Notes on LIBOR Conversion

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This technical note discusses how to arrange for the conversion to new reference rates of legacy financial contracts whose settlement is linked to the London Interbank Offered Rate (LIBOR), which may be disappearing. This conversion depends in part on determining fair-market compensation for future differences between LIBOR and new reference rates. The same principles apply to the conversion of EURIBOR derivatives and potentially other contracts that settle on benchmarks which will eventually be replaced.

1 A Compelling Global Transition

LIBOR is no longer reliable, over the long term, as a benchmark for the settlement of interest rate contracts such as bonds, loans, mortgages, futures, and swaps. The main regulator of LIBOR, the UK Financial Conduct Authority, announced in 2017 that it will end its supervision of LIBOR at the end of 2021. Banks providing information that determines LIBOR each day will no longer be pressured by regulators to continue doing so. Banks are exposed to litigation and reputational risk when providing this information. It is far from assured that LIBOR will be available after 2021.

As 2022 approaches, the systemic risk associated with a large stock of remaining legacy LIBOR contracts could be reduced significantly by converting a large fraction of these contracts to new reference rates. On top of systemic risk concerns, many market participants will prefer to convert their

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contracts in order to better manage the costs and risk associated with a weakening or disappearance of LIBOR benchmarks. The expected cost of conversion to other benchmarks could become significant once the liquidity of LIBOR-based markets drops.

These notes discuss approaches to converting trillions of dollars of legacy LIBOR-linked financial contracts to a new reference rate. The particular new reference rate depends on the currency. For US dollar contracts, the new reference rate is the Secured Overnight Financing Rate (SOFR), an average of the transaction rates of overnight repo contracts collateralized by U.S. treasuries, across a broad spectrum of the treasury repo market. The selection of SOFR as the new benchmark reference rate, and the process of converting the market from LIBOR to SOFR, has been managed by the Alternative Reference Rate Committee (ARRC). The web site of the ARRC is maintained by the Federal Reserve Bank of New York, and provides updates and details. New overnight benchmark reference rates have also been chosen for other currencies that make heavy use of LIBOR. Eurozone authorities have yet to decide how to handle a replacement for EURIBOR, the euro analogue of LIBOR.

Because LIBOR is a significantly higher interest rate than SOFR, the receiver of LIBOR on a legacy contract will require compensation in order to agree to the conversion. The key design problem is how to discover a fair rate of compensation, maturity by maturity, and how to obtain the agreement of contract holders, both the payer and receiver. This note discusses an auction-and-protocol process.

2 Settlement of SOFR-based contracts

Most LIBOR-linked contracts have periodic interest payments that are based on one-month, three-month, or six-month LIBOR. For example, consider the case of market participants who currently make interest payments every three months. After the introduction of the new reference rate, many of these firms are likely to prefer to continue making payments every three months. However, SOFR is an overnight interest rate, as are the new reference rates in all other major currencies.

In order to use SOFR to settle floating-rate coupons at some tenor such as three months, one could in principle use the three-month overnight index swap (OIS) rate S associated with SOFR. For a term of 90 days, for example, an OIS contract commits the fixed-side counterparty to pay $(S-R)\times 90/360$,

