Fragmenting Financial Markets

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Drawing from work with Samuel Antill, Daniel Chen, and Haoxiang Zhu

INFORMS
Markov Lecture

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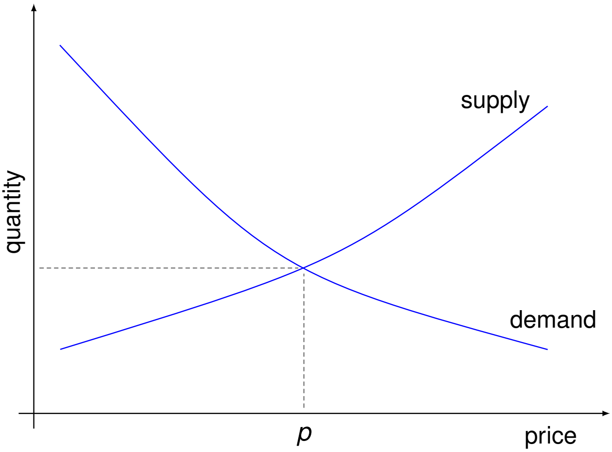
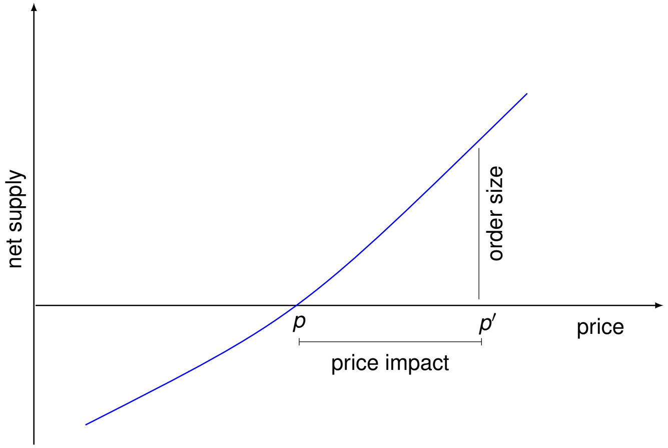
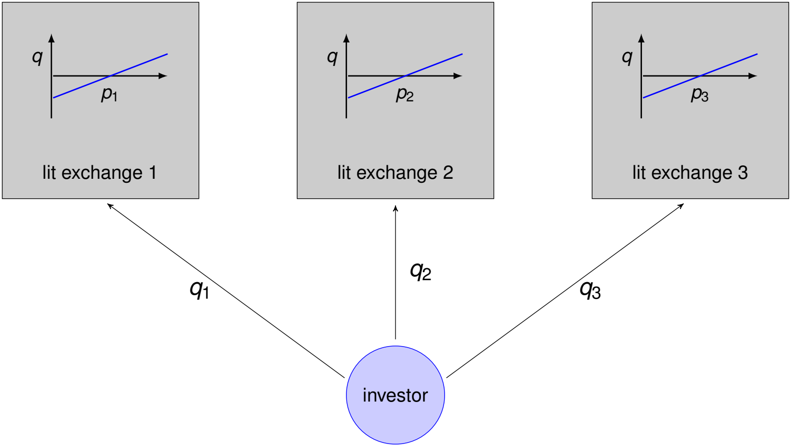
Price determination in an order book market



Price impact

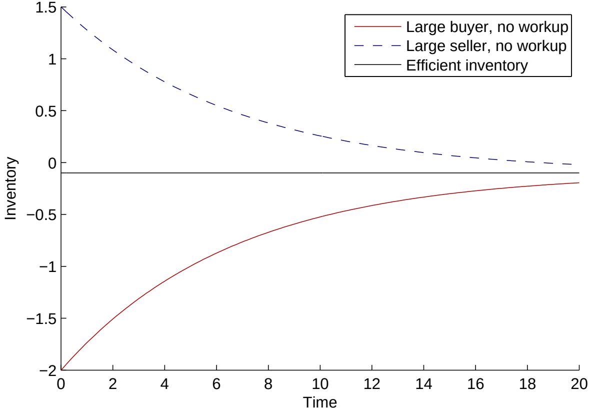


Reducing total price impact by order splitting



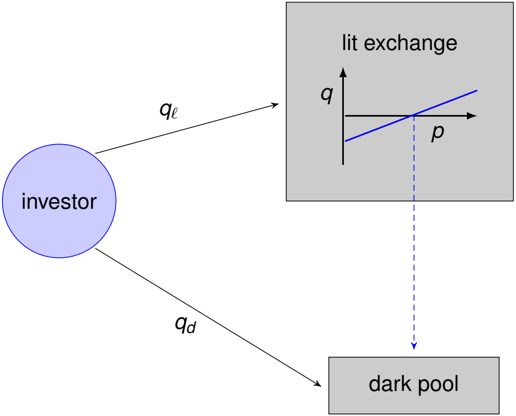
Conditions for fragmentation of trade across exchanges to improve market efficiency are provided in work with Daniel Chen, *American Economic Review*, 2021.

