# Online Appendix of “Benchmarks in Search Markets”

**Darrell Duffie, Piotr Dworczak, and Haoxiang Zhu**

## C Additional Results on Matching Efficiency

This appendix presents results that extend those of Section 4, by *(i)* considering cases not covered in the main body of the paper, and *(ii)* obtaining sharper predictions about matching efficiency when only two dealers are present in the market.

### C.1 Additional analysis of matching efficiency for the case $(1-\hat{\alpha})\gamma\Delta < s < (1-\alpha(1,0))\gamma\Delta$

This subsection extends our analysis of matching efficiency. We maintain the assumptions of Section 4.

We begin with a result concerning partial matching efficiency under the benchmark.

**Proposition 10.** *Suppose that $(1-\hat{\alpha})\gamma\Delta < s < (1-\alpha(1,0))\gamma\Delta$. Then the equilibrium in the benchmark case has the following properties.*

1. *Slow traders enter with probability one.*
2. *High-cost dealers always quote the price $c+\Delta$, and low-cost dealers make offers in an interval whose upper limit is $c+\Delta$.*
3. *Slow traders set a reservation price of $r_c^\star = c+\Delta$. The price $r_c^\star$ is rejected by a slow trader with probability $\theta$, where $\theta \in (0,1)$ solves the equation $s = (1-\alpha(1,\theta))\gamma\Delta$. An offer strictly below $r_c^\star$ is accepted by a slow trader with probability one.*

*Proof.* This follows directly from the derivation in the proof of Proposition 6, found in Section B.1 (case 2.2.1 (b)). □

Under the parameter restrictions of the Proposition, the equilibrium resembles that of Proposition 7 for the case of $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$, but has an unexpected twist. Slow traders follow a reservation-price strategy with $r_c^\star = c+\Delta$, and high-cost dealers always offer to sell at $c+\Delta$. Upon seeing a price offer of $r_c^\star$, slow traders randomize between accepting and rejecting. The equilibrium rejection probability $\theta$ does not depend on $c$ and changes continuously from 1 to 0 as $s$ increases from $(1-\hat{\alpha})\gamma\Delta$ to $(1-\alpha(1,0))\gamma\Delta$. The unique reservation-price equilibrium involves non-trivial randomization at the reservation price. As a consequence, we get partial efficiency in the matching of slow traders to low-cost dealers.

If $s > (1-\alpha(1,0))\gamma\Delta$, then all dealers, including high-cost dealers, sell the asset in a benchmark-based equilibrium. Slow traders search only once on equilibrium path. (See the analysis in Section B.1 and Figure B.1.) In Appendix C.3, for the case of two dealers, we show that without the benchmark, when search costs are sufficiently large, the first-round reservation price is equal to $v$. Because slow traders search only once in this equilibrium, the construction can be immediately extended to the general case of $N$ dealers. It follows that the equilibrium without the benchmark will be exactly as inefficient as the equilibrium with the benchmark. Thus, a sufficiently low search cost is necessary to match slow traders to low-cost dealers, with or without the benchmark.

The equilibrium under the benchmark in the case $s < \kappa(1-\hat{\alpha})\gamma\Delta$ is described in Section B.1. Because it does not achieve the second best, Proposition 8 alone is insufficient to provide a welfare comparison to the no-benchmark case. Moreover, as shown in Section C.3, a reservation-price equilibrium fails to exist in the no-benchmark case if search costs are very low. Given the economic insignificance of the region $s < \kappa(1-\hat{\alpha})\gamma\Delta$ (as explained in Section C.2) and the intractability of non-reservation-price equilibria, we leave open the question of welfare comparison in this parameter range.

## C.2 Supporting analysis of the case $s < \kappa(1-\hat{\alpha})\gamma\Delta$

Here, we provide the supporting analysis of the case $s < \kappa(1-\hat{\alpha})\gamma\Delta$ in the context of Section 4. We show that a low-cost dealer's incentive to quote a high price (leading to higher-than-efficient search by slow traders in equilibrium) disappears as the number $N$ of dealers gets large, in the sense formalized in Lemma 9.

**Lemma 9.** *Letting $\bar{s}(N) = (1-\hat{\alpha})\gamma\Delta$ and $\underline{s}(N) = \kappa(1-\hat{\alpha})\gamma\Delta$, we have*

$$\lim_{N\to\infty} N\bar{s}(N) = \infty \text{ and } \lim_{N\to\infty} N\underline{s}(N) = 0,$$

*where the convergence to 0 in the second equation is exponentially fast.*

The quantity $Ns$ is the upper bound on the search costs incurred by a slow trader. If slow traders adopted the sub-optimal strategy of searching the entire market, we would get the fully efficient outcome of a centralized exchange, before considering the search costs. Thus, $(1-\mu)Ns$ is an upper bound on the potential welfare loss in our setting. Lemma 9 says that the case $s < \kappa(1-\hat{\alpha})\gamma\Delta$ in which case the benchmark fails to achieve the second best can be safely ignored for practical purposes, given that the (rough) upper bound of possible inefficiency goes to 0 exponentially fast[42] with $N$. On the other hand, the search-cost range $(\kappa(1-\hat{\alpha})\gamma\Delta, (1-\hat{\alpha})\gamma\Delta)$ is much more important, as the potential welfare gains or losses are unbounded in this region (if we allow $v$ to get large).

To prove the first claim, we show that $(1-\hat{\alpha})$ converges to zero (as $N \to \infty$) more slowly than $\log(N)/N$. (That $(1-\hat{\alpha})$ converges to 0 follows from Lebesgue Dominated Convergence Theorem.) We have

$$1-\hat{\alpha} = \int_0^1 \frac{a_N\Phi(z)}{1+a_N\Phi(z)}\,dz,$$

where

$$a_N = \frac{N\mu\left(1-(1-\gamma)^{N-1}\right)}{\frac{1-(1-\gamma)^N}{\gamma}(1-\mu) + N\mu(1-\gamma)^{N-1}}.$$

Clearly,

$$1-\hat{\alpha} \geq \int_{\Phi^{-1}(\frac{1}{N})}^1 \frac{a_N\Phi(z)}{1+a_N\Phi(z)}\,dz \geq \left(1-\Phi^{-1}(\frac{1}{N})\right)\frac{a_N}{N+a_N}.$$

[42] For example, if $\mu = \gamma = \frac{1}{2}$, $\kappa \approx 0.019$ for just $N = 10$, and $\kappa \approx 1.5 * 10^{-6}$ for $N = 25$.