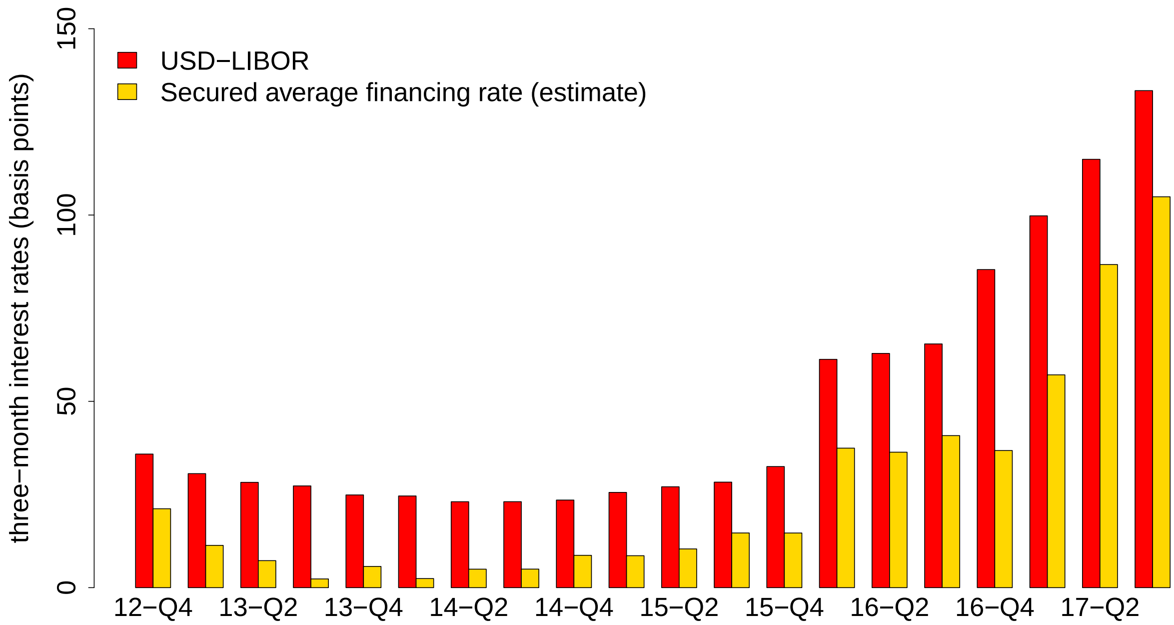

Figure 1: Three-month LIBOR and estimated three-month SAFR. For the purpose of estimating SAFR, SOFR is estimated as the Broad Treasury Financing Rate (BTFR), for dates on which BTFR is available from the web site of the Federal Reserve Bank of New York. For other dates, SOFR is estimated by ordinary least squares regression as -2 basis points $+$ 0.58 GCF treasury repo rate $+$ 0.42 BNYM treasury repo rate ($R^2 = 0.99$). Data for GCF and BNYM repo rates are from Bloomberg.

per dollar of notional position, where R is the compounded overnight rate, defined by

$$1 + R \times \frac{90}{360} = \left(1 + \frac{r_1}{360}\right) \times \left(1 + \frac{r_2}{360}\right) \times \cdots \times \left(1 + \frac{r_{90}}{360}\right),$$

where r_n is SOFR on the n-th of the 90 days during the contract period. Unfortunately, there may not be enough depth in the three-month OIS market to fix S robustly from OIS transactions. If the OIS market is too thin, the fixing of S could be contaminated by noise, or even manipulated. This same concern is likely to apply for other typical contract periods, such as one month and six months.

An alternative is to settle contracts based directly on the compounded rate R, which is known as the Secured Average Financing Rate (SAFR), even though R is not literally an average. While it seems natural to convert legacy LIBOR contracts to SAFR contracts, the rest of this note does not depend on a particular new floating-rate coupon settlement.

Figure 1 shows the behavior of three-month LIBOR and estimates of three-month SAFR over the period 2012-2017. Although SAFR is not scheduled to be published until mid-2018, reasonable estimates of what SAFR would have been in recent years can be obtained from similar market rates that were published during this period. On average, the difference between three-month LIBOR and three-month SAFR is estimated to have been about 0.23% (23 basis points), on average, with a sample standard deviation of about 9 basis points.

3 Overview of an auction-and-protocol approach

This note proposes conversion of legacy contracts by an auction-and-protocol procedure. The key problem is price discovery. The fair market compensation for receiving the new (lower) reference rate in lieu of LIBOR must be determined. A regular market for long-term derivatives on the new reference rate (or basis swaps) will probably not be sufficiently active or deep in time for reliance on that market to set the compensation rates to be used in conversion protocols. It would be risky to rely on thin markets for SOFR-based derivatives to convert a large volume of legacy LIBOR derivatives. Conversion compensation rates that would be based on the term structures of interest rates implied by SOFR derivatives would be noisy and exposed to manipulation.

A natural approach is to determine compensation by using a specially designed auction. In the first step, an auction among the clearing members of a central counterparty (CCP) for interest-rate derivatives, such as swaps or futures, would be based on bids and offers to convert centrally cleared derivatives from LIBOR to the specified new reference rate (NR), such as SAFR.