Dynamic avoidance of price impact by order shredding



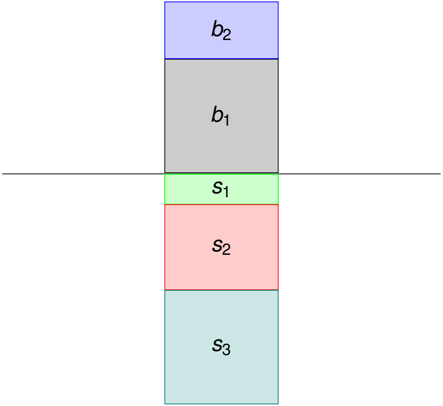
Vayanos (1999), Rostek and Weretka (2015), Du and Zhu (2017), Duffie and Zhu (2017).

A dark pool gets its price from the lit exchange

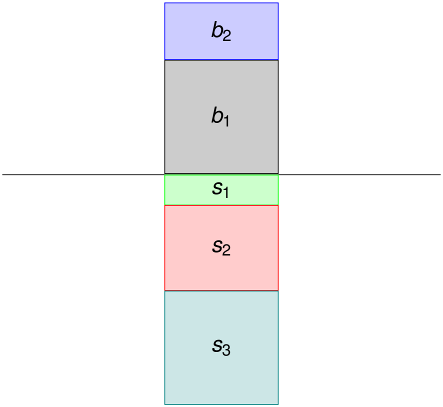


(Haoxiang Zhu, *Review of Financial Studies*, 2014.)

A dark pool is a form of size-discovery venue

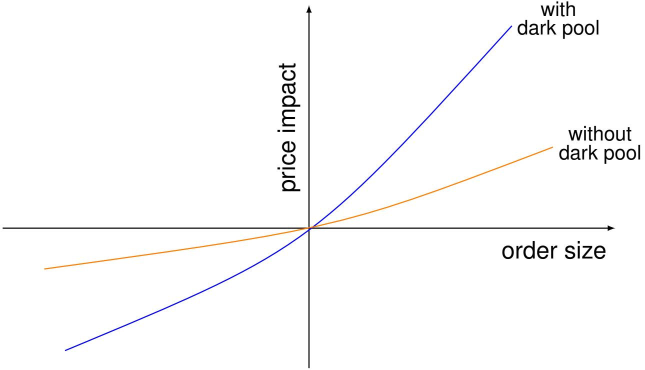


Size discovery by order workup at a frozen price



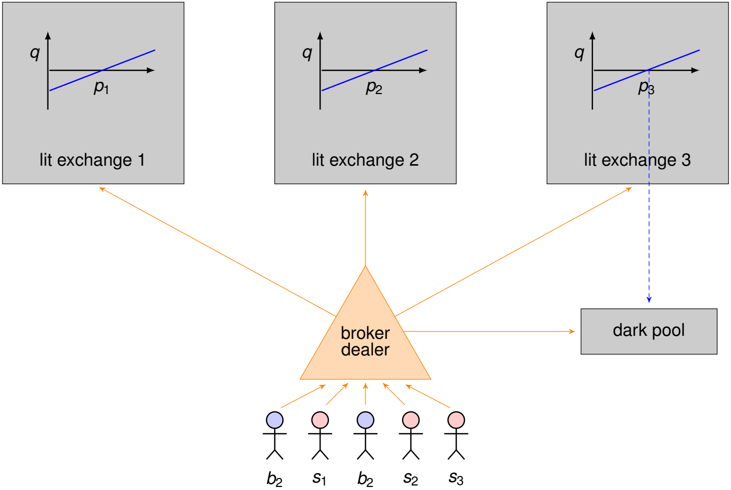
(with Haoxiang Zhu, *Review of Financial Studies*, 2017).

Size discovery reduces market depth at lit exchanges



(with Samuel Antill, *Review of Economic Studies*, 2021.)

