. For a sufficiently high coupon rate, the FVA becomes negative, corresponding to a net funding *benefit* to the dealer. Even though the swap has almost no upfront at a fixed rate of $K = 1.783\%$, its has a positive FVA because of the upward-sloping yield curve. That is, the swap is projected to increase in market value over time, as the net coupons flowing to the dealer are expected (under the valuation measure P^*) to increase over time. In our setting based on Gaussian interest rates, the MVA is invariant to the fixed coupon rate K.

As of January 2016, the bid-offer spread on a 10-year par-coupon plain-vanilla LIBOR swap has been around 0.1 bps to 0.2 bps, or about \$10,000 to \$20,000 in dollar terms. As one can see, the impacts of FVA, DVA, and MVA on equity breakeven swap rates are much larger than these typical bid-offer spreads. The fact that dealers now pay close attention to "XVA optimization," as reported by Sherif (2016a), is therefore not surprising.

VI. Concluding Discussion

We now conclude by briefly recapitulating our main results and then discussing additional implications and new research directions.

A. Summary of Main Results

Based on a neoclassical structural model of the balance sheet of a dealer, we show that the quantity known in practice as the "funding value adjustment" is essentially the cost to the dealer's shareholders for financing up-front counterparty cash payments, variation margin payments, and collateral requirements. This cost to shareholders (which can be negative for swaps that generate positive cash flows to the dealer) is at least partially offset by a change in the value of dealer creditor claims. The total of these value effects on shareholders and creditors is a change in the value of the dealer's frictional financial distress costs.

Our modeling approach is to *(i)* provide a marginal valuation theory for debt and equity benefits associated with financing new investments, *(ii)* derive a pecking order for shareholder financing preferences, *(iii)* apply our framework to the impact on equity and debt values of the unsecured debt financing of swap and CIP basis trades *(iv)* analyze the impact of shareholder preferences on dealer quotations, *(v)* explain the impact of funding value adjustments on the incentive to arbitrage violations of covered interest parity, *(vi)* extend by analogy our simple discrete-time structural model to a reduced-form continuous-time term-structure setting, and *(vii)* for a parametric example of the continuous-time model calibrated to recent interest-rate derivatives, obtain illustrative magnitudes of XVAs and their running-spread equivalents in various examples of interest-rate swaps.

We show that the FVA and its close cousin the margin value adjustment (MVA) can be viewed as debt-overhang costs to shareholders that can easily discourage dealers from offering intermediation, even on terms that may add a positive market value to the dealer's balance sheet. On average across its book of intermediated positions, a dealer's shareholders must be compensated for FVAs, MVAs, and debit value adjustments (DVAs) by counterparty "donations" in the form of pricing terms that imply trading losses to clients. In particular, for a dealer's shareholders to avoid a loss when their firm enters a new position, the pricing terms must imply a gain in the market value of the dealer's positions that is at least as large as the sum of the incremental FVA, MVA, and DVA. In some cases, however, this sum can be negative, implying a gain to shareholders above and beyond the profit-and-loss (P&L) on the trade.

For example, consider the stand-alone $100 million notional interest-rate payer swap of our illustrative numerical example, at a fixed coupon rate of 1.78%. For a term structure of interest rates and swap-rate volatility like those for the US dollar swap market in January 2016, entering this unsecured swap is beneficial to the dealer's shareholders only if the swap terms imply a trading gain to the dealer's balance sheet of $356,000, in roughly equal parts for FVA, MVA (if initial margin is applicable), and DVA. In practice, as we have discussed in the previous section and modeled formally in Appendix F, netting the swap cash flows against those of legacy swaps would typically reduce this required threshold gain significantly, and in proportion to the degree of netting, case by case.

Although the implications of DVA for swap market values are widely treated in the research literature and in practice, as far as we are aware they are for the first time shown in this paper to have a significant incremental impact on shareholder breakeven valuation and breakeven swap quotation.

Others[22] have already noted (although without a supporting structural model) that treat-

[22] We have already cited Hull and White (2012), Cameron (2013), Cameron (2014a), Becker and Sherif (2015), Hull and White (2014), Albanese and Andersen (2014), and Albanese, Andersen, and Iabichino (2015).

ing FVA as an adjustment to the market value of a dealer's swaps causes various logical contradictions. A common informal argument has been that adjusting the market value of a swap for funding costs is a violation of the Modigliani-Miller (MM) Theorem. We confirm that this is the case in the absence of frictional distress costs and provided that valuation impacts are measured in a marginal sense. (Otherwise, MM theory does not precisely apply.[23]) Another inconsistency, emphasized by Burgard and Kjaer (2011), is that an FVA adjustment to swap values violates the simple symmetry condition by which (in the absence of frictional default distress costs) the value to a dealer for entering a swap must be equal and opposite to the value of the swap to the counterparty. These same inconsistencies apply to margin value adjustments (MVAs). In particular, unless there are frictional financial distress costs, it would be impossible for two dealers entering a swap with each other to *both* suffer a loss in the market value of their swap books for the associated margin financing costs, given that the total of the cash flows on the new swap to the two dealers is clearly zero. Further, Hull and White (2012) and Burgard and Kjaer (2011) point out that funding cost adjustments to swap values can imply windfall profits to counterparties or creditors.

B. Market-Making Incentives and Other Strategic Implications of FVAs

Although the common practice of FVA and MVA adjustments to swap market values is inappropriate, it may have arisen from the understandable incentive of large bank holding companies to discourage their swap desks from entering positions that require significant cash financing, given that these are (as we show) a drag on shareholder returns. These funding costs became obvious only after the financial crisis caused significant increases in dealer credit spreads. If dealer valuation practices eventually change so as to reflect the true nature of FVA, some other form of incentives for traders should presumably be substituted. For example, the variable component of swap traders' compensation could be based on their trading P&L, *less* an estimate of the incremental impact of their trading on the firm's FVA, MVA, and DVA.

As we have noted, the clients of dealers must, on average, pay extra, above and beyond the market values of their swap positions, in order to give dealers sufficient incentives to enter swaps with them. Swap clients are often willing to do so because they have motives to enter swaps, such as hedging, that dominate these XVA-related trading losses. To the extent that these XVA "donation effects" are positive, which is the case on average, there is a significant business advantage to relatively highly capitalized dealers. The losses that

[23]The MM principle is that in the absence of distress costs the dealer's total balance-sheet cash flows and thus total market value are invariant to its capital structure. This is not enough on its own to treat the valuation effects of swap financing, given that adding a swap changes the dealer's total cash flows.

clients must incur in order to compensate dealer shareholders for FVA, MVA, and DVA are all roughly proportional to the dealer's credit spread. (Appendix E provides numerical support for the near linearity of these XVAs over a wide range of dealer credit spreads.) For example, if Bank A has a credit spread that is half of that of Bank B, then the shareholders of Bank A can break even with a widening of bid-ask spreads for FVA, MVA, and DVA that is only about half of the corresponding widening of bid-ask spreads that Bank B must quote to its customers. For the average case in which FVA, MVA, and DVA sum to a negative impact on dealer shareholders, this would obviously cause buy-side firms to prefer to trade with Bank A over Bank B, other things equal.[24] Our illustrative numerical examples showed this advantage to Bank A to be quite significant in economic terms. This XVA advantage to Bank A in attracting more clients is further magnified by the increased degree of netting that would be expected with a larger number of swap positions, thus further reducing the XVA-related component of bid-ask spreads quoted by Bank A, with a positive feedback effect. For special cases in which there is a significant funding *benefit* associated with an incremental position, the dealer with the *higher* credit spread would be expected to benefit most from the position, and to bid more aggressively for the trade. This explains recent aggressive bidding by dealers for cross-currency swaps, because of their typically high funding benefits to dealers, as explained by Wood (2016).

The effect of legacy swap positions for the matching of a buyside firm to a dealer on a new swap trade, however, can swamp any credit spread advantage of one dealer over another. The dealer whose netting (and credit spread) result in the lowest *incremental* sum of FVA, MVA, and DVA is the dealer that is most efficiently positioned to get the trade. Search costs and OTC market opaqueness, however, can prevent this most advantaged dealer from actually winning the trade. Even when there are no legacy swap positions with a given client, the dealer may quote for the effect of XVA costs to shareholders on the basis of expected future netting effects with that client.

The accounting disclosures of dealers such as J.P. Morgan (2014) state that FVA adjustments originate primarily from unsecured derivatives positions with non-financial corporate clients. Dealer-to-dealer transactions normally have had little FVA, as they typically exploit a variation-margin mechanism that, as suggested by Piterbarg (2010), provides the effect of "built-in" financing. Starting in late 2016, however, inter-dealer derivatives positions have been required by U.S. regulators to incorporate initial margin, in order to mitigate the risk of missing payments during the closeout period that would follow a dealer's default, as

[24]This assumes that bank's quotes are based on its own funding costs, as in our model. In practice, as noted, some banks quote based on average dealer funding costs (even though this is not optimal from the viewpoint of a given bank's shareholders).

explained by BCBS (2013). European regulators implementated this rule in 2017. Initial margin need not be re-pledgable by either party. The "trapped" portion of initial margin must be financed by dealers.[25] According to ISDA (2013), these new regulations will lock up trillions of additional dollars worth of posted margin. For example, Duffie, Scheicher, and Vuillemey (2015) estimate that new inter-dealer margin requirements will increase the aggregate amount of collateral needed in the CDS market by about 70%, before considering other effects such as central clearing and compression trading.

It is no surprise that some major dealers have initiated "XVA optimization" programs.[26] Some dealers may find it necessary to significantly reduce their swap intermediation businesses. One major dealer, Deutsche Bank, has already eliminated the bulk of its single-name CDS intermediation business, although the precise motive for this decision was not reported. In 2016, another major dealer, Barclays, sold its substantial "non-core" swap portfolio to J.P. Morgan.[27] Our model shows that this novation trade can be motivated by the fact that the associated funding costs to J.P. Morgan's shareholders are lower than those to Barclay's shareholders, given that J.P. Morgan's credit spreads are significantly lower.[28] If FVA were to be treated instead, as suggested by current dealer accounting, as an adjustment to the value of the derivatives themselves, the novation of this swap portfolio to JP Morgan cannot be motivated by any such gain to Barclays' shareholders, who cannot avoid a mark down in the value of their swaps merely by selling the swaps at a reduced market value. Alternatively, and also consistent with our model, there may be cases in which the novation generates better netting for one dealer's shareholders than the other's, and thus beneficial FVA and DVA, when summed across the two dealers.[29]

Our structural model of dealer funding costs also has implications for other areas of asset pricing. We briefly covered the implications for violations of covered interest parity. In another application, based on an extension of our model that allows for the alternative of repo financing of derivatives hedging positions, Song (2016) shows that some supposed "no-arbitrage" pricing relationships frequently break down to an economically important degree

[25] An international accord reported by Financial Stability Board (2013) mandates the central clearing of standardized swaps, subject to rules and exemptions that vary by jurisdiction, will also have an impact on collateral demand. The advent of regulations governing initial margin will soon further reduce systemic risk, as explained by BCBS (2013).