The term $a_N/(N + a_N)$ has a finite and strictly positive limit. It is therefor enough to show that

$$\lim_{N\to\infty} \frac{N}{\log N}\left(1 - \Phi^{-1}(\frac{1}{N})\right) > 0.$$

Using equation (B.4) to invert $\Phi$, and applying d'Hospital rule a few times to simplify the expression, we obtain

$$\lim_{N\to\infty} \frac{N}{\log N}\left(1 - \Phi^{-1}(\frac{1}{N})\right) = \lim_{N\to\infty} \frac{N}{\log N}\left(\left(\frac{1}{N}\right)^{\frac{1}{N}} - 1\right) = \lim_{K\to\infty} K\left(e^{\frac{1}{K}} - 1\right) = 1.$$

To prove the second claim, recall that

$$N\kappa = \frac{N(1-\gamma)^{N-1}}{\mu(1-\gamma)^{N-1} + (1-\mu)\frac{1-(1-\gamma)^N}{N\gamma}} = \frac{1}{\frac{\mu}{N} + (1-\mu)\frac{1-(1-\gamma)^N}{N^2\gamma(1-\gamma)^{N-1}}}.$$

The above expression goes to 0 as quickly as $N^2(1-\gamma)^{N-1}$, that is, exponentially.

Clearly, the result is true in the generalized setting without assuming A.2.

## C.3 Equilibrium in the no-benchmark case with two dealers

In this subsection we explore the special case of two dealers ($N = 2$). Otherwise, we maintain the same assumptions as in Section 4. Proofs of the results of this subsection are provided in Section C.4.

If there are two dealers, traders update their beliefs at most once, so the continuation conditional expected payoff of a slow trader after rejecting the first dealer's offer is easy to calculate. Benabou and Gertner (1993) characterize reservation-price equilibria in their model with two dealers. Our model has a different cost structure for dealers; thus, their results, although fairly general, cannot be applied.

The following result is an analogue of Lemma 1 for the case of two dealers with idiosyncratic costs, $\epsilon_1$ and $\epsilon_2$, for supplying the asset.

**Lemma 10.** *With no benchmark and two dealers, if there exists a reservation-price equilibrium, then the first-round reservation price is either $\underline{c} + \Delta$ or $v$. If*

$$s < (1 - \alpha(1, 0))\gamma\Delta,$$

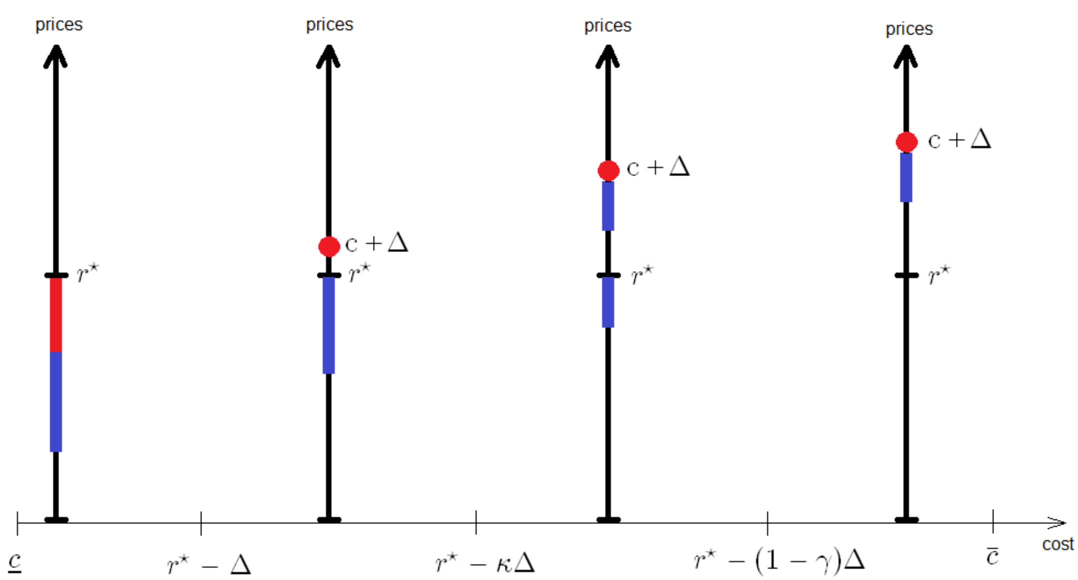
*then the first-round reservation price is $\underline{c} + \Delta$.*

We briefly characterize pricing strategies of dealers in the two equilibria. Remaining details can be found in the proof of the Lemma in Section C.4.1.

In the equilibrium with $r^\star = \underline{c} + \Delta$, because $c + \Delta$ exceeds $r^\star$, high-cost dealers cannot sell immediately when they are the first dealer contacted by a trader. As a consequence, it can be shown that they post a price offer equal to their cost $c + \Delta$ and make zero profits. Low-cost dealers always use a continuous distribution, but there are three regions for the outcome of the cost $c$ that lead to qualitatively different offer distributions, as illustrated in the three diagrams (counting from the right) in Figure C.1. When the cost $c$ is very low, all offers lie below $r^\star$. For the middle range of $c$, the support of prices consists of two intervals, one below $r^\star$, and one above $r^\star$. Thus, a low-cost dealer sells to a slow trader upon the trader's first contact if and only if an offer from the lower interval is drawn. Finally, when $c$ is high, all offers lie above $r^\star$. Conditional on this outcome of $c$, traders never buy from the dealer on their first contact.

In the equilibrium with $r^\star = v$, we can distinguish two qualitatively different regions of cost $c$. (See the two diagrams (counting from the left) in Figure C.1.) When $c < v - \Delta$, which is always the case under Assumption A.2, both high-cost and low-cost dealers make positive profits, and quote prices according to continuous distributions with adjacent supports lying below $r^\star$. (In the opposite case of $c > v - \Delta$, which does not satisfy Assumption A.2 but is nonetheless interesting, there are no gains from trade between traders and high-cost dealers. Low-cost dealers use a continuous distribution with upper limit $r^\star$.)

Fig. C.1: Supports of the distributions of prices (as a function of $c$)
(Low-cost dealers: blue. High-cost dealers: red. A dot denotes an atom.)




Although the proof of Lemma 10 assumes $N = 2$ dealers, the equilibrium with reservation price $r^\star = v$ is easy to characterize even if $N$ is arbitrary. Because each slow trader contacts at most one dealer in equilibrium, the only difference is that we need to correctly adjust the posterior probability of a fast trader. Using arguments along the lines of the proof of Proposition 3, we can also show existence of reservation-price equilibrium if search costs are sufficiently high.

In line with the objective of Section 4, which analyzes matching efficiency when search costs are low, in the remainder of this appendix we focus on the equilibrium with $r^\star = \underline{c} + \Delta$.