Strategic order routing and market design



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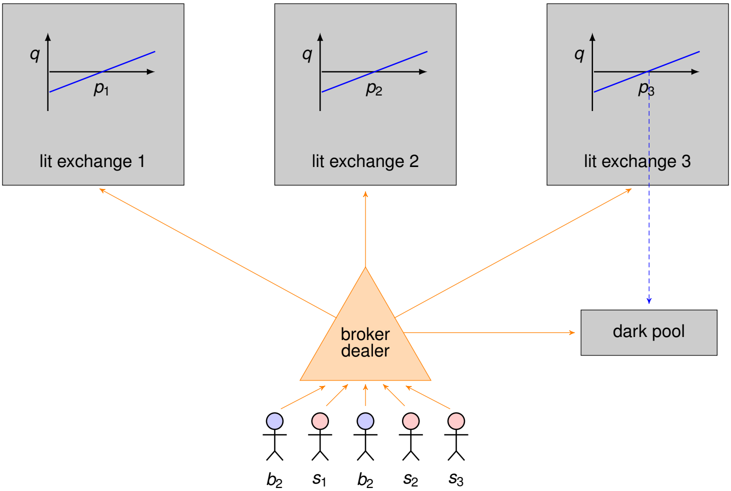
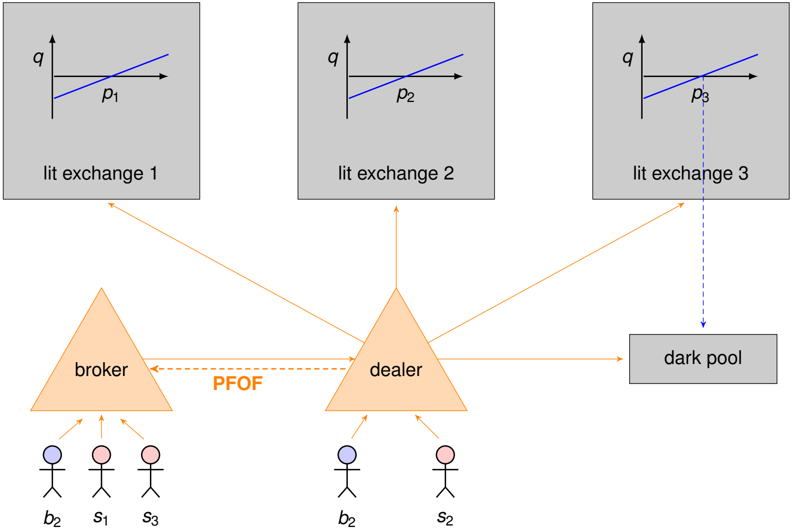


U.S. equities are traded on 16 exchanges and 33 alternative trading systems such as dark pools. Blocks often trade over the counter with broker-dealers. (SIFMA, 2021)

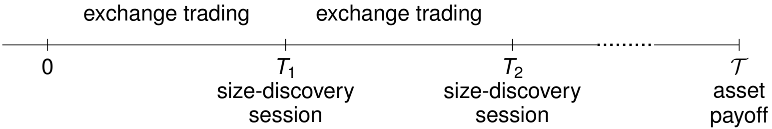
Internal crossing and payment for order flow



How size discovery degrades market efficiency

From work with Samuel Antill

model timeline



Trader inventory costs

- Fix a probability space. Each trader's inventory is private information.
- $n \geq 3$ traders with initial excess inventories $z_{10}, \ldots, z_{n0}$.
- Cumulative inventory shocks of trader i: a zero-mean Lévy process H_i.
- At time $\mathcal{T} \sim exp(r)$, the asset pays v, independent.
- The Almgren-Chriss holding cost for inventory process z is $\gamma \int_0^{\mathcal{T}} z_t^2 \, dt$.
- Without trade, the total value to trader i is therefore

$$E\left(vz_{i\mathcal{T}} - \gamma \int_0^{\mathcal{T}} z_{it}^2 \, dt\right),$$

where $z_{it} = z_{i0} + H_{it}$.

The exchange: A dynamic double-auction market

- By submitting a demand schedule $\mathcal{D}_{it}(\,\cdot\,)$ (e.g., with a package of limit orders), if price p clears the market in state ω at time t, then trader i is allocated the asset at the quantity rate $\mathcal{D}_{it}(\omega, p)$.