[26] See Sherif (2016a) and Sherif (2017).

[27] See Morris (2016) and Parsons (2016).

[28] As explained by Sherif (2016a), and consistent with our model, "For banks trying to estimate other banks' FVA costs, SG CIB's Lascar describes a rule-of-thumb method that involves using their five-year credit default swap (CDS) spread. The bank would take its own FVA, divide it by its own CDS spread, and then multiply the result by the other bank's CDS spread."

[29] In this case, however, the novation can also be motivated in part by the associated reduction through netting in deadweight expected financial distress costs.

in the presence funding costs to derivatives dealers' shareholders for carrying and hedging dealing inventory. In particular, Song (2016) shows that put-call parity must be adjusted significantly for longer-dated options in order to obtain reasonable synthetic pricing for equity dividend strips. He shows that a failure to do so may have lead to a potentially important bias in prior research on the term structure of S&P 500 equity risk premia.

C. Adjustments for Use of Regulatory Capital

In addition to their FVA and MVA adjustments, some banks have recently begun to make further valuation adjustments, so as to factor the effects of regulatory capital requirements into their accounting valuations. As discussed by Sherif (2015a) and Sherif (2016a), a "capital value adjustment" known as "KVA" is purportedly a markdown of the market value of the dealer's swaps associated with the amount of capital needed to support derivatives trading, whether to meet economic risk management requirements or regulatory capital rules. In practice, KVAs are not based on any sort of coherent valuation model. Our basic theory in Section II does indeed imply that when swap or other positions calls for additional equity capital, there is an associated cost to shareholders, which we calculate. In Section IV, we provided a simple example for a trade designed to take advantage of a violation of covered interest parity. This "KVA effect" is not, however, an adjustment to the value of the positions themselves, but rather to the value of equity and debt claims on the dealer. We have also ignored the incremental costs to shareholders for swap or other new positions associated with meeting the Liquidity Coverage Ratio rule (which may trigger the need to finance additional High Quality Liquid Assets), the Net Stable Funding Ratio, and stress tests (such as CCARs). These rules imply incremental costs to legacy shareholders, and thus have implications for dealer quotation and trader compensation analogous to, but structurally different from, those that we have analyzed in this paper. We leave these KVA and other related implications to future work.

D. Final Remarks

Also left for future research are models determining optimal intermediation strategies, from the viewpoint of dealer shareholder value maximization, given the implications that we have shown for a divergence between market values of new positions (in the form of "P&L") and the associated changes in the equity value of the dealer's firm. For example, it is interesting to note that two banks are able to execute trades with each other at prices that can improve the shareholder values of both firms, especially in the context of MVA. Margin lending strategies, as explained by Albanese, Andersen, and Iabichino (2015), can give dealers access to comparatively cheap funding, and provide efficient collateralized funding for lower-rated banks. We believe this is also a topic that will increase in recognized importance.

In general, the management of various "XVA costs" to bank shareholders will test the ability of financial market participants to adapt to a new reality in which a variety of previously under-appreciated financing and regulatory costs to dealer shareholders must be managed in order for robust over-the-counter market intermediation by regulated dealers to remain viable. A potential market adaptation for those OTC financial instruments that are broadly and frequently traded is the introduction of all-to-all trading, for example on an exchange operator's central limit order book.

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Appendix A A Pecking Order of Financing Choices

Section II considers the primary case of debt financing. Here, we consider the alternatives of equity financing and of financing with cash from the firm's balance sheet.

First, in the context of the model setup of Section II, we consider the funding of an investment paying Y by issuing equity. Because investors in a competitive market for newly issued equity break even on their purchase of shares, the incremental effect on the valuation of the legacy shareholders' equity is $\delta E^*[(A+qY-L)^+]-\delta E^*[(A-L)^+]-U(q)$. A calculation shown in Appendix B implies that the marginal value to the legacy shareholders of entering the position is in this case

$$G^0 = \delta E^*(1_{D^c} Y) - u. \tag{24}$$

The calculation (24) of G^0 reflects the fact that legacy shareholders must give up the entire valuation of the incremental cash flows that arise from the investment when the firm defaults.

As another alternative financing choice, if the firm is able to, and does, finance the position by using cash from its balance sheet, the initial equity valuation is $\delta E^*[(A-U(q)R+qY-L)^+]$. The marginal value of entering the position to the shareholders is shown in Appendix B to be

$$G_0 = \delta E^*(1_{D^c} Y) - uP^*(D^c). \tag{25}$$

Details underlying the calculations shown for G^0 and G_0 are omitted for brevity because they are similar to that shown in Appendix B for the calculation of G (the debt financing case) in the proof of Proposition 1.

PROPOSITION 4: *A PECKING ORDER OF FINANCING PREFERENCES. Suppose that the firm's probability of default is not zero and that the marginal investment cost u is strictly positive. The marginal value G_0 to the firm's existing shareholders of financing the investment with existing cash is strictly higher than the marginal value G under debt financing, which in turn is strictly higher than the marginal value G^0 under equity financing. That is, $G^0 < G < G_0$.*

To prove this result, we will show that $G^0 \leq G \leq G_0$, and that the inequalities are strict if the dealer's default probability is positive. By the fact that

$$G = \delta E^*[1_{D^c}(Y - u(R+S))] = \delta E^*(1_{D^c} Y) - \delta u(R+S)E^*(1_{D^c}),$$

we have $G_0 \geq G$. Moreover, $G_0 > G$ if the credit spread S is strictly positive.

By the fact that $G^0 = \delta E^*(1_{D^c}Y) - u$, it suffices to show that $u \geq \delta u(R+S)E^*(\mathbf{1}_{D^c})$ in order to see that $G \geq G^0$. We recall that $S = RE^*(\phi)/(1 - E^*(\phi))$ and $\phi = 1_D(L - \kappa A)/L$. Thus,

$$1 - E^*(\phi) \geq P^*(D^c),$$

which is equivalent to $1 \geq \delta(R+S)P^*(D^c)$. Again, the inequality is strict if S is positive.

For the case in which the investment cost u is strictly negative, the strict pecking order shown in Proposition 4 is reversed. If $u = 0$, meaning there is no up-front cash flow to finance, then $G^0 = G = G_0$.

Appendix B Proofs and Calculations for Sections II and III

This appendix supplies proofs of Propositions 1, 4, and 2, and a supplementary calculation of the marginal valuation of the swap transaction package to legacy creditors in Appendix C.C. For generality, we consider two cases: *(i)* there are finite states of the world and $P(A = L) = 0$, and *(ii)* (A, L) have continuous joint density function or A has a continuous density and L is a constant. In either case, we assume that A, L, and some given random payoff Y have finite expectations with respect to P^*.

A *Proof of Proposition 1*

Because we have assumed a competitive capital market with complete information, creditors offering the new debt break even. That is, the market credit spread $s(q)$ on the new debt, which is issued to finance the cost $U(q)$ of the new position, solves

$$U(q) = \delta E^* \left[\mathbf{1}_{\mathcal{D}^c(q)} U(q)(R + s(q)) + \mathbf{1}_{\mathcal{D}(q)} \frac{\kappa(A + qY_1 + qY_2^+)}{L + U(q)(R + s(q)) + qY_2^-} U(q)(R + s(q)) \right],$$

where we recall $\mathcal{D}^c(q)$ is the dealer's survival event $\{A + qY - L - U(q)(R + s(q)) \geq 0\}$. By letting q go to zero, one can easily see from the equation that $\lim_{q \to 0} s(q)$ exists, and that $\lim_{q \to 0} s(q) = S = RE^*(\phi)/(1 - E^*(\phi))$, where $\phi = \mathbf{1}_D(L - \kappa A)/L$.

If the dealer finances the position by issuing new debt, the marginal value of the asset purchase to shareholders is defined by

$$G = \left. \frac{\partial E^*[\delta(A + qY - L - U(q)(R + s(q)))^+]}{\partial q} \right|_{q=0}.$$

We intend to show that the derivative exists and is given by

$$G = \delta E^*[\mathbf{1}_{D^c}(Y - u(R+S))].$$

By definition,

$$\begin{aligned} G &= \lim_{q\to 0} \delta \frac{E^*[1_{\mathcal{D}^c(q)}(A + qY - L - U(q)(R+s(q)))] - E^*[1_{D^c}(A-L)]}{q} \\ &= \lim_{q\to 0} \delta \frac{E^*[1_{\mathcal{D}^c(q)}(qY - U(q)(R+s(q)))] + E^*[(1_{\mathcal{D}^c(q)} - 1_{D^c})(A-L)]}{q}. \end{aligned}$$

We know

$$\begin{aligned} \lim_{q\to 0} \delta \frac{E^*[1_{\mathcal{D}^c(q)}(qY - U(q)(R+s(q)))]}{q} &= \lim_{q\to 0} \delta E^*[\mathbf{1}_{\mathcal{D}^c(q)}(Y - U(q)/q(R+s(q)))] \\ &= \delta E^*[1_{D^c}(Y - u(R+S))], \end{aligned}$$

where the last equality is due to that $\lim_{q\to 0} U(q)/q$ and $\lim_{q\to 0}(R+s(q))$ exist, and that A, L, and Y have finite expectations, allowing interchangeability of the limit and expectation. We only need to show

$$\lim_{q\to 0} \delta \frac{E^*[|(1_{\mathcal{D}^c(q)} - 1_{D^c})(A-L)|]}{q} = 0. \tag{26}$$

There are two cases to be considered:

(i) If the set of possible states of the world is finite, then there exists a q_0 such that for any $q < |q_0|$, $\mathbf{1}_{\mathcal{D}^c(q)} - \mathbf{1}_{D^c} = 0$. Thus, (26) is immediate.