We let $\Psi(p; r^\star)$ denote the expected benefit of an additional search (that is, the expected benefit of visiting the second dealer), before considering the search cost, after observing a price offer of $p$, based on assumed reservation price $r^\star$. In the proofs of Lemma 10 and Proposition 11 we give a closed-form expression for $\Psi(p;\, r^\star)$ and show that $\Psi(p;\, r^\star)$ is continuous in $p$ on $[\underline{p}^l_{\underline{c}},\, r^\star)$ and on $(r^\star,\, v]$. Moreover, for $r^\star = \underline{c} + \Delta$, $\Psi(p;\, r^\star)$ jumps up at $p = r^\star$. This implies that the incentives to search adjust in the "correct" direction "locally" around $r^\star$. Define $\underline{s} = \sup\left\{\Psi(p;\, r^\star) : p \in [\underline{p}^l_{\underline{c}},\, r^\star)\right\}$ and $\bar{s} = \inf\{\Psi(p;\, r^\star) : p \in (r^\star,\, v]\}$. We have the following result.

**Proposition 11.** *Suppose that $s < (1 - \alpha(1,\, 0))\gamma\Delta$. Then, a reservation-price equilibrium in the no-benchmark case exists if and only if the search cost $s$ is in the interval $[\underline{s},\, \bar{s}]$. Moreover, $\bar{s} \le (1 - \alpha(1,\, 1))\gamma\Delta$.*[43]

While the constants $\underline{s}$ and $\bar{s}$ are directly computable, the associated analytic formulas are complicated. It can be shown, however, that the interval $[\underline{s},\, \bar{s}]$ is not empty if the distribution $G$ of costs is uniform and $\gamma$ is not too large. It is also easy to find examples (similar to an example considered by Janssen, Pichler, and Weidenholzer 2011) when the interval is empty.

Proposition 11, together with Lemma 10, implies that there is no reservation-price equilibrium when the search cost $s$ is in the interval

$$((1 - \alpha(1,\, 1))\gamma\Delta,\, (1 - \alpha(1,\, 0))\gamma\Delta)\,.$$

Moreover, and perhaps surprisingly, a reservation-price equilibrium fails to exist when the search cost $s$ is sufficiently small.

The above analysis implies the following Corollary.

**Corollary 1.** *Fix a search cost $s \ge \kappa(1 - \hat{\alpha})\gamma\Delta$, and suppose that in all equilibria (with and without the benchmark) there is full entry. If introducing the benchmark does not increase welfare, it must be the case that the equilibrium in the no-benchmark case is not a reservation-price equilibrium.*

*Proof.* This follows directly from Proposition 7, Lemma 10, and Proposition 11. □

The Corollary can be interpreted as saying that the no-benchmark setting cannot lead to a better matching efficiency than the benchmark setting if slow traders are using reservation-price strategies in equilibrium.

With two dealers and a search cost $s$ in the interval $[\underline{s},\, \bar{s}]$, the social cost of not having a benchmark does not arise from inefficient matching between slow traders and high-cost dealers in equilibrium. Rather, the inefficiency is caused by having slow traders engage in superfluous search. Indeed, unless the realization of the dealer's common cost $c$ is very small, low-cost dealers make offers above the reservation price $\underline{c} + \Delta$, and might not trade with the first low-cost dealers that they encounter. The following corollary (proved in Section C.4.3) expresses the welfare gain from introducing a benchmark by comparing it to the gain that would be achieved if the market were organized as a centralized exchange with no search costs (that is, if all traders had zero search cost).

[43] We interpret the interval $[a,\, b]$ as the empty set when $a > b$, and as the singleton $\{a\}$ when $a = b$. The proof that we provide also implies that for $s \le (1 - \alpha(1,\, 1))\gamma\Delta$ we cannot have a reservation-price equilibrium with $r^\star = \underline{c} + \Delta$ (even if we allow $s \ge (1 - \alpha(1,\, 0))\gamma\Delta$).

**Corollary 2.** *With $N = 2$ dealers, and for any search cost $s$ in the interval* $[\max\{\kappa(1-\hat{\alpha})\gamma\Delta,\, \underline{s}\}, \bar{s}]$, *introducing a benchmark eliminates at least a fraction*

$$\frac{(1-G(\underline{c}+\gamma\Delta))\,\gamma}{2-\gamma+(1-G(\underline{c}+\gamma\Delta))\,\gamma}$$

*of the total loss in social surplus that is induced by search frictions (relative to a setting with a centralized exchange).*

## C.4 Proofs for Section C.3

### C.4.1 Proof of Lemma 10

**Outline for the main steps of the proof.** The proof is long and tedious, unlike the proof of Lemma 1. We outline the main steps below. First, we characterize the equilibrium response of dealers as a function of the first-round reservation price $r^\star$. We obtain four regions of cost $c$ with different qualitative pricing strategy of dealers (see Figure C.1). When costs are low, both low-cost and high-cost dealers post price offers according to continuous distributions with adjacent supports below the reservation price $r^\star$ and make positive profits. When $c$ is in the lower-middle region, low-cost dealers continue to mix below $r^\star$, while high-cost dealers bid $c + \Delta > r^\star$ and make zero profits. In the upper-middle region, the support of the distribution of low-cost dealers consists of two disjoint intervals, $[\underline{p}^l_c,\, r^\star]$ and $[\hat{p}^l_c,\, c+\Delta)$, where $\hat{p}^l_c > r^\star$. Finally, when costs are highest, low-cost dealers bid exclusively in the upper interval that lies above $r^\star$. Second, we analyze the optimal search policy of slow traders. The proof proceeds by finding a contradiction when $r^\star \notin \{v,\, \underline{c}+\Delta\}$. Clearly, $r^\star$ cannot be larger than $v$. When $r^\star \in (\underline{c}+\Delta,\, v)$, we show that the posterior distribution of costs conditional on observing a price $p$ converges to an atom at $r^\star - \Delta$ as $p$ converges to $r^\star$. Because search behavior of traders is different on the two sides of $r^\star$, there is a discontinuity in expected price as $c$ crosses the level $r^\star - \Delta$ (the price distribution is impacted through the posterior probability of a fast trader). As a result, the benefits from search for a slow trader jump down discontinuously at $r^\star$, a contradiction. Finally, when $r^\star < \underline{c} + \Delta$, we can show that upon observing a price just above $r^\star$, a slow trader believes with probability one that this is (nearly) the best price that she can get, and thus wants to accept, a contradiction.

**The proof.** Let the reservation price (in the first round of search) be $r^\star$ and probability of entry be $\lambda^\star$.[44] We first characterize the equilibrium response of dealers. We focus on the case $c < v - \Delta$, as the other case requiring only minor modifications.[45] As the derivation of dealers' strategies is similar to the benchmark case, we skip some of the details. We also summarize the conclusions in Lemma 11 and Figure C.1 below for the convenience of the reader.