Figure 2 represents the structure of a swap market with participation by major dealers and certain other large firms as the clearing members of a central counterparty (CCP). Centrally cleared swaps are ideal for the purpose of conversion via an auction or protocol, because centrally cleared contracts of a given type are fungible across multiple counterparties, thus avoiding the need to obtain pair-wise conversion agreements.

3.1 The auction

In the most basic form of the auction, a bid is a pair (r, q) consisting of the maximum compensation rate r the bidder is willing pay (annualized over the life of the contract) to convert legacy pay-LIBOR contracts of up to the notional quantity q to a new contract that pay NR. An offer (r, q) likewise

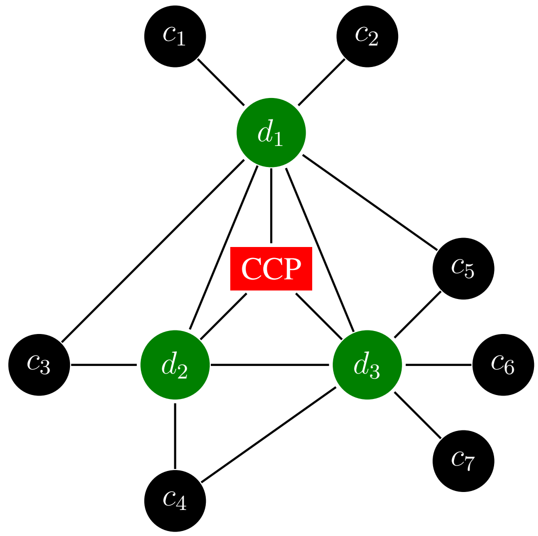

Figure 2: Schematic of a centrally cleared swap market. Dealers and other clearing members, marked in green, novate their trades to a central counterparty (CCP), which becomes the buyer to each seller and the seller to each buyer. Centrally cleared contracts of a given type with different clearing members are fungible, in that they are all obligations of the same type to the same counterparty, the CCP. This fungibility allows submissions of contracts from many long and short position holders to be netted, which is not possible with bilateral contracts.

consists of the minimum compensation rate r that would be accepted to convert up to q units (notional) of legacy receive-LIBOR contracts into new contracts that will receive the new rate, NR. If a bid or offer is accepted, any converted contract will pay, in lieu of LIBOR, interest payments at the rate NR $+ r^*$, where r^* is the rate at which the total quantity of bids to pay compensation at rates at or above r^* is equal to the quantity of offers to receive compensation at rates no higher than r^*. As usual, market clearing can be assured by pro-rata allocations of quantities bid at r^*.

Figure 3 illustrates the total demand and supply schedules that were realized in a hypothetical auction. For example, as illustrated, a clearing member with legacy swaps that receive LIBOR was willing to convert up to $q =$ \$19 billion notional if compensated at a rate of at least $r = 21$ basis points (bps) running. This clearing member will be awarded all \$19 billion and be compensated at the auction clearing rate $r^* = 30$ bps. Another clearing member with legacy swaps that pay LIBOR, was willing to convert up to $q =$ \$15 billion notional if required to pay in compensation no more than $r = 20$ basis points running. None of this bid was executed, because 20 bps $< r^*$.

An auction participant would be permitted to place as many different such (q, r) limit orders as desired. Many sorts of rules could be imposed on the auction design. For example, an auction participant could be permitted to place both buy and sell orders, subject to not expanding that bidder's