(ii) If there are infinitely many states of the world, under which A and L have a joint continuous density function, then we know

$$\lim_{q\to 0} P^*(\mathcal{D}^c(q)) = P^*(D^c).$$

It is easy to see that

$$1_{\mathcal{D}^c(q)} - 1_{D^c} = 1_{\mathcal{D}^c(q)\cap D} - 1_{\mathcal{D}(q)\cap D^c},$$

and that $|A-L| \le q|Y - (r+s(q))U(q)/q|$ on the events $D^c(q) \cap D$ and $D(q) \cap D^c$. Thus,

$$\begin{aligned} \lim_{q\to 0} \delta \frac{E^*[|(1_{\mathcal{D}^c(q)} - 1_{D^c})(A-L)|]}{q} &\le \lim_{q\to 0} \delta \frac{E^*[|1_{\mathcal{D}^c(q)\cap D}(A-L)|] + E^*[|1_{\mathcal{D}(q)\cap D^c}(A-L)|]}{q} \\ &= \lim_{q\to 0} \delta E^*[|(1_{\mathcal{D}^c(q)\cap D} + 1_{\mathcal{D}(q)\cap D^c})(Y - U(q)/q(R+s(q)))|]. \end{aligned}$$

By the Lebesgue Dominated Converge Theorem,

$$\lim_{q\to 0} E^*[|(1_{\mathcal{D}^c(q)\cap D} + 1_{\mathcal{D}(q)\cap D^c})Y|] = E\left[\lim_{q\to 0} |(1_{\mathcal{D}^c(q)\cap D} + 1_{\mathcal{D}(q)\cap D^c})Y|\right] = 0.$$

Since $\lim_{q\to 0} U(q)/q$ and $\lim_{q\to 0}(r + s(q))$ exist, we have

$$\lim_{q\to 0} E^*\left[(1_{\mathcal{D}^c(q)\cap D} + 1_{\mathcal{D}(q)\cap D^c})\frac{U(q)}{q}(R + s(q))\right] = \lim_{q\to 0} E^*[(1_{\mathcal{D}^c(q)\cap D}+1_{\mathcal{D}(q)\cap D^c})]\frac{U(q)}{q}(R+s(q)) = 0.$$

Thus,

$$\lim_{q\to 0} \delta\frac{E^*[|(1_{\mathcal{D}^c(q)} - 1_{D^c})(A - L)|]}{q} = 0,$$

and we have shown that

$$G = \delta E^*[\mathbf{1}_{D^c}(Y - u(R + S))].$$

B *Marginal Valuation for Legacy Creditors*

We also characterize the marginal valuation of the new position to the dealer's legacy creditors. Recall that $Y = Y_1 + Y_2$, where Y_1^- is secured and Y_2^- is unsecured. For an investment of q units, the dealer's assets at time 1 are

$$\mathcal{A}(q) = A + qY_2^+ + qY_1,$$

and the dealer's total liabilities due at time 1 are

$$\mathcal{L}(q) = L + qY_2^- + U(q)(R + s(q)).$$

Thus, the marginal value of the transaction package to the existing creditors is defined by

$$H = \left.\frac{\partial \delta E^*\left[(1 - \mathbf{1}_{\mathcal{D}(q)})L + \mathbf{1}_{\mathcal{D}(q)}\frac{\kappa\mathcal{A}(q)}{\mathcal{L}(q)}L\right]}{\partial q}\right|_{q=0},$$

where we recall $\mathcal{D}(q)$ is the dealer's default event with the new position. Thus,

$$\begin{aligned} H &= \lim_{q\to 0} \delta\frac{E^*\left[(1 - \mathbf{1}_{\mathcal{D}(q)})L + \mathbf{1}_{\mathcal{D}(q)}\frac{\kappa\mathcal{A}(q)}{\mathcal{L}(q)}L\right] - E^*[(1 - \mathbf{1}_D)L + \mathbf{1}_D\kappa A]}{q} \\ &= \lim_{q\to 0} \delta\frac{E^*[\mathbf{1}_{\mathcal{D}(q)}(\mathcal{A}(q) - A)] - (1 - \kappa)E^*[\mathbf{1}_{\mathcal{D}(q)}\mathcal{A}(q) - \mathbf{1}_D A] - E^*\left[\mathbf{1}_{\mathcal{D}(q)}\frac{\mathcal{L}(q)-L}{\mathcal{L}(q)}\kappa\mathcal{A}(q)\right]}{q}, \end{aligned}$$

where the last equality is due to that $\lim_{q\to 0} E^*[(\mathbf{1}_{\mathcal{D}^c(q)} - \mathbf{1}_{D^c})(A-L)]/q = 0$, as we have shown. For simplicity, we write

$$\psi \equiv \lim_{q\to 0} \frac{E^*\left(\mathcal{A}(q)\mathbf{1}_{\mathcal{D}(q)}\right) - E^*\left(A\mathbf{1}_D\right)}{q} = E^*[\mathbf{1}_D(Y_2^+ + Y_1)] + \lim_{q\to 0} \frac{E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]}{q},$$

where we write $\gamma = (1-\beta)\mathbf{1}_B$. There are two cases to be discussed:

(i) In the finite-space case, $\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D = 0$ for sufficiently small q. Thus, $\lim_{q\to 0} E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]/q = 0$, and

$$\psi = E^*[\mathbf{1}_D(Y_2^+ + Y_1)].$$

(ii) In the infinite-state space case,

$$\psi = E^*[\mathbf{1}_D(Y_2^+ + Y_1)] + J,$$

with $J \equiv \lim_{q\to 0} E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]/q$. The existence of J is guaranteed by the fact that A and L have a continuous joint density.

Thus, the marginal value of the package to the existing creditors is

$$\begin{aligned} H &= \delta E^*[\mathbf{1}_D(Y_2^+ + Y_1)] - \delta E^*\left[\mathbf{1}_D \frac{\kappa A}{L}(Y_2^- + u(R+S))\right] - \delta(1-\kappa)\psi \\ &= \delta E^*[\mathbf{1}_D(Y - uR)] + \delta E^*(\phi Y_2^-) + \delta E^*(\mathbf{1}_{D^c} uS) - \delta(1-\kappa)\psi, \end{aligned}$$

where the last equality is due to that $E^*(\mathbf{1}_D uR) - E^*[\mathbf{1}_D u(R+S)\kappa A/L] = E^*(\mathbf{1}_{D^c} uS)$. In the special case that the deadweight frictional loss is zero (that is, if $\kappa = 1$), then

$$H = \delta E^*[\mathbf{1}_D(Y - uR)] + \delta E^*(\phi Y_2^-) + \delta E^*(\mathbf{1}_{D^c} uS).$$

C The Asset-Substitution Effect

Proposition 1 captures only the first-order component $\delta \operatorname{cov}(1_D, Y)$ of the asset-substitution impact on the firm's valuation, based on the covariance of the incremental cash flow Y with the firm's default event. Here, we calculate the second-order impact on shareholder value. By the usual Taylor-series argument, the second-order asset-substitution effect is always dominated by the first-order effect for sufficiently small incremental positions.

In order to get simple explicit expressions, we treat only settings in which (A, L, Y) has a continuous joint density function, and where Y has a finite variance. The existence of the derivative $s'(q)$ of the credit spread with respect to the new position size q is guaranteed by the joint continuous density function for (A, L, Y). All other assumptions are maintained.

We first calculate the marginal shareholder value

$$G(q) \equiv \delta \frac{\partial E^*[(A + hY - L - U(h)(R + s(h)))^+]}{\partial h}\bigg|_{h=q}.$$

By definition,

$$\begin{aligned}
G(q) &= \lim_{h\to q} \delta \frac{E^*\left[(A - L + hY - U(h)(R + s(h))\mathbf{1}_{\mathcal{D}^c(h)}\right] - E^*\left[(A - L + qY - U(q)(R + s(q)))\mathbf{1}_{\mathcal{D}^c(q)}\right]}{h-q} \\
&= \lim_{h\to q} \delta \frac{E^*\left[((h-q)Y - (U(h) - U(q))R)\mathbf{1}_{\mathcal{D}^c(h)}\right]}{h-q} - \lim_{h\to q} \delta \frac{E^*\left[(U(h)s(h) - U(q)s(q))\mathbf{1}_{\mathcal{D}^c(h)}\right]}{h-q} \\
&+ \lim_{h\to q} \delta \frac{E^*\left[(A - L + qY - U(q)(R + s(q)))\left(\mathbf{1}_{\mathcal{D}^c(h)} - \mathbf{1}_{\mathcal{D}^c(q)}\right)\right]}{h-q} \\
&= \delta E^*[(Y - U'(q)R)\mathbf{1}_{\mathcal{D}^c(q)}] - \delta E^*[\mathbf{1}_{\mathcal{D}^c(q)}(U(q)s'(q) + s(q)U'(q))] + \Pi(q),
\end{aligned}$$

where

$$\Pi(q) = \lim_{h\to q} \delta \frac{E^*[(A - L + qY - U(q)(R + s(q)))(\mathbf{1}_{\mathcal{D}^c(h)} - \mathbf{1}_{\mathcal{D}^c(q)})]}{h-q}.$$

By arguments similar to those above, $\Pi(q) \equiv 0$. Thus,

$$G(q) = \delta E^*[(Y - U'(q)R)\mathbf{1}_{\mathcal{D}^c(q)}] - \delta E^*[\mathbf{1}_{\mathcal{D}^c(q)}(U(q)s'(q) + s(q)U'(q))].$$

We have shown that

$$G(0) = \delta E^*[\mathbf{1}_{D^c}(Y - uR)] - \delta E^*(u\mathbf{1}_{D^c}S).$$

The second derivative of shareholder value with respect to position size q is

$$\begin{aligned}
g &= \lim_{q\to 0} \frac{G(q) - G(0)}{q} \\
&= \lim_{q\to 0} \delta \frac{E^*[\mathbf{1}_{\mathcal{D}^c(q)}(Y - u(R + s(q)))] - E^*[\mathbf{1}_{D^c}(Y - u(R + S))]}{q} - \delta E^*[\mathbf{1}_{D^c} us'(0)] \\
&= \lim_{q\to 0} \delta \frac{E^*\left[\left(\mathbf{1}_{\mathcal{D}^c(q)} - \mathbf{1}_{D^c}\right)(Y - u(R + S))\right]}{q} - 2\delta E^*[\mathbf{1}_{D^c} us'(0)] \\
&= \delta E^*[(Y - u(R + S))^2 f(L \mid L, Y)] - 2\delta us'(0)p^*, \qquad (27)
\end{aligned}$$

where $p^* = P^*(D^c)$ is the risk-neutral survival probability and $f(x \mid L, Y)$ denotes the risk-neutral probability density at x of A conditional on (L, Y), and

$$s'(0) = \frac{R\kappa}{(1 - E^*(\phi))^2} \; E^*\left[\mathbf{1}_D \left(\frac{Au(R + S) - YL}{L^2}\right) - \frac{L - \kappa E^*(A)}{\kappa L}(Y - u(R + S))f(L \mid L, Y)\right].$$

For the rest of this section, we restrict attention for sake of simplicity to the case in which L is a constant and Y is independent (under P^*) of A. We then have

$$g = \delta f(L) E^*[(Y - u(R+S))^2] - 2p^* \delta u s'(0), \tag{28}$$

and we can write

$$s'(0) = \frac{R\kappa(a-b)}{(1 - E^*(\phi))^2},$$

where

$$a = \frac{u(R+S)E^*(\mathbf{1}_D A) - E^*(Y) L P^*(D)}{L^2}$$

and

$$b = \frac{L - \kappa E^*(A)}{\kappa L} f(L) \left(E^*(Y) - u(R+S)\right).$$

In general, $s'(0)$ can be positive, negative, or zero. However, for the case of a trade that is "breakeven" after debt servicing costs, in the sense that $\delta E^*(Y) - \delta u(R+S) = 0$, we have

$$s'(0) = \frac{R\kappa E^*(Y) E^*[\mathbf{1}_D(A-L)]}{(1 - E^*(\phi))^2 L^2} < 0.$$

For a non-zero investment position q, the second-order Taylor series approximation of the incremental gain to equity value associated with the investment can now be computed explicitly as

$$qG + \frac{q^2}{2} g, \tag{29}$$

where G is given by (5).