First, consider $c < r^\star - \Delta$. In that case high-cost dealers make positive profits and we

[44] When the benchmark is not present, there is no need to consider the parameter $\theta$ that was a relevant part of the strategy in the benchmark case.

[45] The main difference is that when $c > v - \Delta$, the upper limit of the distribution of prices for low-cost dealers will be $v$ in all cases when it was $c + \Delta$.

have a situation analogous to case 2.2.2 with the benchmark. The cdf $F_c^h(p)$ for high-cost dealers must satisfy

$$\left[1 - q(\lambda^\star,\, 0) + q(\lambda^\star,\, 0)(1-\gamma)(1 - F_c^h(p))\right](p - c - \Delta) = \left[1 - q(\lambda^\star,\, 0)\right](r^\star - c - \Delta).$$

Solving, we obtain

$$F_c^h(p) = 1 - \left(\frac{\lambda^\star(1-\mu)}{2\mu(1-\gamma)} \frac{r^\star - p}{p - c - \Delta}\right),$$

with upper limit $\bar{p}_c^h = r^\star$, and lower limit

$$\underline{p}_c^h = \frac{\lambda^\star(1-\mu)}{2\mu(1-\gamma) + \lambda^\star(1-\mu)} r^\star + \frac{2\mu(1-\gamma)}{2\mu(1-\gamma) + \lambda(1-\mu)}(c + \Delta).$$

Then, $F_c^l(p)$ must satisfy

$$\left[\lambda^\star(1-\mu) + 2\mu\left[(1-\gamma) + \left(1 - F_c^l(p)\right)\gamma\right]\right](p - c) = \left[\lambda(1-\mu) + 2\mu(1-\gamma)\right](\underline{p}_c^h - c).$$

Solving for $F_c^l(p)$ we get

$$F_c^l(p) = 1 - \frac{\lambda^\star(1-\mu) + 2\mu(1-\gamma)}{2\gamma\mu} \, \frac{\underline{p}_c^h - p}{p - c},$$

with upper limit $\bar{p}_c^l = \underline{p}_c^h$ and lower limit

$$\underline{p}_c^l = \frac{\lambda^\star(1-\mu) + 2\mu(1-\gamma)}{\lambda^\star(1-\mu) + 2\mu} \underline{p}_c^h + \frac{2\gamma\mu}{\lambda^\star(1-\mu) + 2\mu} c.$$

Second, consider the case $c > r^\star - \Delta$. Now high-cost dealers cannot make positive profits, so they post a deterministic offer price equal to $c + \Delta$. It is easy to show (using arguments familiar from previous derivations) that the upper limit of the distribution of prices for low-cost dealers must be either $r^\star$ or $c + \Delta$. Thus, the support of the distribution is either (i) an interval with upper limit $r^\star$, (ii) an interval above $r^\star$ with upper limit $c + \Delta$, or (iii) a sum of intervals from (i) and (ii). We analyze these possibilities below.

If low cost dealers bid on an interval with upper limit $r^\star$, then the distribution $F_c^l(p)$ must solve

$$\begin{aligned}\left[1 - q(\lambda^\star,\, 1) + q(\lambda^\star,\, 1)\left[(1-\gamma) + \left(1 - F_c^l(p)\right)\gamma\right]\right](p - c) \\ = \left[1 - q(\lambda^\star,\, 1) + q(\lambda^\star,\, 1)(1-\gamma)\right](r^\star - c),\end{aligned}$$

and is thus given by

$$F_c^l(p) = 1 - \frac{\lambda^\star(2-\gamma)(1-\mu) + 2\mu(1-\gamma)}{2\gamma\mu} \, \frac{r^\star - p}{p - c},$$

with upper limit $\bar{p}_c^l = r^\star$, and lower limit

$$\underline{p}_c^l = \frac{\lambda^\star(2-\gamma)(1-\mu) + 2\mu(1-\gamma)}{\lambda^\star(2-\gamma)(1-\mu) + 2\mu} r^\star + \frac{2\gamma\mu}{\lambda^\star(2-\gamma)(1-\mu) + 2\mu} c.$$

The profit of a low-cost dealer conditional on a contact is

$$\frac{\lambda^\star(2-\gamma)(1-\mu)+2\mu(1-\gamma)}{\lambda^\star(2-\gamma)(1-\mu)+2\mu}(r^\star - c).$$

To verify optimality of the pricing strategy, we need to check that a low-cost dealer cannot improve upon the above profit by bidding just below $c+\Delta$ (this is the most profitable deviation). The expected profit (conditional on a visit) under that deviation can get arbitrarily close to

$$\frac{2(1-\gamma)(\mu+\lambda^\star(1-\mu))}{\lambda^\star(2-\gamma)(1-\mu)+2\mu}\Delta.$$

Comparing the two expressions, we conclude that the above price distribution constitutes an equilibrium if and only if

$$c < r^\star - \frac{2(1-\gamma)(\mu+\lambda^\star(1-\mu))}{\lambda^\star(2-\gamma)(1-\mu)+2\mu(1-\gamma)}\Delta.$$

We will denote[46]

$$\kappa = \frac{2(1-\gamma)(\mu+\lambda^\star(1-\mu))}{\lambda^\star(2-\gamma)(1-\mu)+2\mu(1-\gamma)}.$$

Clearly, $r^\star - \kappa\Delta > r^\star - \Delta$.

When the cost $c$ is above $r^\star - \kappa\Delta$, we must have one of the cases (ii) or (iii). We explore the possibility of case (iii) below.

The two intervals in the support of the price distribution of low-cost dealers will be denoted by $[\underline{p}^l_c,\, r^\star]$ and $[\hat{p}^l_c,\, c+\Delta)$. Let $\zeta_c$ be the probability of bidding in the lower of the two intervals. Note that under such price distribution we need to adjust the posterior probability that a visiting trader is a slow trader. Moreover, the fact of being visited is informative of the price posted by a competing dealer.

If the support consists of two intervals, then the dealer must be indifferent between posting $r^\star$ and $c+\Delta-\epsilon$ (for $\epsilon \to 0$) which gives us the condition

$$\left[\mu(1-\gamma\zeta_c)+\lambda^\star(1-\mu)\left(\frac{1}{2}+\frac{1}{2}(1-\gamma\zeta_c)\right)\right](r^\star - c) = (1-\gamma)(\mu+\lambda^\star(1-\mu))\Delta.$$

Solving for $\zeta_c$ we obtain

$$\zeta_c = \frac{2\mu+2\lambda^\star(1-\mu)}{2\mu+\lambda^\star(1-\mu)}\left[1-\frac{1-\gamma}{\gamma}\frac{c+\Delta-r^\star}{r^\star-c}\right].$$

Because $\zeta_c$ must lie in $[0,\, 1]$, we can determine the maximal interval of costs for which the conjectured price distribution might arise in equilibrium. Simple calculation shows that this interval is $[r^\star - \kappa\Delta,\, r^\star - (1-\gamma)\Delta]$. (The interval is always non-empty because $\kappa > 1-\gamma$.)