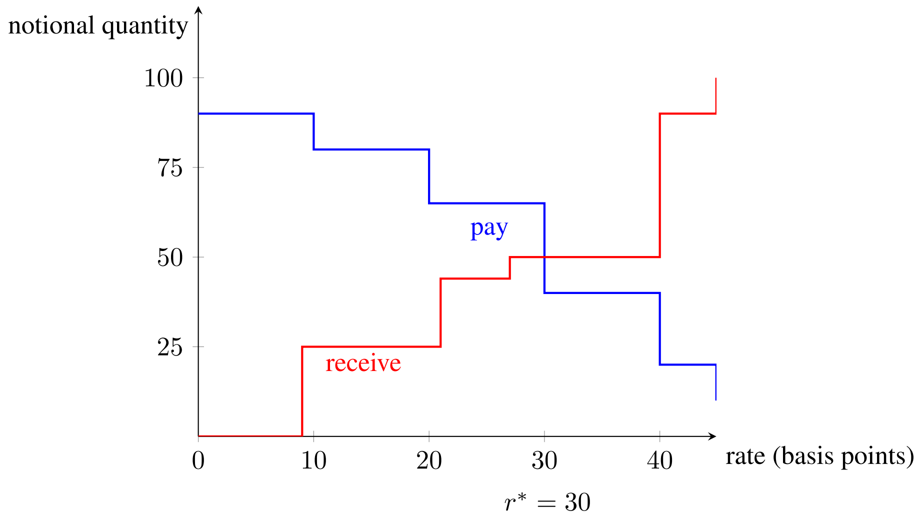

Figure 3: The total demand and supply schedules associated constructed from bids and offers. The market-clearing compensation rate is $r^* = 30$ basis points.

total absolute amount of LIBOR swaps.

Such an auction would be run simultaneously for each of a list of standard maturities. A later section of this note discusses the conversion of contracts at non-standard maturities, using a compression algorithm based on interpolation of rates between maturities.

3.2 Approaches to protocol conversion

In the second step of the procedure, additional contracts would be converted from LIBOR to NR, with compensation at the rate r^* determined by the auction, before considering any conversion fees. These additional protocols conversions could be arranged by one or more the following approaches:

A. By non-competitive bids. In advance of the auction, authorized market participants could submit centrally cleared positions of stipulated quantities, positive or negative, for conversion at whatever turns out to be the market-clearing compensation rate r^*. The market clearing compensation rate is then determined as that rate at which the net demand (including the total of non-competitive bids) is zero.

B. By bilateral agreement between the counterparties of derivatives and other financial instruments. These agreements could be binding advance contractual commitments to convert a given quantity of the specified instrument (including perhaps bonds and loans) at the compensation rate r^* to be determined in the auction. The alternative of arranging agreement after the auction has revealed the compensation rate r^* seems likely to produce a lower volume of of conversions. An advantage of this method is that it can be used whether or not the financial instrument involved is centrally cleared. Interpolation of compensation rates between standard auction maturities would be based on an agreed formula.

C. By submission of requests for conversion of centrally cleared derivatives by clearing members, and perhaps also by clients of clearing members whose derivatives have been centrally cleared on their behalf by a clearing member using the FCM approach. Because the total quantity of submitted legacy contracts that pay LIBOR would generally differ from the total quantity of submitted contracts that receive LIBOR, the "heavy-side" submissions would be rationed. A typical pro-rata rationing approach, such as that used by dark pools, could be adopted. Other rationing approaches include time prioritization (first come first served), or via fee bidding, with priority based on bids.

D. By some mechanism design to be determined. A specific mechanism design is discussed in the next section.

Protocol conversion fees could be paid to auction participants in order to encourage auction participation (deepening the auction market) and to discourage excessive free riding on auction price discovery. This is a significant aspect of the design problem.

An additional concern to be addressed is the potential for manipulation of the auction market clearing rate r^*, with the intent of giving up some potential gain (or even incurring an intentional loss) in the auction with the goal of strategically influencing the terms of compensation in the protocol step.

The overall design problem includes the timing of auctions and protocols, which could be staged periodically, for example once every three months between 2019 and the end of 2021.

Nothing yet rules out the superiority of designs that avoid a two-step auction-and-protocol approach.

4 Preliminary modeling approaches

Here, I explore some preliminary modeling approaches.[1] Suppose there are two groups of investors. Group A participates in both the auction and the conversion protocol. Group B participates only in the protocol. Suppose that investor i, whether in Group A or B, has a legacy contract position of size z_0^i, a finite-variance random variable that is observable by investor i and not necessarily by other investors, and which can be positive (receive LIBOR) or negative (pay LIBOR). The unconditional expected value to investor i for a total conversion cash payment of c and a final legacy contractual position of z is assumed to be of the form

$$E[c - f(z)],$$

where $z \mapsto f(z) > 0$ is a symmetric and strictly convex cost associated with holding an unconverted position of size z. For example, we could take $f(z) = \gamma z^2$, for a constant parameter $\gamma > 0$.