We can use the result of Breeden and Litzenberger (1978) that $\delta f(L)$ is equal to the "gamma" (second derivative) $\mathcal{E}''(L)$ of the equity value function $\mathcal{E}(\,\cdot\,)$, treated as the function mapping the strike price L to the equity value $\delta E^*[(A-L)^+]$. Thus,

$$g = \mathcal{E}''(L) m_2 - 2p^* \delta u s'(0), \tag{30}$$

where $m_2 = E^*[(Y - u(R+S))^2]$ is the second moment of the payoff of the investment net of the total financing payback.

The first term of g in (30) now appears clearly as the volatility impact of asset substitution, namely the product of the "equity gamma" $\mathcal{E}''(L)$ and the second moment m_2 of the net marginal payoff to shareholders.

To further interpret the "asset-substitution" effect g, we can consider the case in which

A is log-normally distributed, in that

$$A = A_0 \exp\left(\log R - \frac{\sigma^2}{2} + \sigma \mathcal{W}\right),$$

where A_0 is a positive constant, $\mathcal{W}$ is standard normal under P^*, and σ is the volatility of the firm's existing assets. Applying the Black-Scholes formula, we have the explicit equity gamma

$$\mathcal{E}''(L) = \frac{\delta\, \mathcal{N}'(d_2)}{L\sigma},$$

where $\mathcal{N}$ is the probability density of the standard normal distribution and

$$d_2 = \frac{\log A_0 - \log L + \log R - \frac{1}{2}\sigma^2}{\sigma}.$$

One can calculate that

$$s'(0) = \frac{R\kappa(a-b)}{(1 - E^*(\phi))^2},$$

where

$$a = \frac{u(R+S)RA_0\,\mathcal{N}(-d_2 - \sigma) - E^*(Y)L\,\mathcal{N}(-d_2)}{L^2}$$

and

$$b = \frac{L - \kappa R}{\kappa L} f(L)\left(E^*(Y) - u(R+S)\right).$$

In the shareholder-breakeven case $\delta E^*(Y) = \delta u(R+S)$, we can further simplify to

$$s'(0) = \frac{R\kappa E^*(Y)(RA_0\,\mathcal{N}(-d_2 - \sigma) - L\,\mathcal{N}(-d_2))}{(1 - E^*(\phi))^2 L^2}.$$

For our examples of a dealer financing an uncollateralized client swap hedged with a collateralized inter-dealer swap, or financing a CIP basis arbitrage, the asset-substitution effect is extremely small because the incremental asset payoff Y is risk-free or nearly risk-free. Even for relatively risky asset purchases, the asset-substitution effect is small relative to the FVA impact, at typical major-dealer credit spreads. This point is illustrated by the following numerical example.

Consider a dealer with a constant liability of L of \$1 trillion. The risk-free one-year gross return is $R = 1.02$. We take an initial dealer asset value A_0 of \$1.14 trillion, the fractional default recovery $\kappa = 0.5$, and the asset volatility $\sigma = 7.01\%$. These parameters imply that the dealer's one-year credit spread S is 50 basis points and that the dealer's initial equity volatility is 30%, based on the Black-Scholes delta method.[30]

Under P^*, the added asset payoff Y, uncorrelated with A, has a mean of $m = \$100$ million, and a standard deviation that we vary parametrically from 0 to \$50 million. The asset is purchased at a price u that equates the expected payoff m with the net financing payback $u(R + S)$. The discrete amount q of the asset purchase is 1 unit. The first-order FVA for this purchase is $qp^* \delta u S = \$469,000$.

Even at a standard deviation of \$50 million, the asset-substitution impact of the purchase, $q^2 \mathcal{E}''(L) m_2/2$, is only about \$800. The second-order FVA impact $-2q^2 p^* \delta u s'(0)/2$ for this example is about \$20. If the purchased quantity q is scaled up by a factor of 10, so that the mean asset payoff is \$1 billion, the asset-substitution effect scales by a factor of $q^2 = 100$ to about \$80,000, still only a small fraction of the first-order FVA impact of \$4.69 million. Although the second moment m_2 factor of the asset-substitution term is large in this example, the gamma factor $\mathcal{E}''(L)$ is tiny because, at a credit spread of 50 basis points, the equity option to default is far out of the money. The first-order asset-substitution effect depends on the risk-neutral correlation between Y and the default indicator 1_D. Assuming this correlation is -0.5, which is of the sign beneficial to shareholders, the first-order asset substitution benefit to shareholders is $-\delta \operatorname{cov}(1_D, Y) = \delta \times 0.5 \times 0.136 \times \$50$ million, which is \$3.34 million dollars. This first-order asset-substitution benefit, while much larger than the second-order benefit, is still dominated by the FVA cost to shareholders.

D Proof of Proposition 2

The proof has the following three parts.

(i) We have characterized the net cash flow of the package of transactions if the dealer finances the upfront payment $U(q)$ by issuing new debt. The net cash flow at time 1, from the viewpoint of the dealer, is

$$\mathcal{C}(q) = q(X - K) - q(1 - \beta)(X - K)^+ \mathbf{1}_B + (1 - \kappa \rho(q)) q (X - K)^- \mathbf{1}_{\mathcal{D}(q)},$$

where $\mathcal{D}(q) = \{A - L + qY - U(q)(R + s(q)) < 0\}$ is the event of the dealer's default with $Y = X - K - (1 - \beta)(X - K)^+ \mathbf{1}_B$, and $\rho(q)$ is the asset-to-debt payoff ratio

$$\rho(q) = \frac{A}{L + U(q)(R + s(q)) + q(X - K)^-}.$$

[30]That is the volatility is $\sigma A_0 \Delta / W$, where Δ is the Black-Scholes delta and W is the Black-Scholes value of equity, treated as an option on the final asset value A, struck at L.

Thus, the market value of the package of transactions is

$$\mathcal{V}(q) = \delta E^*[\mathcal{C}(q)].$$

(ii) Suppose the dealer finances the initial investment by issuing new equity. The dealer's default event in this case is $\mathcal{D}^0(q) = \{A - L + qY < 0\}$. The cash flow $q(X - K)$ of the unsecured client-to-dealer is not paid in full at time 1 in either of the two events: (a) the event that the client defaults and $q(X-K) > 0$, in which case the dealer receives $\beta q(X-K)$ from the client, and (b) the event that the dealer defaults and $q(X - K) < 0$, in which case the client is pari passu with other creditors of the dealer, and the proportional recovery rate is

$$\kappa\rho^0(q) = \frac{\kappa A}{L + q(X - K)^-}.$$

Thus, the net cash flow at time 1, from the viewpoint of the dealer, is

$$\mathcal{C}^0(q) = q(X - K) - q(1 - \beta)(X - K)^+ \mathbf{1}_B + (1 - \kappa\rho^0(q))q(X - K)^- \mathbf{1}_{\mathcal{D}^0(q)}.$$

The market value of the package of transactions is

$$\mathcal{V}^0(q) = \delta E^*[\mathcal{C}^0(q)].$$

(iii) If the dealer finances the initial investment by using cash from its balance sheet, the dealer's default event is $\mathcal{D}_0(q) = \{A + qY - L - U(q)R < 0\}$. Thus, the net cash flows at time 1, from the dealer's perspective, is

$$\mathcal{C}_0(q) = q(X - K) - q(1 - \beta)(X - K)^+ \mathbf{1}_B + q(1 - \kappa\rho_0(q))(X - K)^- \mathbf{1}_{\mathcal{D}_0(q)},$$

where $\rho_0(q) = (A - U(q)R)/(L + q(X - K)^-)$. Thus, the market value of the package of transactions is

$$\mathcal{V}_0(q) = \delta E^*[\mathcal{C}_0(q)].$$

It is easy to see that whether the dealer finances the initial investment by issuing debt, by issuing equity, or by using existing cash, the marginal value of the package to the dealer is

$$V = \lim_{q \to 0} \frac{\mathcal{V}(q)}{q} = \lim_{q \to 0} \frac{\mathcal{V}^0(q)}{q} = \lim_{q \to 0} \frac{\mathcal{V}_0(q)}{q} = \delta(X - K) + \delta E^*[\phi(X - K)^-] - \delta E^*[\gamma(X - K)^+],$$

where the last equality is due to the fact that A, L, and Y have finite expectations, allowing interchangeability of the limit and expectation.

E *Marginal Swap Valuation to Legacy Creditors*

We first calculate the marginal value H of the package of transactions to the legacy creditors by assuming the dealer finances the initial investment by issuing new debt. For an investment of q units, the dealer's assets at time 1 are

$$\mathcal{A}(q) = A + q(X-K)^+ - q\mathbf{1}_B(1-\beta)(X-K)^+ + q(\tilde{K}-X) + qIR.$$

The dealer's total liabilities due at time 1 are

$$\mathcal{L}(q) = L + q(X-K)^- + qI(R+s(q)).$$

As in the proof of Proposition 1, we can show that the marginal value of the package to the existing creditors is

$$H = \delta P^*(D)(\tilde{K}-K) + \Lambda + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma \mathbf{1}_D(X-K)^+] - \delta(1-\kappa)\psi,$$

where

(i) in the finite-state case,

$$\psi = E^*[\mathbf{1}_D((X-K)^+ + (\tilde{K}-X) + IR)] - E^*[\gamma \mathbf{1}_D(X-K)^+].$$

(ii) in the infinite-state case,

$$\psi = E^*[\mathbf{1}_D((X-K)^+ + (\tilde{K}-X) + IR)] - E^*[\gamma \mathbf{1}_D(X-K)^+] + J,$$

where $J = \lim_{q\to 0} E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]/q$. The existence of J is guaranteed by the fact that A and L have a continuous joint density.

If the dealer instead finances the initial investment by issuing new equity, it can be shown similarly that the marginal value of the package of transactions to the dealer's legacy creditors H^0 is

$$H^0 = \delta P^*(D)(\tilde{K}-K) + E^*(\mathbf{1}_D)I + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma \mathbf{1}_D(X-K)^+] - \delta(1-\kappa)\psi^0,$$

where

(i) in the finite-state case

$$\psi^0 = E^*[\mathbf{1}_D(\tilde{K} - X + IR + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+].$$

(ii) in the infinite-state case,

$$\psi^0 = E^*[\mathbf{1}_D(\tilde{K} - X + IR + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+] + \hat{J},$$

where $\hat{J} = \lim_{q \to 0} E^*[A(\mathbf{1}_{\mathcal{D}^0(q)} - \mathbf{1}_D)]/q$.

Finally, if the dealer finances the initial investment by using cash on the balance sheet, the marginal value of the package of transactions to the dealer's legacy creditors H_0 is

$$H_0 = \delta P^*(D)(\tilde{K} - K) + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma \mathbf{1}_D(X-K)^+] - \delta(1-\kappa)\psi_0,$$

where

(i) in the finite-state case,

$$\psi_0 = E^*[\mathbf{1}_D(\tilde{K} - X + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+].$$

(ii) in the infinite-state case,

$$\psi_0 = E^*[\mathbf{1}_D(\tilde{K} - X + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+] + \tilde{J},$$

where $\tilde{J} = \lim_{q \to 0} E^*[A(\mathbf{1}_{\mathcal{D}_0(q)} - \mathbf{1}_D)]/q$.