Given the structure of the support, the upper part of the distribution $F^l_c(p)$ must satisfy

$$\left[1-\gamma+\gamma(1-F^l_c(p))\right](p-c) = (1-\gamma)\Delta,$$

[46] This coincides with the previous definition of $\kappa$ when $\lambda^\star = 1$.

for all $p \in [\hat{p}^l_c,\, c+\Delta)$, so that

$$F^l_c(p) = 1 - \frac{1-\gamma}{\gamma}\frac{c+\Delta-p}{p-c}.$$

To determine the cutoff $\hat{p}^l_c$, we use the fact that $F^l_c(\hat{p}^l_c) = \zeta_c$ to obtain the equation

$$1 - \frac{1-\gamma}{\gamma}\frac{c+\Delta-\hat{p}^l_c}{\hat{p}^l_c - c} = \zeta_c,$$

which gives

$$\hat{p}^l_c = c + \frac{1-\gamma}{1-\gamma+\gamma(1-\zeta_c)}\Delta.$$

Note that when $c = r^\star - \kappa\Delta$, so that $\zeta_c = 1$, we get $\hat{p}^l_c = c + \Delta = r^\star + (1-\kappa)\Delta$. On the other hand, when $c = r^\star - (1-\gamma)\Delta$, we have $\zeta_c = 0$, and $\hat{p}^l_c = r^\star$. As $c$ increases, the lower limit of the upper interval converges to $r^\star$ from above. This observation will be important in a later part of the proof.

To solve for the lower part of the distribution $F^l_c(p)$, we write down the indifference condition between bids in $[\underline{p}^l_c,\, r^\star]$:

$$[2(1-\gamma) + (2-\zeta_c)\lambda^\star(1-\mu)\gamma + 2\mu\gamma(1-F_l(p))]\,(p-c)$$

$$= [2\mu(1-\gamma\zeta_c) + (2-\zeta_c\gamma)\lambda^\star(1-\mu)]\,(r^\star - c).$$

This gives us

$$F^l_c(p) = \zeta_c - \frac{2\mu(1-\gamma\zeta_c) + \lambda^\star(1-\mu)(2-\zeta_c\gamma)}{2\mu\gamma}\;\frac{r^\star - p}{p-c},$$

with the lower limit

$$\underline{p}^l_c = \frac{(\lambda^\star(1-\mu)+2\mu)(1-\gamma)\Delta}{2\mu(r^\star-c)+\lambda^\star(1-\mu)(1-\gamma)\Delta}r^\star + \frac{2\mu(r^\star-c)-2\mu(1-\gamma)\Delta}{2\mu(r^\star-c)+\lambda^\star(1-\mu)(1-\gamma)\Delta}c.$$

Finally, we consider the case $c > r^\star - (1-\gamma)\Delta$. Since we have shown that neither of cases (i) and (iii) is possible, we explore case (ii), that is, we solve for the distribution $F^l_c(p)$ with support that lies above $r^\star$. The usual indifference condition is

$$\left[1-\gamma+\gamma(1-F^l_c(p))\right](p-c) = (1-\gamma)\Delta,$$

so we get

$$F^l_c(p) = 1 - \frac{1-\gamma}{\gamma}\;\frac{c+\Delta-p}{p-c},$$

with lower limit

$$\underline{p}^l_c = c + (1-\gamma)\Delta.$$

By comparing the profit of a low-cost dealer under this price distribution to the profit from a deviation to $r^\star$ (which is the most profitable deviation), we conclude that we have an equilibrium best response precisely when $c \geq r^\star - (1-\gamma)\Delta$.

This concludes the characterization of the best-response of dealers to a reservation-price strategy $r^\star$. We summarize the most important observations in the Lemma below. Figure C.1 depicts the qualitative features of the supports for different cost ranges.

**Lemma 11.** *The equilibrium response of dealers to slow traders playing a reservation-price strategy $(r^\star,\, \lambda^\star)$ is payoff-unique. When $c < r^\star - \Delta$, low-cost and high-cost dealers use continuous distributions of price offers with adjacent supports that lie below $r^\star$. When $c > r^\star - \Delta$, high-cost dealers offer $c+\Delta$, and low-cost dealers offer according to a continuous distribution. When $c \in (r^\star - \Delta,\, r^\star - \kappa\Delta)$ the support is $[\underline{p}^l_c,\, r^\star]$. For $c \in (r^\star - \kappa\Delta,\, r^\star - (1-\gamma)\Delta)$ the support is $[\underline{p}^l_c,\, r^\star] \cup [\hat{p}^l_c,\, c+\Delta]$. Finally, for $c > r^\star - (1-\gamma)\Delta$ the support is $[\underline{p}^l_c,\, c+\Delta]$ with $\underline{p}^l_c > r^\star$.*

Having determined the equilibrium pricing policy of dealers, we can turn to the analysis of the search behavior of slow traders. Our goal is to exclude the possibility that $r^\star \notin \{\underline{c}+\Delta,\, v\}$.

The posterior distribution of cost $c$ conditional on observing a price $p$ in the first search is

$$G(c \,|\, p) = \frac{\int_{\underline{c}}^{c} \left[\gamma f^l_y(p) + (1-\gamma) f^h_y(p)\right]\, dG(y)}{\int_{\underline{c}}^{\bar{c}} \left[\gamma f^l_y(p) + (1-\gamma) f^h_y(p)\right]\, dG(y)},$$

where $f^i_c(p)$ denotes the density corresponding to $F^i_c(p)$, for $i \in \{l,\, h\}$. Here, whenever $F^h_c(p)$ is a step function with jump at $c+\Delta$, $f^h_c(p) = \delta_{c+\Delta}(p)$, that is, we interpret the density as a dirac delta measure at $c+\Delta$. As in Section C.3, we define $\Psi(p;\, r^\star)$ to be the benefit from search after observing price $p$. The second argument $r^\star$ emphasizes that the whole function changes with $r^\star$ because the distribution of prices changes. Since there are only two dealers, we can calculate $\Psi(p;\, r^\star)$ explicitly:

$$\Psi(p;\, r^\star) = \int_{\underline{c}}^{\bar{c}} \left[\int_{\underline{p}^l_c}^{p} (v-p)\left[\gamma f^l_c(p) + (1-\gamma) f_h(p)\right] dp\right] dG(c \,|\, p)$$

$$-(v-p) \int_{\underline{c}}^{\bar{c}} \left[\gamma F^l_c(p) + (1-\gamma) F^h_c(p)\right]\, dG(c \,|\, p).$$

Because the function $\Psi(p;\, r^\star)$ does not need to be continuous in general, we can no longer without loss of generality use the condition that $\Psi(r^\star;\, r^\star) = s$, that is, the indifference of a slow trader between buying and searching at $p = r^\star$. Rather, a necessary and sufficient condition for a reservation-price strategy $r^\star$ to be optimal is that

$$\begin{aligned} s &\geq \Psi(p;\, r^\star), \quad p \leq r^\star, \\ s &\leq \Psi(p;\, r^\star), \quad p \geq r^\star. \end{aligned} \tag{C.1}$$

The strategy for the rest of the proof is to show that the condition (C.1) fails when $r^\star \notin \{\underline{c}+\Delta,\, v\}$.