For instance, if investor i converts the quantity q_1 (which can be positive or negative) in the auction at a price of p, and converts quantity q_2 in the protocol with a total cash compensation (net of fees) of c, then the final value of investor i is

$$f(z_0^i - q_1 - q_2) + pq_1 + c.$$

Assuming that cash payments must be funded entirely by participating investors, it is socially efficient to minimize $E[\sum_i f(z_i)]$, where z_i is the final position of investor i. By strict convexity of $f(\,\cdot\,)$, the unique first-best allocation has $z_i = \bar{Z}$ for all i, where $\bar{Z} = \sum_i z_0^i / n$.

A naïve protocol design, method C above, would set $c = pq_2$, with pro-rata rationing of positions submitted for conversion. (Whether positions are submitted before or after the auction, or by some contingent formula, is a design issue.) This assumes there is actually an equilibrium in the two-stage auction-and-protocol game, which is a key issue.

An alternative protocol design is the mechanism proposed by Antill and Duffie (2017), by which investor i reports a desired quantity $\hat{z}^i$ to convert, after the auction determines a price p. The list $\hat{z} = (\hat{z}^1, \ldots, \hat{z}^n)$ of reports of the n investors participating in the protocol then generates a conversion

[1] I fix a probability space $(\Omega, \mathcal{F}, P)$. The information held by investor i is represented by a sub-σ-algebra $\mathcal{F}_i$ of $\mathcal{F}$.

for investor i of the quantity

$$Y^i(\hat{z}) = \hat{z}^i - \frac{\sum_{j=1}^n \hat{z}^j}{n}, \tag{1}$$

along with cash compensation of

$$T^i(\hat{z}, p) = \kappa_0 \left(-n\eta p + \sum_{j=1}^{n} \hat{z}^j \right)^2 + \alpha p(\hat{z}^i - \eta p) + \frac{\alpha^2 p^2}{4\kappa_0 n^2} \tag{2}$$

where $\kappa_0 < 0$, α, and η are constants.

Aside from efficiency concerns, a key issue is whether there is even an equilibrium in the game determined by the auction-and-protocol process. In fact, the results of Antill and Duffie (2017) raise concerns about existence of equilibrium, because of the incentives of auction bidders to strategically avoid price impact in the auction, and to manipulate the auction price p in order to improve their terms of trade in the protocol. One might address these concerns by including protocol conversion fees, which discourage excessive reliance on the protocol. The protocol fees could be paid to auction participants, based on some sharing formula that would increase the incentives for contributing to a robust auction price.

From this point, we consider the quadratic-cost case $f(z) = \gamma z^2$. We take as given an auction price p that is observable before the protocol game is played. That is, for now, we ignore the issue of existence of equilibrium in the two-stage auction-and-protocol game, and consider only the second-stage protocol game, with given investor positions $z_1, \dots, z_n$ at the beginning of the protocol stage game. Equilibrium in the two-stage game, auction and protocol, is put off for later consideration.

As shown by Antill and Duffie (2017), the protocol game determined by (Y, T) is a direct-revelation mechanism. Moreover, we can choose the constants α and η so that for any on-the-equilibrium-path auction price p, participation is individually rational[2] and budget feasible, in that $\sum_i T^i(\hat{z}, p) \leq 0$ for all $\hat{z}$. Finally, for an appropriate κ_0, truth telling is a dominant strategy.

Beginning the protocol stage with a position of z_i, and given the previously determined price p, the final value to investor i is

$$V(z_i; \hat{z}, p) = \gamma(z_i - Y^i(\hat{z}))^2 + T^i(\hat{z}, p). \tag{3}$$

[2]Individual rationality is not assured in the protocol stage for off-equilibrium-path auction prices.

In the direct-revelation equilibrium for the protocol mechanism, taking the price p as given, the equilibrium value of investor i is

$$V(z_i; \hat{z}, p) = U(z_i, p, \bar{Z}) \equiv \alpha + \beta_0 z_i p + \beta_1 (z_i - \bar{Z})^2 + \beta_2 p Z + \beta_3 p^2, \tag{4}$$

for some constants α, and $\beta_0, \beta_1, \beta_2, \beta_3$ that do not depend on the investor.