F *Modigliani-Miller Invariance in the Absence of Distress Costs*

If there are no deadweight frictional losses at the dealer's default (that is, if $\kappa = 1$), we have the following result.

PROPOSITION 5: MODIGLIANI-MILLER INVARIANCE. *If the fractional default recovery rate κ is 1, then the total marginal value of the forward portfolio to the dealer is invariant to how the collateral is financed, and identical to the market value of the forward portfolio. That is,*

$$G + H = G_0 + H_0 = G^0 + H^0 = v.$$

Appendix C Secured or Hedged Swaps

This appendix extends the results of Section III.A to treat cases involving variation margin, hedged swap positions, and margin value adjustments.

A Variation Margin and Inter-Dealer Hedging

When a dealer trades an unsecured swap with a client, the dealer is likely to combine the position with a suitable hedge. In practice, two separate hedges would typically be used. One hedge would mitigate the risk of default of the swap counterparty, for instance using a credit default swap (CDS) referencing the counterparty. Another position would be taken as a hedge against the market risk exposure of the floating-side payment X.

Using the setup in Section III.A, we can incorporate the effect of hedging a swap by assuming that the hedge simply takes the form of an offsetting position paying $-Y$, where Y is the net payout given by (6). As an abstract simplification, this covers both the counterparty risk and the underlying market risk X. The hedge is executed with another dealer, called the "hedge dealer." As is standard practice in inter-dealer transactions, the hedge requires the posting of variation margin, a running exchange of collateral that is sufficient to cover the entire present value of the transaction. In addition to providing default protection for both dealers, the variation margin mechanism provides an automatic source of cash funding of the hedge position, as we mentioned earlier.

In our one-period model, we can capture the effect of a running posting of variation margin in the following simplified way.

- At time 0, the dealer receives a cash payment from the hedge dealer equal to the market value $\delta E^*(Y)$. The dealer immediately posts this cash amount back to the hedge-dealer as a variation margin payment, earning the risk-free rate on the associated posting of collateral. As the two initial cash payments cancel, neither the dealer nor the hedge-dealer needs any financing to instantiate the hedge transaction.
- At time 1, but before other cash flows at time 1 are paid, the collateral is refreshed. That is, the dealer receives $E^*(Y)$ back from the hedge dealer. (This is margin posted at time zero, plus the risk-free interest.) The dealer pays Y to the hedge-dealer. The hedge-dealer is assumed to be paid with priority over all other creditors.[31] As the

[31] This effective priority over standard debt claims follows from exemptions for swaps from automatic stays in bankruptcy or other insolvency proceedings. Even under proposed methods for resolving the failure of a systemically important dealer that would apply the effect of an automatic stay on swap terminations, the dealer's swaps would likely retain priority over ordinary creditors, who would be "bailed in." This would fully prioritize swap counterparties except in the most extreme scenarios, in which even the cancellation of all debt subject to bail-in is insufficient to re-capitalize the dealer.

swap itself pays Y, given this assumed priority, the dealer will always be able to make this payment. This abstracts from some potential loss of priority that might apply in extreme practical cases, for example in an administrative failure resolution process that could override contractual termination rights.

Netting the cash flows, the total package consisting of an unsecured asset and the hedge will pay the dealer $E^*(Y)$ at time 1, an amount that is known at time zero. As desired, the hedge removes the variability of the payment Y, replacing it with its fair-market forward value.

Assuming that the dealer finances the purchase of the client asset by issuing debt, we can now repeat the funding cost analysis shown in Section III.A. The results, found in Appendix B, are obvious. Because the hedge removes net payout variance, the covariance term in (5) disappears, and the FVA for the package consisting of the asset and its hedge is simply $g(v - d) = -\Phi$.

As we have explained, the assumption of a perfectly offsetting hedge payout of $-Y$ is an idealization. In practice, the risk associated with the client swap payoff is not completely extinguished. This allows small default covariance terms to creep back into the breakeven price v^*. Further, inter-dealer hedge swaps are virtually always executed at par, that is, at a fixed rate of $\tilde{K} = E^*(X)$, rather than at an arbitrary rate of K. We deal with this minor complication in the next section.

B Par Swaps and Forward Swap Rates Without Margin

In practice, the fixed swap rate K is typically negotiated so that there is no upfront payment. In this case, the swap is known as a "par-valued swap." The resulting fixed rate K is often known as the "forward swap rate." In our setting, three different forward swap rates are of interest:

- The forward swap rate $\tilde{K}$ for a fully collateralized dealer-to-dealer swap. The swap has a market value of $\delta E^*(X - \tilde{K})$, so the fair forward swap rate $\tilde{K} = E^*(X)$ reflects no credit risk component. This is the benchmark forward swap rate typically shown on standardized trading screens. In practice, the risk-neutral probability measure P^* used by dealers for market valuation would typically be calibrated so as to match the risk-neutral expected payment $E^*(X)$ to the "screen rate" $\tilde{K}$, and likewise for other liquidly traded financial instruments.
- The forward swap rate $\hat{K}$ for an unsecured client swap that is executed at fair-market pricing. If we express v in (7) as $v = \eta(K)$, then $\hat{K}$ is the solution in K of the equation $\eta(K) = 0$.

- The forward swap rate K' for an unsecured client swap that leaves shareholders indifferent to the trade. From (6) and (8), K' is determined by the equation $E^*(1_{D^c} y(K')) = 0$.

Neither $\hat{K}$ nor K' depend on the financing strategy used by the dealer. Without an upfront, no financing is required, putting aside for now the issue of initial margin, which we will get to later in this section. Here, $\hat{K}$ and K' differ only because the DVA benefit on the swap is excluded from K'.

LEMMA 1: *ORDERING OF FORWARD SWAP RATES. Suppose that either (a) the dealer's default indicator 1_D is uncorrelated (under P^*) with the swap payment Y, or (b) the swap position is fully hedged by an inter-dealer swap. Then $K' \leq \hat{K}$ and $K' \leq \tilde{K}$.*

In a model with several time periods, even a position with no upfront cash payment may involve a funding value adjustment. For example, consider a position entered a time zero with no upfront payment, requiring a significant positive expected cash payment by the dealer at some intermediate date or dates, before compensating payments are later received by the dealer. A common example of this is a long-dated swap issued in an environment with a steeply sloped yield curve. As we will explain in more detail in Section V, such a position can be associated with a substantial funding value adjustment.

C Par Swaps with Initial Margin, and Margin Value Adjustment (MVA)

Par-valued swaps require no upfront funding and therefore have no FVA in our one-period setting. This situation changes with the introduction of initial margin, whether on the client swap itself or on the hedge swaps. In fact, it is becoming increasingly common to encounter swap agreements that require one or both counterparties to post risk-based initial margin, providing an additional layer of credit risk protection beyond variation margin. For instance, such agreements are routinely required by CCPs and are now mandatory under the Dodd-Frank Act and European MiFID regulations (see BCBS (2013)). Because initial margin always implies a positive initial cash outlay, even for par-valued swaps, funding valuation adjustments for margin will inevitably result in costs to dealer shareholders.

To be concrete, we consider the funding cost impact on the shareholders of a swap dealer that hedges an unsecured par-valued swap with a par-valued hedge transaction that requires the dealer to post initial margin. In summary, the swaps dealer in question is contemplating a pair of transactions consisting of:

(i) An uncollateralized swap with a client, by which the dealer pays a fixed rate K in exchange for a floating payment X, for a net contractual receivable at time 1 of $X - K$.

We take K as given for now, and assume that the client swap terms involve no initial exchange of cash. The terms of trade for the swap are thereby captured entirely by the fixed-side payment K.

(ii) A hedge-motivated fully collateralized swap with another dealer or a central counterparty, by which the dealer has a net receivable at time 1 of $\tilde{K} - X$, at the fair forward swap rate $\tilde{K} = E^*(X)$. As before, we suppose that the hedge swap involves variation margin and no net initial payment. In this case, however, the swap additionally requires the dealer to post a specified cash initial margin of $I > 0$. The recipient of the margin, typically either a CCP or a third-party custodian, invests the margin in risk-free assets, paying the dealer RI back at time 1 (unless the dealer defaults). As a simplification, we assume that the margin agreement is sufficient to ensure that both of the counterparties to the hedge swap are fully secured against loss.

The hedge swap payout $\tilde{K} - X$ is not an exact match for the client swap, except in the unlikely case that $K = \tilde{K}$. We do not consider a CDS hedge against default, but our results can be trivially extended to this case.[32] Our results are unaffected if the initial margin I_q for a position of size q is not necessarily proportional to q, provided that the per-unit margin has some limit $I \equiv \lim_{q \downarrow 0} I_q/q$. Likewise, our results remain as stated if the swap fixed-side terms K and $\tilde{K}$ depend on q, provided only that they converge with q to limits denoted K and $\tilde{K}$, respectively. These generalizations are avoided merely for notational simplicity.

We carry over all notation from Section III.A. Once again, the effect of any pre-existing positions between the swap counterparties is considered only in the appendix. We model variation margin in the same manner as in Section C.A, so that the net payment at time 1 on the hedge swap is $E^*(X - \tilde{K}) - (X - \tilde{K}) = \tilde{K} - X$. Before considering the impact of dealer default, the package of swap transactions therefore has a per-unit cash flow to the dealer at time 1, including the return of the margin with interest, of

$$Y = RI + \tilde{K} - K - \gamma(X - K)^+.$$

The initial required per-unit cash investment u is merely the initial margin I, because the swaps themselves are all executed without upfront payments.

Assuming that the initial margin is funded by debt issuance, Proposition 1 implies that the marginal value of the transaction to the dealer's shareholders is

$$G = \delta P^*(D^c)(\tilde{K} - K) - \delta E^*[1_{D^c}\gamma(X - K)^+] - \Lambda, \tag{31}$$

[32]As we have already seen in Section C.A, adding a CDS hedge essentially removes the covariance effects in the CVA term. For instance, the term $\delta E^*[1_{D^c}\gamma(X - K)^+]$ in (31) would become $\delta P^*(D^c)E^*[\gamma(X - K)^+]$.

where $\Lambda = \delta P^*(D^c)SI$ is the funding cost adjustment for the payment of initial margin, known in industry practice as the margin value adjustment (MVA). In this simplest of settings, the value adjustment Λ for initial margin is the initial market value of the component of net margin-funding interest expense SI that is borne by shareholders at time 1. The shareholders bear the entire expense SI if the dealer does not default, and bear none of the expense if the dealer defaults.