- For a given price process ϕ, the total payment by trader i in state ω is thus
$$\int_0^{\mathcal{T}} \phi_t(\omega)\mathcal{D}_{it}(\omega, \phi_t(\omega))\, dt.$$

- Without size discovery, the inventory process of trader i is
$$z_{it} = z_{i0} + \int_0^t \mathcal{D}_{it}(\phi_s)\, ds + H_{it}.$$

Size-discovery sessions

- When a session opens, trader i submits an inventory report $\hat{z}_i$.
- Given the latest exchange price p and the inventory reports $\hat{z} = (\hat{z}_1, \ldots, \hat{z}_n)$, trader i receives cash transfer $T_i(\hat{z}, p)$ and asset transfer $Y_i(\hat{z}, p)$.
- Taking the observed price p and the reporting strategies of other traders as given, trader i solves the reporting problem

$$\sup_{\mu} \ \mathbb{E}\left[V_i(z_{it} + Y_i((\mu, \hat{z}^{-i})), p) + T_i((\mu, \hat{z}^{-i}), p) \,|\, \mathcal{F}_{it}, \right],$$

where $V_i(z, p)$ is the continuation value of trader i.

Size-discovery design

- Size-discovery sessions are held at the event times of a Poisson process N with mean arrival rate λ.
- We focus on size-discovery designs (T, Y) that rebalance inventories perfectly at each session.
- Among several such demonstrated examples is a standard proportional-rationing dark-pool.

Exchange trading augmented with size discovery

- Trader i chooses a strategy $\eta = (\mathcal{D}_i, \hat{z}_i)$ generating the excess inventory process

$$z_{it}^{\eta} = z_{i0} + \underbrace{\int_0^t \mathcal{D}_i(\phi_s^{\eta})\, ds}_{\text{exchange trade}} + \underbrace{\int_0^t Y_i(\hat{z}_s, \phi_s^{\eta})\, dN_s}_{\text{size-discovery trade}} + H_{it}.$$

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- The stochastic control problem of trader i, given other traders' strategies, is

$$\sup_{\eta} \; E\left[z_{i\mathcal{T}}^{\eta} v - \int_0^{\mathcal{T}} \phi_t^{\eta} \mathcal{D}_i(\phi_t^{\eta})\, dt + \int_0^{\mathcal{T}} T_i(\hat{z}_t, \phi_t^{\eta})\, dN_t - \gamma \int_i^{\mathcal{T}} (z_{it}^{\eta})^2\, dt \right].$$

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- Solving the HJB equation (with verification) allows explicit calculation of equilibrium strategies.

- Equilibrium: market clearing, consistent conjectures, and agent optimality (including rational participation).

Size-discovery reduces allocative efficiency

The key intuition:

Waiting for the low price impact of size-discovery sessions reduces market depth and delays the efficient matching of buyers and sellers.

Proposition

1. Above a stated mean frequency $\bar{\lambda}$ of size-discovery sessions, exchange trading breaks down.
2. For any $\lambda < \bar{\lambda}$, there are 2 linear equilibria. For the more efficient equilibria, *every* trader's value is strictly decreasing in λ.

Policy-related observations

- With competing platform operators, entry of a size-discovery platform is profitable but (in our model) socially harmful.
- As size-discovery sessions become more frequent, exchange volume and depth decline.
- If size discovery is available, traders will use it even though they are all better off (in our model) if it is banned.
- Scope for regulation. In Europe, MiFiD II caps dark-pool trading volume.
- The policy-relevant empirical evidence is limited to equities, and mixed. See: Buti, Rindi, and Werner (2011), DeGryse, De Jong, and Kervel (2015), Nimalendran and Ray (2014), Farley, Kelley, and Puckett (2017).

Pandemic dysfunctionality of U.S. Treasury markets

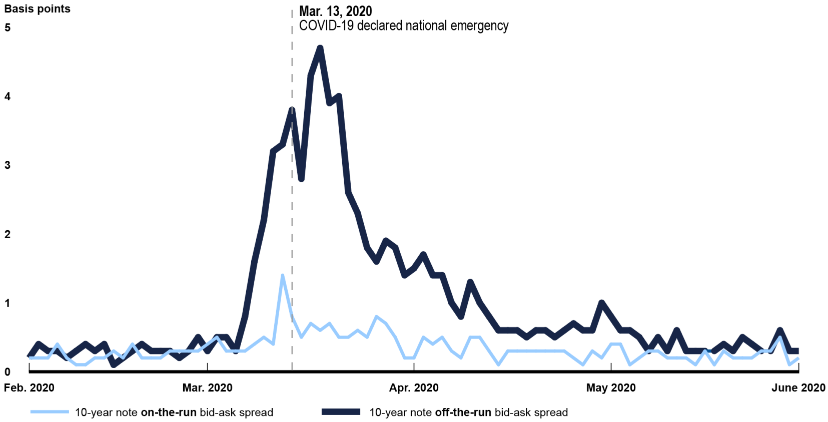

Figure: Source: Congressional General Accounting Office, August, 2021. The underlying data source is Bloomberg Financial LP. Bloomberg.

Typical two-tiered bond market structure