Now we can consider the preceding auction stage of the game, in which an investor i in Group A submits a demand function $\mathcal{D}^i : \Omega \times \mathbb{R} \to \mathbb{R}$, implying the purchase of the quantity $\mathcal{D}^i(\omega, p)$ in state ω if the auction price p is chosen. In practice, any piece-wise continuous demand function can be well approximated by a combination of limit orders of the form considered in the previous section.[3]

Here, bidding is in the form of demand schedules denominated in the price p as an up-front payment, per unit of the notional position that is converted. Bids could be equivalently be formulated in terms of annualized compensation rates that are paid in periodic installments until final maturity, as in described the previous section.[4]

We only consider equilibria in which demand functions are of the affine form

$$\mathcal{D}^i(\omega, p) = a + bp + c z_0^i(\omega), \tag{5}$$

for constants a, $b < 0$, and c, to be determined, that do not depend on i. Although investors are not restricted to affine demand functions, they optimally choose affine demand functions if they assume that other investors do so.

Given the demand-function coefficients (a, b, c), a price p is chosen by a trade platform operator to clear the market.

Lemma 1. *Fix any demand-function coefficients (a, b, c) with $b < 0$, and some trader i. For any candidate demand $d \in \mathbb{R}$ by trader i, there is a unique price p with $d + \sum_{j \neq i}(a + bp + c z_t^j) = 0$. This clearing price is calculated as*

$$p = \Phi_{(a,b,c)}(d; Z^{-i}) \equiv \frac{-1}{b(n-1)} \left(d + (n-1)a + c Z^{-i} \right), \tag{6}$$

[3]This is just the Stone-Weierstrass Theorem.

[4]Given time-value discount factors $d_1, \ldots, d_T$ for the T payment dates associated with a given financial contract, a bid or offered of compensation rate r is equivalent to the bid or offered conversion price $p = r \sum_{t=1}^{T} d_t \Delta_t$, where Δ_t is the length of the t-th contract period. Default risk is ignored, given the typically high credit quality of CCPs.

where $Z^{-i} = \sum_{j \neq i} z_0^j$.

Thus, for any non-degenerate affine demand function used by $n-1$ of the traders, there is a unique market clearing price for each quantity chosen by the remaining trader. This approach, generalized in Klemperer and Meyer (1989), allows any given trader i to simplify his or her strategic bidding problem to the selection of a demand D^i, which then determines the market clearing price $\Phi_{(a,b,c)}(D^i; Z^{-i})$. A demand D^i is optimal for trader i given the demand coefficients (a, b, c) of the other traders if D^i solves

$$\sup_{D \in \mathcal{A}^i} E\left[U\left(z_{i0} + D, \Phi_{(a,b,c)}(D; Z^{-i}), \bar{Z}\right) - \Phi_{(a,b,c)}\left(D; Z^{-i}\right) D \;\middle|\; \mathcal{F}_0^i \right], \tag{7}$$

where $\mathcal{F}_0^i$ is the initial information set of investor i and $\mathcal{A}^i$ is the set of finite-variance random variables that are measurable with respect to $\mathcal{F}_0^i$.

Demand coefficients (a, b, c) with $b < 0$ are said to constitute a symmetric affine equilibrium if, for any trader i, given (a, b, c), the demand $D^i = a + bp + cz_0^i$ is optimal, where

$$p = \frac{1}{-b}(a + c\overline{Z}),$$

which is the associated equilibrium price.

Taking as given the indirect utility U from (4), this definition of equilibrium in the first-stage auction game implies market clearing, individual trader optimality given the assumed demand functions of other traders, and consistent conjectures about the demand functions used by other traders.

The existence of equilibria in the two-stage game is far from obvious. The main concern, which can be intuited from the more complicated setting of Antill and Duffie (2017), is that auction participants anticipate that they will be able to obtain conversion at the same price, but with *reduced price impact of their own bids*, in the second-stage protocol. So, they have a low incentive to bid aggressively in the auction, and could even wish to manipulate the auction price so as to advantage themselves in the protocol. In fact, unreported and very preliminary calculations by Sam Antill show that there appears to be a degenerate sort of equilibrium. In this symmetric equilibrium, auction participants are indifferent the sizes of their bids! Moreover, although the total of the cash payments made by participants in the second-stage protocol mechanism is never negative, this total can be strictly positive with positive probability.