We also calculate the total market value of the package of swap transactions. For a position of q units, the initial margin payment generates cash flow of $-qI$ to the dealer at time 0. At time 1, the payment of the hedging swap, including the return of margin with interest, is $q(\tilde{K} - X) + qIR$. The payment of the client-to-dealer swap to the dealer is $q(X - K)$ before considering default. The cash flow $q(X - K)$ is not paid in full at time 1 in either of two events: *(i)* the client defaults and $q(X - K) > 0$, in which case the dealer receives $\beta q(X - K)^+$ from the client; and *(ii)* the dealer defaults and $q(X - K) < 0$, in which case the client is pari passu with the other creditors of the dealer, and the swap client receives $\mathcal{R}(q)q(X - K)^-$, where, based on (2),

$$\mathcal{R}(q) = \frac{\kappa(A + q(\tilde{K} - X) + qIR)}{L + q(X - K)^- + qI(R + s(q))}$$

is the fractional recovery of the dealer's assets in default on the event that $X - K < 0$. The numerator of $\mathcal{R}(q)$ is the amount of the dealer's assets that are recovered if the dealer defaults and $X - K < 0$. The denominator is the aggregate liabilities of the dealer, which include the legacy liabilities L, the liabilities due to financing the initial margin, which is $qI(R + s(q))$, and the liabilities to the swap client, which is $q(X - K)^-$. By assumption, $A + qIR + q\,(X - K)^+$ is always sufficient to pay the amount $q(\tilde{K} - X)^-$ due on the secured hedge.

Following the definitions of Section II.B, the net actual cash flow at time 1 of the package of swap transactions is

$$\hat{\mathcal{C}}(q) = q(\tilde{K} - X) + qIR + q(X - K) - q\gamma(X - K)^+ + q1_{\hat{\mathcal{D}}(q)}(1 - \mathcal{R}(q))(X - K)^-,$$

where

$$\hat{\mathcal{D}}(q) = \{A + q(\tilde{K} - K) - q\gamma(X - K)^+ - L - qIs(q) < 0\}$$

is the event of the dealer's default.

The total market value of the package of transactions is

$$\mathcal{V}(q) = -qI + \delta E^*(\hat{\mathcal{C}}(q)).$$

One can see that the initial payment I of margin at time 0 and the return payment of RI at time 1 have offsetting impacts on the total market value of the swap. When considering the marginal value of the transaction to shareholders, however, the computation (eq:G3) shows the crucial impact on shareholder value of of financing the initial margin.

Similar to the case of Proposition 2, the marginal value of the swap,

$$v = \left.\frac{\partial \mathcal{V}(q)}{\partial q}\right|_{q=0} = \delta(\tilde{K} - K) - \delta E^*[\gamma(X-K)^+] + \delta E^*[\phi(X-K)^-], \tag{32}$$

is decomposed into the present value of the gross swap spread $\tilde{K} - K$, less the CVA, plus the DVA. As anticipated, the per-unit market value v of the combined swap position does not depend on the amount I of required initial margin, nor does v depend on how the margin was financed. As we have noted, however, this invariance of valuation to the financing of initial margin is contrary to current dealer valuation practice.

Appendix B calculates the impact of the value H of the package on the legacy creditors. If there are no default distress costs, we have usual value-conservation identity $H + G = v$.

The fair-market level of the spread $\tilde{K} - K$ between the two swap rates, obtained from (32) by setting v equal to zero, is

$$\mathcal{S} = E^*[\gamma(X-K)^+] - E^*[\phi(X-K)^-], \tag{33}$$

which is merely the net risk-neutral expected default loss on the client swap (loss from client default net of loss from dealer default). The swap spread $\mathcal{S}' = \tilde{K} - K$ that makes the dealer's shareholders indifferent to the trade is instead obtained from (31) by setting $G = 0$, leaving

$$\mathcal{S}' = SI + \frac{E^*[1_{D^c}\gamma(X-K)^+]}{P^*(D^c)}.$$

In order to generate positive shareholder returns in this setting, the dealer must be able to identify hedged swap positions at fixed swap rates that improve on fair-market rates by $\mathcal{S}' - \mathcal{S}$. In gauging how difficult this may be for the dealer's swap desk, we suppose that the dealer's default event is uncorrelated under P^* with the client default loss $\gamma(X-K)^+$. The dealer must then be able to improve on fair-market swap rates by at least

$$\mathcal{S}' - \mathcal{S} = SI + E^*(\phi)E[(X-K)^-].$$

For the typical (small) credit spreads of major dealers, and for small risk-free interest rates (that is, R near 1), we have the Taylor approximation $S \simeq E^*(\phi)$, and thus

$$\mathcal{S}' - \mathcal{S} \simeq S\left(I + E^*[(X-K)^-]\right), \tag{34}$$

where the first term originated from the margin funding costs and the second from the DVA. This is the adjustment to the swap quote necessary to overcome effect of value impact on shareholders shown by Equation (12).

Because initial margins set by CCPs or in the inter-dealer swap market are standardized, the right hand side of (34) is the dealer's credit spread S multiplied by some positive swap-specific amount that does not depend on the identity of the dealer.

Appendix D Multi-Period Model

We generalize the basic model of Section III to 2 periods with 3 dates $t = 0, 1, 2$. New information is revealed at the interim date 1 through observation of a collect Z of random variables. All uncertainty is resolved at date 2. We let E_1^* denote expectation under P^* conditional on Z. We assume that the one-period gross risk-free returns are R_0 and R_1 at time 0 and 1, respectively. We don't require R_1 to be constant. Thus, the market value of the cash flows $\{C_t\}_{t=1}^2$ is defined as $\sum_{t=1}^2 E^*(\delta_t C_t)$, where $\delta_1 = 1/R_0$ and $\delta_2 = 1/(R_0 R_1)$.

We consider a dealer whose pre-existing assets have payoffs at time 2 are given by some random variable A. The firm has short-term liabilities L_1 that expire at time 1 and long-term liabilities L_2 that expire at time 2. We assume that the dealer liquidates a portion of its legacy assets to pay back the maturing liabilities L_1 at time 1 and pay out dividend π_1, which is also a random variable. If the liquidation value of asset is not enough to cover L_1, the dealer defaults, which we denote the event as D_1. We let W denote the payoff at time 2 of the liquidated assets. As a result, the firm defaults at time 2 in the event $D_2 = \{A - W < L_2\}$. In the dealer's default events D_1 and D_2, we assume all liabilities are pari passu with each other, and the recovery rates of assets are some constant κ_1 and κ_2, respectively. We let τ_D denote the dealer's default time. If the dealer survives at time 2, that is, $\tau_D = \infty$, the firm is liquidiated and the remaining cash flows are attributed to shareholders after paying back creditors. Thus, the total value of the firm's equity is $E^*[\delta_1 \mathbf{1}_{\{\tau_D>1\}}\pi_1] + E^*[\delta_2 \mathbf{1}_{\{\tau_D>2\}}(A - W - L_2)]$. The total value of the dealer's liabilities is

$$E^*[\delta_1 \mathbf{1}_{\{\tau_D>1\}} L_1 + \delta_1 \mathbf{1}_{\{\tau_D=1\}} \kappa_1 E_1^*(A)/R_1] + E^*[\delta_2 \mathbf{1}_{\{\tau_D>2\}} L_2 + \delta_2 \mathbf{1}_{\{\tau_D=2\}} \kappa_2 (A - W)].$$

We assume either *(i)* finite states of the world, or *(ii)* infinitely many states of the world with standard continuity conditions of (A, W, L_1, L_2) as in Section II. As in Section II, the dealer's marginal credit spread at time 0 for short-term (one-period) debt is

$$S_0 = \frac{E^*(\phi_1)R_0}{1 - E^*(\phi_1)},$$

where $\phi_1 = \mathbf{1}_{D_1}(L_1 + E_1^*(L_2)/R_1 - \kappa_1 E_1^*(A))/(L_1 + E_1^*(L_2)/R_1)$. If the dealer survives at 1, the dealer's marginal credit spread at time 1 for one-period debt is

$$S_1 = \frac{E_1^*(\phi_2)R_1}{1 - E_1^*(\phi_2)},$$

where $\phi_2 = \mathbf{1}_{D_2}(L_2 - \kappa_2(A - W))/L_2$.

In this two-period setting, a swap is a contract promising *(i)* floating payment X_1 in exchange for fixed payment K_1 at time 1, and *(ii)* floating payment X_2 in exchange for fixed payment K_2 at time 2, before considering the effect of counterparty default. We let $C_1 \equiv X_1 - K_1$ and let $C_2 \equiv X_2 - K_2$. We focus on the payer swap, that is, the positive cash flow of this contract is an asset to the dealer, whereas the negative cash flow is a contingent liability. A swap position of size q requires the dealer to make an upfront payment of $U(q)$. We assume $u = \lim_{q \downarrow 0} U(q)/q$ exists. Results for the reverse case are obvious by analogy.

The supporting calculations for the following results are similar to Appendix B and are omitted for brevity.

A Valuing Unsecured Swaps with Upfront

In this section, we extend the results in Section III.A. That is, the client swap is assumed to be fully unsecured. For simplicity, we assume that at the interim period, swap counterparties default after the coupon payment.[33] We let τ_C denote the swap client's default time. At the client's default, the dealer recovers a fraction β_1 and β_2 of any remaining contractual amount due to the dealer at time 1 and time 2, respectively. We also suppose that there are no pre-existing positions between the swap client and the dealer. The effect of netting the new swap flows against those of the legacy positions with the same client is analyzed in Appendix F.

We have the following natural extension of the basic one-period swap valuation model in Section III.A.

PROPOSITION 6: *Whether the dealer finances any net payments by issuing debt, issuing equity, or using existing cash on its balance sheet, the marginal market value of the swap is*

[33] This assumption is valid for the purpose of marginal analysis.

well-defined by

$$v = E^*\left(\sum_{t=1}^{2}\delta_t C_t - u\right) + E^*\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_D=t,\tau_C>t-1\}}\phi_t V_t^-\right) - E^*\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t,\tau_D>t-1\}}(1-\beta_t)V_t^+\right), \quad (35)$$

where $V_1 = E_1^*(C_2)/R_1$ *and* $V_2 = C_2$.

As in the single-period model, the swap value (35) includes two credit-related adjustments for the default free value, $V_0 = E^*(\delta_1 C_1) + E^*(\delta_2 C_2)$, for default. The CVA is

$$E^*\left[\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t,\tau_D>t-1\}}(1-\beta_t)V_t^+\right]$$

and the DVA is $E^*\left[\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_D=t,\tau_C>t-1\}}\phi_t V_t^-\right]$. The market value of the same swap from the viewpoint of the swap client is of course $-v$.

Now, we analyze the marginal value of the new swap to shareholders of the dealer, we assume that the positive financing requirement is financed by issuing short-term (one-period) debt. Likewise, any net positive cash flow to the dealer is used to retire short-term debt.

PROPOSITION 7: *If the dealer issues debt to finance net payments and uses received cash to retire outstanding debt, then the marginal value of the swap to the dealer's shareholders is well defined by*

$$G = E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2}\delta_t C_t - u\right)\right] - E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t\}}(1-\beta_t)V_t^+\right)\right] - \Phi(u), \quad (36)$$

where

$$\Phi(u) = E^*\left[\delta_1 \mathbf{1}_{\{\tau_D>1\}}uS_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}}uR_0S_1\right] - E^*\left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}}C_1S_1)\right],$$

is the debt funding valuation adjustment.