Clearly, we cannot have $r^\star > v$. The lemma below deals with the case $r^\star \in (\underline{c}+\Delta,\, v)$.

**Lemma 12.** *When $r^\star \in (\underline{c}+\Delta,\, v)$, the posterior distribution of costs $G(\,\cdot \,|\, p)$ converges to an atom at $r^\star - \Delta$ as $p$ converges to $r^\star$.*

*Proof.* First, suppose that $p$ converges to $r^\star$ from the left. By the above derivation, the support of the posterior distribution must be contained in $[r^\star - \Delta, r^\star - (1-\gamma)\Delta]$ in the limit. We have

$$\lim_{p \to r^\star} G(c \,|\, p) = \lim_{p \to r^\star} \frac{\int_{\underline{c}}^{c} \left[\gamma f_y^l(p) + (1-\gamma) f_y^h(p)\right] \, dG(y)}{\int_{\underline{c}}^{\bar{c}} \left[\gamma f_y^l(p) + (1-\gamma) f_y^h(p)\right] \, dG(y)}. \tag{C.2}$$

Letting

$$\phi(p) = \frac{p - \phi(\lambda^\star) r^\star}{1 - \phi(\lambda^\star)} - \Delta$$

and

$$\phi(\lambda^\star) = \frac{\lambda^\star(1-\mu)}{2\mu(1-\gamma) + \lambda^\star(1-\mu)},$$

we note that the integral

$$\int_{\underline{c}}^{\bar{c}} f_y^h(p) \, dG(y) = \frac{\lambda^\star(1-\mu)}{2\mu(1-\gamma)} \int_{\underline{c}}^{\phi(p)} \frac{r^\star - y - \Delta}{(p - y - \Delta)^2} \, dG(y),$$

diverges to $\infty$ as $p \nearrow r^\star$. Thus, the numerator in expression (C.2) is going to $\infty$. For any $\epsilon > 0$, the integral

$$\int_{r^\star - \Delta + \epsilon}^{\bar{c}} \left[\gamma f_y^l(p) + (1-\gamma) f_y^h(p)\right] \, dG(y)$$

is finite. Therefore, $G(c \,|\, p) \to \mathbf{1}_{\{c \geq r^\star - \Delta\}}$.

The intuition for this result is simple: As $c$ gets closer to $r^\star - \Delta$, the distribution of prices for high-cost dealer gets "squeezed" on a very small interval below $r^\star$, and the density explodes. This is never the case for low-cost dealers. Thus, upon observing a price offer $p$ just below $r^\star$, a trader believes that it is much more likely that it has been posted by a high-cost dealer in which case the cost must be close to $p - \Delta$ (and exactly $r^\star - \Delta$ in the limit).

Second, consider the case when $p$ converges to $r^\star$ from the right. In this case, if the offer is posted by a high-cost dealer, then the cost must be equal to $p - \Delta$. The numerator in expression (C.2) is finite and bounded away from zero. Moreover, we have, for any $\epsilon > 0$,

$$\int_{\underline{c}}^{\bar{c}} f_y^l(p) \, dG(y) = \underbrace{\int_{r^\star - \Delta}^{r^\star - (1-\gamma)\Delta - \epsilon} f_y^l(p) \, dG(y)}_{I_1} + \underbrace{\int_{r^\star - (1-\gamma)\Delta - \epsilon}^{r^\star - (1-\gamma)\Delta + \epsilon} f_y^l(p) \, dG(y)}_{I_2} + \underbrace{\int_{r^\star - (1-\gamma)\Delta + \epsilon}^{\bar{c}} f_y^l(p) \, dG(y)}_{I_3}. \tag{C.3}$$

As $p \searrow r^\star$, integrals $I_1$ and $I_3$ become zero at some point, because $p$ falls out of the support of $f_c^l(p)$. And because $f_c^l(p)$ (as a function of $c$) is bounded in the neighborhood of $r^\star - (1-\gamma)\Delta$, integral $I_2$ can be made arbitrarily small. It follows once again that $G(c \,|\, p) \to \mathbf{1}_{\{c \geq r^\star - \Delta\}}$.

The intuition this time is a little more tricky. The key observation is that (i) prices just above $r^\star$ are in the support of the distribution of low-cost dealers only when $c$ is close to $r^\star - (1-\gamma)\Delta$, and (ii) even when $c = r^\star - (1-\gamma)\Delta$, prices very close to $r^\star$ are unlikely (density is bounded). For high-cost dealers, prices just above $r^\star$ are in the support only when $c$ is close to $r^\star - \Delta$, but conditional on $c = r^\star - \Delta$, the price is $r^\star$ with probability one. □

**Lemma 13.** *When $r^\star \in (\underline{c} + \Delta,\, v)$, $\Psi(p;\, r^\star)$ jumps down discontinuously at $p = r^\star$.*

*Proof.* By Lemma 12, as the observed price $p$ converges to $r^\star$, the posterior distribution of costs converges to an atom at $r^\star - \Delta$. Moreover, by the inspection of the proof, when $p$ converges to $r^\star$ from the left, the probability mass converges to an atom at $r^\star - \Delta$ from the left, and when $p$ converges to $r^\star$ from the right, the probability mass converges to an atom at $r^\star - \Delta$ from the right.[47] The price distribution of high-cost dealers is continuous (for example, in the Lévy-Prokhorov metric) in $c$. The lemma will be thus proven if we can show that the price distribution of low-cost dealers changes discontinuously at $c = r^\star - \Delta$ in such a way that expected benefits from search jump down at $p = r^\star$. By the above derivation and direct calculation, we show that $F^l_c(p)$ for $c$ in the right neighborhood of $r^\star - \Delta$ strictly first-order stochastically dominates $F^l_c(p)$ for $c$ in the left-neighborhood of $r^\star - \Delta$, and the difference between the cdfs is bounded away from zero. This means that expected continuation value of search jumps down at $c = r^\star - \Delta$. □