These clues suggest some need for modifications to the rules of two-stage conversion design.

Perhaps protocol fees should be re-designed and paid (by some formula to be determined) to auction participants, in order to obtain more aggressive (non-degenerate) bidding in the auction. Perhaps the game should also be modified by obtaining reports and assessing protocol fees *before* the auction. This would simplify the bidding, because it would make observable to auction participants the pool of cash fees that are available to pay winning bidders.

Clearly, a significant amount of design work remains to be done.

5 Compression-conversion auctions

Running separate auctions and protocols for each maturity-class of swaps is not efficient, given the associated loss in potential netting of long and short positions across different maturities that are submitted for conversion. A CCP could adapt an existing sort of cross-maturity compression algorithm to obtain significant increases in conversion volumes. This could be done at both the auction and protocol stages.

A hypothetical illustrative example of a limit order for a buyer in a compression auction could be of the following form.

I am willing to pay a duration-weighted compensation rate of up to 24 basis points for conversion of up to $50 billion total notional of the legacy pay-LIBOR swaps listed on my bid form (which have actual maturities between 9 and 11 years). My post-conversion swaps could have final maturities that differ from those of converted legacy swaps, subject to compensation for any change in effective portfolio duration of my combined unconverted-plus-converted swap position of at least 2.5 basis points per year per unit notional. The post-conversion duration of my combined positions (unconverted submissions and converted submissions) must remain within 0.25 years of the pre-conversion duration.

One could in principle also allow bids to include some specified minimum compensation for changes in position convexity (weighted squared maturity), with limits on changes in convexity.

Swaps at the same CCP that are to be converted by protocol could also be included in the same compression step, subject to risk tolerances specified in the protocol submissions of the type described in the above illustrative bid. This further increases conversion efficiency.

Given all of the bids and protocol submissions, the CCP (or its contractor) would then solve a convex optimization problem in order choose the conversion compensation rates $r_1, \ldots, r_k$ at respective standard maturities $t_1, \ldots, t_k$, and an allocation of legacy and new-rate swaps to the respective bidders. The objective of this optimization problem could be:

Maximize the total notional volume converted, subject to (i) compatibility with respect to bid constraints, (ii) market clearing in both legacy and new swaps, and (iii) cash budget balancing.

Cash balancing in future settlement payments could be assisted by allowing upfront payments. This approach to compression designs for the two-stage auction-and-protocol conversion is obviously very preliminary and incomplete. Compression and auction experts should get involved here.

Appendix

Tables 1 and 2 show, respectively, estimated averages differences between various three month rates, over the period 2012-2017. SOFR is estimated as BTFR (FRBNY) when available, and otherwise by regression as -2 bps $+$ 0.58 GCF treasury repo rate $+$ 0.42 BNYM tri-party treasury repo rate ($R^2 = 0.99$). Estimates of SAFR are obtained by compounding the estimated SOFR, daily for 3 months. Compounded FF is the federal funds rate, compounded daily for 3 months.

Table 1: Sample mean differences, in basis points, in various three-month rates (2012 Q4-2017 Q3). Data sources: Bloomberg, Bank of New York-Mellon, and Federal Reserve Bank of New York.

	LIBOR	OIS	Compounded FF	SAFR
LIBOR	0	19.24	19.61	23.02
OIS	-19.24	0	0.37	3.77
Compounded FF	-19.61	-0.37	0	3.41
SAFR	-23.02	-3.77	-3.41	0

Table 2: Sample standard deviations, in basis points, of differences in various three-month rates (2012 Q4-2017 Q3). Data sources: Bloomberg, Bank of New York-Mellon, and Federal Reserve Bank of New York.

	LIBOR	OIS	Compounded FF	SAFR
LIBOR	0	7.75	6.83	8.96
OIS	7.75	0	1.99	4.89
Compounded FF	6.83	1.99	0	4.29
SAFR	8.96	4.89	4.29	0

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