As in Section III.B, if the swap is executed at the "conventional" upfront,

$$u^* = V_0 - c^* = V_0 - E^*\left(\sum_{t=1}^{2}\mathbf{1}_{\{\tau_C=t\}}\delta_t(1-\beta_t)V_t^+\right),$$

then the marginal value of the swap portfolio to the dealer's shareholders is

$$G = \text{cov}\left(\mathbf{1}_{\{\tau_D>2\}}, \sum_{t=1}^{2}\delta_t C_t - \sum_{t=1}^{2}\mathbf{1}_{\{\tau_C=t\}}\delta_t(1-\beta_t)V_t^+\right) - \Phi(u^*). \quad (37)$$

In practice, c^* is often known as *Unilateral Credit Valuation Adjustments* (UCVA),[34] and it is different from the CVA in (35) as it does not take into account the dealer's default. In the case that the dealer's default is independent of the swap cash flows, the shareholder value is

$$G = -\Phi(u^*).$$

By analogy with (12), for a small spread S, the dealer's indifference quote is approximately $u^* - \Phi(u^*)$.

B Inter-Dealer Hedges, Initial Margin, and MVA

In this subsection, we consider a swap dealer hedges the unsecured swap with a fully collateralized inter-dealer swap that requires the dealer to post both initial margin and variation margin. We assume that the hedge-motivated collateralized swap with another dealer or a central counterparty has a net receivable of $-C_1 = K_1 - X_1$ at time 1 and a net receivable of $-C_2 = K_2 - X_2$ at time 2. The hedging swap requires the dealer to post both cash initial margin of I_0 and I_1, and variation margin M_0 and M_1 at time 0 and time 1, respectively. We follow the same variation margin mechanism as in Section C.A,and we assume that $M_0 = V_0 = E^*\left(\sum_{t=1}^{2} \delta_t (X_t - K_t)\right)$ and $M_1 = V_1 = E_1^*(X_2 - K_2)/R_1$, the standardized margin payment that equal to the market value of the hedging swap. We assume this hedging swap is transacted at the fullly collateralized value V_0.

We have the following natural extension of the basic one-period swap valuation model with inter-dealer hedge.

PROPOSITION 8: *If the dealer issues debt to finance margin payments and uses received margin to retire outstanding short-term debt obligations, then the marginal value of the swap portfolio to the dealer's shareholders is well defined by*

$$G = E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(V_0 - u\right)\right] - E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2} \delta_t \mathbf{1}_{\{\tau_C=t\}}(1-\beta_t)V_t^+\right)\right] - \Phi(u) - \Psi,$$

where

$$\Phi(u) = E^*\left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} u S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} V_1 S_1\right] + E^*\left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}}(u - V_0)\right],$$

[34]See Albanese and Andersen (2014) for details on UCVA.

is the funding value adjustment, and

$$\Psi = E^* \left(\delta_1 \mathbf{1}_{\{\tau_D > 1\}} I_0 S_0 \right) + E^* \left(\delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} I_1 S_1 \right)$$

is the margin value adjustment.

In the special case that the unsecured swap is executed at the default-free market value, that is, $u = V_0$, the FVA is

$$\Phi(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} V_0 S_0 + \delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} V_1 S_1 \right].$$

C Imperfect Variation Margin and FVA

So far, we have assumed that the client swap is fully unsecured. It is also of interest to consider the case that the client swap requires both counterparties to post some variation margin. To be concrete, we assume the client swap requires some "imperfect" variation margin, so that m_0 and m_1 are the amount of variation margin in the dealer's possenssion at time 0 and time 1, respectively. We assume this client swap is hedged with the same fully collateralized inter-dealer swap in Section D.B.

By direct algebra, the FVA in this case is

$$\begin{aligned} \Phi(u) = & E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} (V_0 - m_0) S_0 + \delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} (V_1 - m_1) S_1 \right] \\ & + E^* \left[\delta_1 \mathbf{1}_{\tau_D > 1} (u - V_0) \right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} (u - V_0) \right]. \end{aligned}$$

In the case that the client swap is execuated at the default-free market value V_0, then the FVA is

$$\Phi(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} (V_0 - m_0) S_0 + \delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} (V_1 - m_1) S_1 \right].$$

If the "imperfect" margin becomes "perfect", that is, if $m_0 = V_0$ and $m_1 = V_1$, then the FVA $\Phi(V_0) = 0$.

D Cash Management Strategy and Asymmetric FVA

Our denition of FVA is symmetric, in the sense that cash inflows and outflows are assumed to be financed or to reduce financings, respectively, at a spread of S. For the case of cash inflow, this implicitly assumes that there is always some short-term unsecured debt to roll over whose total amount can be reduced by swap cash inflows.

Now, we consider the case that the cash outflows are financed with unsecured debt and cash inflows are invested at the risk-free rate. All else are equal as in Section D.B.

Correspondingly, we can calcuate the "asymmetric funding value adjustment" (AFVA) as

$$\tilde{\Phi}(u) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} u^+ S_0\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} (V_1 + u - V_0)^+ S_1\right].$$

If the unsecured swap is executed at $u = V_0$, then the AFVA is

$$\tilde{\Phi}(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} V_0^+ S_0\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} V_1^+ S_1\right].$$

Appendix E The Continuous-Time Reduced-Form Model

This appendix provides additional details underlying the continuous-time reduced-form model of Section IV.

A *Technical Assumptions*

We fix our probability space, $(\Omega, \mathcal{F}, P^*)$ and a filtration $\{\mathcal{F}_t : t \geq 0\}$ of sub-σ-algebras of $\mathcal{F}$ satisfying the usual conditions, as defined by Protter (2005). We take the short-rate process $r = \{r_t : t \geq 0\}$ to be progressively measurable and adapted, and such that $\int_0^t |r_s|\, ds$ is finite almost surely for all t. As usual, we let E_t^* denote conditional expectation with respect to $\mathcal{F}_t$.

All probabilistic statements to follow are with respect to our valuation probability measure P^*. This means, by definition, that the market value at time t of a fully collateralized claim to some payment C at some bounded stopping time $T \geq t$ is by definition $E_t^*(\delta_{t,T}\, C)$, where $\delta_{t,u} = e^{-\int_t^u r(s)\, ds}$ for any times t and $u \geq t$. Here, C is measurable with respect to $\mathcal{F}_T$ and such that $e^{-\int_0^T r_s\, ds} C$ has a finite expectation with respect to P^*.

Before considering the effect of incremental cash flows associated with a new position, the derivatives dealer defaults at a stopping time τ_D with intensity process[35] λ_D. An unsecured claim of size C on the dealer's estate at default is paid $(1 - \ell_D(\tau_D))C$, for some proportional loss process[36] ℓ_D taking outcomes in $[0, 1]$. This implies that the dealer's short-term credit

[35] The default time τ_D of the dealer is doubly stochastic driven by a sub-filtration $\{\mathcal{G}_t : t \geq 0\}$ of $\{\mathcal{F}_t : t \geq 0\}$ to which the short-rate process and all payment processes that we consider are adapted. See Duffie (2001), Chapter 11, for details.

[36] We assume that ℓ_D is a predictable process. One can generalize so as to get essentially the same result, under mild regularity, by replacing ℓ_D with the dual predictable projection of a loss-given-default random variable.

spread at time t is $S_t = \lambda_D(t)\ell_D(t)$. That is,[37] each unit of the dealer's short-term unsecured debt can be continually renewed, or "rolled over," until any fixed time U, or until default, whichever comes earlier, by making continual floating-rate interest payments at the adjusting rate $r_t + S_t$, and by making a final payment of 1 at time U in the event that default occurs after time U. In the event that the default time τ_D is before U, each unit of this debt recovers $1 - \ell_D(\tau_D)$ at default.

Similarly, a given client swap counterparty has default risk characterized by a default time τ_C with intensity process[38] λ_C, and by a proportional loss-given-default process ℓ_C.

The CVA and DVA definitions and calculations shown in Section IV.A, from Duffie and Huang (1996), differ from the so-called "unilateral" CVA and DVA, which are given, respectively, by

$$E^* \left(1_{\{T>\tau_C\}}\delta_{0,\tau_C}\ell_C V_t^+\right)$$

and

$$E^* \left(1_{\{T>\tau_D\}}\delta_{0,\tau_D}\ell_D V_t^-\right).$$

See Albanese and Andersen (2014) for details. The unilateral definitions abstract from the fact that the dealer's default is irrelevant if the customer has already defaulted, and vice versa.

B Computational Analysis

We provide the computational analysis underlying the numerical examples in Section V.B of XVAs for an unsecured semi-annual plain-vanilla interest rate swap. We assume the swap has a maturity of 10 years and that the coupon payment dates are $\{t_i\}_{i=0}^N$, where $t_i = i\Delta$ with $\Delta = 0.5$. At time t_i, a payer swap to the dealer has a contractual payment of $C_i = \Delta(X_{i-1} - K)$, where X_{i-1} is the LIBOR rate fixed at time t_{i-1} and K is the fixed coupon rate. The first LIBOR fixing is assumed to take place at $t_0 = 0$, and the last coupon time is $t_N = 10$.

We use the overnight index swap (OIS) rate as a benchmark for the instantaneous risk-free

[37]This follows from the fact that a martingale $\mathcal{M}$ is defined by

$$\mathcal{M}_t = E_t^* \left(\int_0^U \delta_{t,u}(r_u + S_u)1_{\{\tau_D>u\}}\,du + 1_{\tau_D>U}\delta_{0,U} + 1_{\tau_D\le U}\,\delta_{t,\tau_D}\ell_D(\tau_D) \right).$$

The same result applies if U is any given bounded stopping time relative to the driving sub-filtration $\{\mathcal{G}_t : t \ge 0\}$.

[38]The counterparty default time τ_C is jointly doubly stochastic with τ_D, and driven by the same sub-filtration $\{\mathcal{G}_t : t \ge 0\}$.

rate r_t, corresponding to a risk-free discount of

$$p(t,u) = E_t^*(\delta_{t,u}) = E_t^*\left(e^{-\int_t^u r_s\,ds}\right),$$

where E_t^* denotes conditional expectation at time t under P^*. As a result, the default-free market value of the payer swap is

$$V_t = 1_{t<t_{\eta(t)}}\Delta\left(X_{\eta(t)-1}\right) - K\big)\,p\left(t, t_{\eta(t)}\right) + E_t^*\left(\sum_{i=\eta(t)}^{N-1} e^{-\int_t^{t_{i+1}} r_u\,du}\Delta(X_i - K)\right).$$

C Term Structure Model

We use a one-factor Hull-White term structure model for the short rate r_t, as given in Section V.B, implying that r_t is normally distributed with conditional distribution given r_s of $\mathcal{N}\left(m(s,t), v(s,t)\right)$, where, with $f_t = -d\log(p(0,t))/dt$,

$$m(s,t) = f_t + e^{-\alpha(t-s)}(r_s - f_s) + e^{-\alpha t}\frac{\sigma^2}{2\alpha^2}\left(e^{\alpha t} - e^{\alpha s} + e^{-\alpha t} - e^{-\alpha s}\right),$$

and $v(s,t) = \sigma^2/(2\alpha)\left(1 - e^{-2\alpha(t-s)}\right)$. The associated discount factor at time t for cash flows at $T > t$ is

$$p(t,T) = \frac{p(0,T)}{p(0,t)}e^{-\frac{1}{2}G(t,T)^2\theta_t - (r_t - f_t)G(t,T)}, \tag{38}$$

where θ_t was defined in (23) and $G(t,T) = (1 - e^{-\alpha(T-t)})/\alpha$.