The intuition behind Lemma 13 is as follows. When $c < r^\star - \Delta$, high-cost dealers sell when contacted by a slow trader. Thus, low-cost dealers attach a higher probability to the trader being a fast trader and as a result they quote smaller prices. When $c > r^\star - \Delta$, high-cost dealers do not sell when they are visited by a slow trader. Thus, slow traders search more, the posterior probability of a fast trader falls, and low-cost dealers quote higher prices. Thus, there is a discontinuity in expected price, which jumps up at $c = r^\star - \Delta$. When a slow trader sees $p$ just below $r^\star$, she thinks that $c$ is just below $r^\star - \Delta$ and prices are low. When a slow trader sees a price just above $r^\star$, she thinks that $c$ is just above $r^\star - \Delta$ and prices are high. The value of taking the offer is almost the same in both cases, but the value of search is clearly more attractive in the first. Thus, if a slow traders does not want to search at $p$ below $r^\star$, she definitely does not want to search for $p$ just above $r^\star$. As a result, condition (C.1) must fail, that is, we cannot have an equilibrium for $r^\star \in (\underline{c} + \Delta,\, v)$.

Now we deal with the case $r^\star < \underline{c} + \Delta$. Because $r^\star < c + \Delta$ for all $c$, all prices $p$ below $r^\star$ observed on equilibrium path must be posted by low-cost dealers (in equilibrium there has to be positive probability of observing a price below $r^\star$). Moreover, by the characterization above, prices $p \in (r^\star,\, r^\star + \epsilon)$ for small $\epsilon < \underline{c} + \Delta - r^\star$ can only be quoted by low-cost dealers when $c$ is close to $r^\star - (1-\gamma)\Delta$. In other words, upon observing $p = r^\star + \epsilon$ for small $\epsilon > 0$, a slow trader believes that $c$ is within $\epsilon$ of $r^\star - (1-\gamma)\Delta$. But in this case, price $p$ is within $\epsilon$ of the best possible price given the beliefs. Therefore, the benefit from search drops to zero as $p$ crosses $r^\star$ from left to right. Clearly, this contradicts existence of an equilibrium.

We have thus shown that $r^\star = \underline{c} + \Delta$ or $r^\star = v$ in a reservation-price equilibrium.

[47] We do not formalize what we mean by "probability mass converges from the left/ right" although this could be done easily. The point is that the probability mass is centered around $p - \Delta$ for $p$ close to $r^\star$.

To conclude the proof of Lemma 10, we argue that when $s < (1-\alpha(1,0))\gamma\Delta$, we cannot have an equilibrium with $r^\star = v$. We prove this by showing that in this case a slow trader wants to search when observing a price offer at or slightly below $v$. By the argument used in Lemma 12, we show that the posterior cost distribution converges to an atom at $v - \Delta$ as $p$ converges to $v$. We can then calculate the benefit from search explicitly using the price distribution derived above. We obtain $\Psi(v; v) = (1-\alpha(1,0))\gamma\Delta$, which is also the right limit of $\Psi(p; v)$ as $p \nearrow v$. This produces a contradiction with condition (C.1).

### C.4.2 Proof of Proposition 11

The proposition follows directly from what has been shown above. We know that when $s < (1-\alpha(1,0))\gamma\Delta$, we cannot have an equilibrium with $r^\star = v$. Thus, we can only have $r^\star = \underline{c} + \Delta$. The condition $s \in [\underline{s}, \bar{s}]$ is equivalent to condition (C.1). High-cost dealers quote $c + \Delta$, and low-cost dealers use a continuous distribution corresponding to the cases $c \in [r^\star - \Delta, r^\star - \kappa\Delta]$, $c \in [r^\star - \kappa\Delta, r^\star - (1-\gamma)\Delta]$, and $c \in [r^\star - (1-\gamma)\Delta, \bar{c}]$. Entry can be analyzed in the same way as in previous equilibrium constructions (it can be shown that slow traders enter with probability one under the assumptions that we have imposed).

When $r^\star = \underline{c} + \Delta$, continuity of $\Psi(p; r^\star)$ at all points $p$ in the support other than $r^\star$ is easy to show by direct inspection. We prove that $\Psi(p; r^\star)$ jumps up at $p = r^\star$.

When $p$ converges to $r^\star$ from the right, the posterior distribution of costs converges to an atom at $r^\star - \Delta = \underline{c}$. In that case, we can calculate $\lim_{p \searrow r^\star} \Psi(p; r^\star)$ explicitly. We have

$$\lim_{p \searrow r^\star} \Psi(p; r^\star) = (1-\alpha(1,1))\gamma\Delta.$$

This means that $\bar{s} \leq (1-\alpha(1,1))\gamma\Delta$.

When $p$ converges to $r^\star$ from the left, the benefit from search must converge to a number that is strictly lower than $(1-\alpha(1,1))\gamma\Delta$. The reason is that in this case the trader believes that the offer has been posted by a low-cost dealer, and thus the posterior distribution of costs will be atomless with support $[\underline{c}, r^\star - (1-\gamma)\Delta]$. Since prices are increasing with costs, the trader expects that prices are higher than in the case in which the cost $c$ is equal to $\underline{c}$. Unfortunately, a closed form solution for $\lim_{p \nearrow r^\star} \Psi(p; r^\star)$ is hard to obtain because the expected price becomes non-linear in $c$ for $c \in [r^\star - \kappa\Delta, r^\star - (1-\gamma)\Delta]$.

### C.4.3 Proof of Corollary 2

Under the assumption $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$, the equilibrium with the benchmark achieves the second best. In the no-benchmark case, when $s \in [\underline{s}, \bar{s}]$, we have a reservation-price equilibrium with $r_0^\star = \underline{c} + \Delta$. By the derivation of equilibrium pricing strategies from the proof of Lemma 10, whenever $c > \underline{c} + \gamma\Delta$, low-cost dealers quote prices above $\underline{c} + \Delta$. With probability $\gamma$ a slow trader visits a low-cost dealer in the first search round. Therefore, the expected surplus loss in the no-benchmark case relative to the second best is at least

$$(1 - G(\underline{c} + \gamma\Delta))(1-\mu)\gamma s. \tag{C.4}$$

On the other hand, the surplus gain from moving from the second best to centralized exchange is $(1-\mu)(2-\gamma)s$. Dividing (C.4) by the sum of (C.4) and $(1-\mu)(2-\gamma)s$ we conclude the proof of the Corollary.