For simplicity, we assume that the spread ϵ between the LIBOR rate and the OIS rate is constant over time. Thus, the LIBOR rate is

$$X_i = \Delta^{-1}\left(p(t_i, t_{i+1})^{-1}(1 + \epsilon\Delta) - 1\right).$$

For notational simplicity, we define an annuity factor by

$$a(t;j) = \sum_{i=j}^{N-1} p(t, t_{i+1})\Delta,$$

an OIS forward yield by

$$y(t;j) = \frac{p(t,t_j) - p(t,T_N)}{a(t;j)},$$

as well as a LIBOR forward yield by $y_L(t;j) = \epsilon + (1+\epsilon\Delta)y(t;j)$. By direct algebra, the default-risk-free version of the swap has market value

$$V_t = 1_{t<t_{\eta(t)}} \left[\Delta \left(X_{\eta(t)-1}\right) - K\right) p(t, t_{\eta(t)})\right] + a(t,\eta(t)) \left(y_L(t,\eta(t)) - K\right). \tag{39}$$

D CVA, DVA, FVA, and MVA Calculations

For the numerical examples in Section V.B, we assume that the swap client has a constant default intensity of $\lambda_C = 4\%$. We also assume that the dealer has a constant default intensity of $\lambda_D = 2\%$. We assume that the proportional loss process ℓ_C and ℓ_D are also constant, and $\ell_C = \ell_D = 50\%$. This implies a credit spread $S_D = 1\%$ for the dealer. We further assume that dealer default and client default are independent of each other and of the state of interest rates. Thus, the CVA, DVA and FVA are, respectively,

$$\begin{aligned}
\text{CVA} &= \ell_C \lambda_C \int_0^{T=10} E^*(\delta_{0,t} V_t^+) e^{-(\lambda_C+\lambda_D)t}\, dt, \\
\text{DVA} &= \ell_D \lambda_D \int_0^{T=10} E^*(\delta_{0,t} V_t^-) e^{-(\lambda_C+\lambda_D)t}\, dt, \\
\Phi &= S_D \int_0^{T=10} E^*(\delta_t V_t) e^{-(\lambda_C+\lambda_D)t}\, dt.
\end{aligned}$$

As V_t is driven by a single-factor Gaussian model, the expected values in these integrals are easy to compute from equations (38) and (39); they are shown in Figure 1 below for the payer swaps in our numerical example, using three different fixed coupon levels.

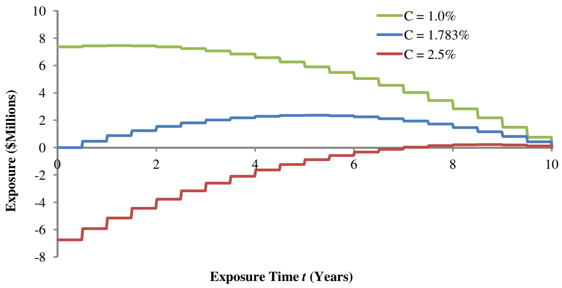
We write

$$F_t \equiv E_t^* \left(\sum_{i=\eta(t)}^{N-1} e^{-\int_t^{t_{i+1}} r_u\, du} \Delta(X_i - K) \right),$$

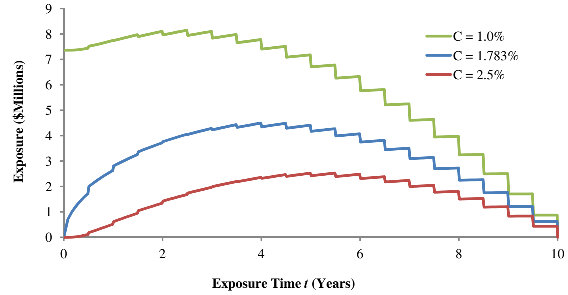
and $\mathcal{D}_{t,t+l} \equiv F_{t+l} - E_t^*(F_{t+l})$, where l is assumed to be two weeks. When calculating the MVA, we assume that the margin I_t is set as the 99th percentile of $\mathcal{D}_{t,t+l}$. As $\mathcal{D}_{t,t+l}$ is here very closely approximated by Gaussian random variable, the computation of I_t is straightforward. The resulting MVA is

$$\Psi = S_D \int_0^{T=10} E^*(\delta_t I_t) e^{-(\lambda_C+\lambda_D)t}\, dt.$$

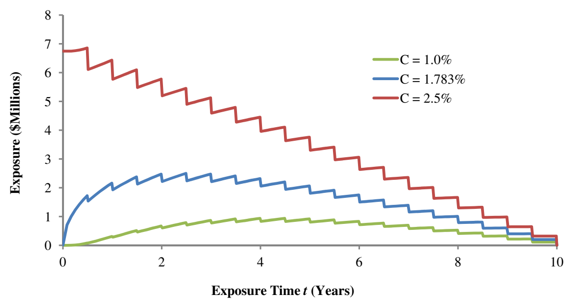
In Figure 2 we show $E^*(\delta_t I_t)$ for our numerical example. Notice how the initial margin decreases over time as the duration of the swap shrinks as it approaches the final maturity.



(a) $E^*(\delta_{0,t}V_t)$



(b) $E^*(\delta_{0,t}V_t^+)$



(c) $E^*(\delta_{0,t}V_t^-)$

Figure 1: Exposure profiles for 10-year payer swap.

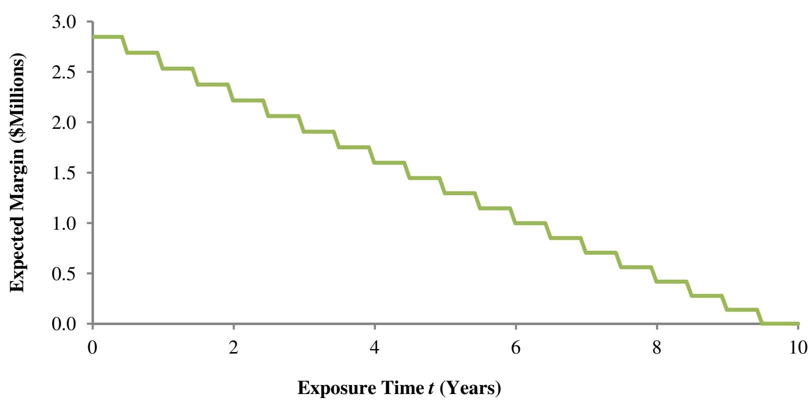


Figure 2: Margin exposure profile $E^*(\delta_{0,t}I_t)$ for 10-year payer swap.

Appendix F The Effect of Netting with Legacy Positions

In this section, we extend the results in Section III.A to the case in which the dealer has a pre-existing swap position with the swap client.

The dealer purchases a new unsecured swap from a client, which is identical to that in Section III.A. This same client already has a legacy swap position with the dealer, whose contractually promised payment is c_0 and requires the dealer to make an upfront payment of u_0. As has been our convention, the positive cash flow of this contract is an asset to the dealer, whereas the negative cash flow is a contingent liability.

As in the main context, we characterize the marginal value of the new swap investment for the dealer's legacy shareholders and legacy creditors (excluding the swap counterparty). We also characterize the marginal market value of the new swap investment. As we have noted, this first-order valuation approach is sufficiently accurate to analyze the investment, except for the cases in which the size of the the investment is large relative to the dealer's entire balance sheet. To this end, we compute the first-order valuation effects of the aggregate positions and the legacy swap with the client. The difference between the two is the first-order valuation of the new swap investment.

A Market Value

As explained by Mengle (2010), standard ISDA agreements specify close-out netting at default of either counterparty. We let B denote the client's default event, which is assumed to be independent under P^* of the floating-side swap payment X. By direct analogy with calculations in Appendix B, the marginal market value of the new swap is well defined by

$$V = -u + \delta \left(E^*(X - K) + E^* \left[\phi \left((X - K + c_0)^- - c_0^- \right) \right] - E^* \left[\gamma \left((X - K + c_0)^+ - c_0^+ \right) \right] \right), \tag{40}$$

and V is invariant to whether the dealer finances the swap by issuing debt, issuing equity, or using existing cash on its balance sheet. That is, $\delta E^*[\gamma((X - K + c_0)^+ - c_0^+)]$ and $\delta E^*[\phi((X - K + c_0)^- - c_0^-)]$ are the incremental CVA and DVA due of the new swap position, respectively.

B Shareholder Value

We focus on the case in which the dealer finances swap positions by issuing new debt. From Proposition 1, the first-order valuation effect to shareholders of the swap portfolio is

$$G_a = \delta E^*[\mathbf{1}_{D^c}(X - K + c_0)] - \delta E^*[\mathbf{1}_{D^c}(u_0 + u)(R + S)] - \delta E^*[\mathbf{1}_{D^c}\gamma(X - K + c_0)^+].$$

Similarly, the first-order valuation effect of the legacy swap to shareholders is

$$G_0 = \delta E^*(\mathbf{1}_{D^c}c_0) - \delta E^*[\mathbf{1}_{D^c}u_0(R + S)] - \delta E^*(\mathbf{1}_{D^c}\gamma c_0^+).$$

Thus, the marginal value of the new swap to the shareholders is

$$G = G_a - G_0 = \delta E^*[\mathbf{1}_{D^c}(X - K)] - \delta E^*[\mathbf{1}_{D^c}u(R + S)] - \delta E^*[\mathbf{1}_{D^c}\gamma((X - K + c_0)^+ - c_0^+)].$$

C Legacy Creditor Value

We also consider the marginal value of the new swap to the dealer's existing creditors (excluding the swap client). To this end, we characterize the first-order effect of the legacy swap, and we characterize the first-order effect of the swap portfolio. Thus, the marginal value of the new swap to the dealer's legacy creditors is

$$\begin{aligned} H = {} & \delta E^*[\mathbf{1}_D(X - K)] - \delta E^*(\mathbf{1}_D uR) + \delta E^*(\mathbf{1}_{D^c}uS) + \delta E^*[\phi((X - K + c_0)^- - c_0^-)] \\ & - \delta E^*[\gamma \mathbf{1}_D((X - K + c_0)^+ - c_0^+)] - \delta(1 - \kappa)J, \end{aligned}$$

where

$$J = \lim_{q \to 0} E^* \left(\frac{\mathcal{A}(q)\mathbf{1}_{\mathcal{D}(q)} - \mathcal{A}_0(q)\mathbf{1}_{\mathcal{D}_0(q)}}{q} \right),$$

and J is well defined by the same argument used in Appendix B.

In the special case of no distress costs ($\kappa = 1$), we have $V = G + H$.