# D Completion of the Analysis of Section 3.5—The Case of Heterogeneous Dealers' Costs

If $\gamma \in (0,\, 1)$, a reservation-price equilibrium with $r^\star = v$ exists in the no-benchmark setting if search costs are sufficiently large. (See the discussion under Lemma 10 in Section C.3.) Here, we might use this equilibrium to ask if it is possible that the introduction of the benchmark benefits one type of the dealers but not the other (through the entry channel, that is, when search costs are relatively high).

The answer turns out to be generally no.[48] When the benchmark increases entry sufficiently, both types of dealers benefit. When the benchmark fails to encourage entry (for example because we have already full entry without the benchmark), profits of both types of dealers are harmed. This is intuitive. Under parameter restrictions that guarantee full entry and existence, reservation-price equilibria will have no search in both cases, so the volumes of trade remain the same for both types of dealers. Prices generally decrease. Thus, if entry does not increase, introducing a benchmark acts as a transfer of surplus from dealers to traders.

There is however one case in which high-cost dealers would opt for a benchmark while low-cost dealers would not. Just as in the homogeneous-cost case, dealers prefer to trade under the benchmark if the search cost of slow traders exceeds a certain cutoff. The threshold for high-cost dealers will be slightly lower that for low-cost dealers. To understand this observation, recall from Section 3 that the benchmark has the effect of increasing entry especially in the case when gains from trade are large. Also in this case (that is, when gains from trade are large) high-cost dealers trade in equilibrium. In the opposite case (when $c > v - \Delta$), high-cost dealers cannot trade anyway, and are thus not harmed by relatively smaller entry in the benchmark case for high cost realizations.

Figure D.1 illustrates the above point. We take the same numerical example as in Section 4. When search costs are relatively small, the profits of dealers are larger when there is no benchmark. When $s$ gets bigger, the positive effect of benchmarks on entry gets strong enough for dealers to benefit from increased volume of trade. Once $s$ crosses 0.32, high-cost dealers would like to introduce the benchmark. For low-cost dealers, the corresponding threshold is slightly above 0.33.

# E Generalization of Theorems 1 and 2

This appendix generalizes Theorems 1 and 2 by relaxing the assumption that $\gamma = 1$.

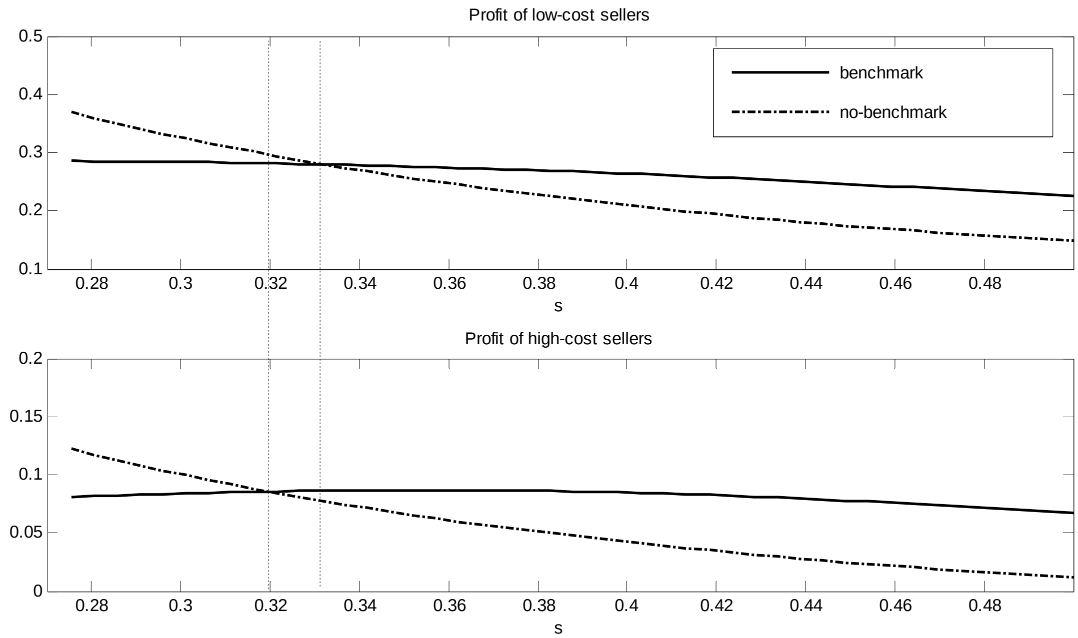
**Theorem 7.** *Consider the model with heterogeneous dealers' costs.*[49] *Suppose that (i) $s \geq \gamma\Delta$ and (ii)*

$$\frac{s - (1 - \alpha(1,\, 0))\gamma\Delta}{\gamma\,(1 - \phi(1)\alpha(1,\, 0)) + (1-\gamma)\,(1 - \alpha_h(1))} \geq v - \underline{c} - \Delta.$$

[48] For a formal result, see Appendix E.

[49] That is, $\gamma \in (0,\, 1)$, $\Delta > 0$.

Fig. D.1: The profits of low-cost and high-cost dealers (in reservation-price equilibria for intermediate and large $s$)




*Then, a reservation-price equilibrium in the no-benchmark case (if it exists) yields a lower social surplus than in the setting with the benchmark. Moreover, if $\bar{c} \leq v - \Delta$, then the expected profits of both high-cost and low-cost dealers are higher in the setting with the benchmark. Condition (ii) holds if there are sufficiently many dealers or if the fraction $\mu$ of fast traders is small enough.*

*Remark.* Notice that when $\gamma \to 1$, condition (ii) boils down to

$$\frac{s}{(1 - \alpha(1, 0))} \geq v - \underline{c}.$$

Because $\alpha(1, 0) = \bar{\alpha}$, this is exactly condition (i) from Theorem 1 and Theorem 2.

*Proof.* Because the logic of the proof is the same as in the case of homogeneous dealers' costs, we sketch the main arguments and omit most calculations. Unless stated otherwise, the symbols that we use have the same meaning as in the proof of Theorem 1. We begin by describing the welfare and profits in the two settings. Without loss of generality we assume that $\bar{c} \leq v$.[50]

**Benchmark setting.** By the equilibrium characterization from the proof of Proposition 6 in Appendix B (cases 1 and 2.2.2), under the parameter restrictions of the Theorem we can

[50] In the opposite case, the comparison can only be even more favorable for the benchmark case.

have two types of equilibria with the benchmark. When

$$s - (1 - \alpha(0, 0))\gamma\Delta \geq x - \Delta,$$

we have no entry of slow traders (that is, $\lambda(x) = 0$). Therefore, social welfare is equal to $\mu\left(x - (1-\gamma)^N \Delta\right)$. High-cost dealers make no profits, and low-cost dealers have expected profits equal to $(1-\gamma)^{N-1} \min\{\Delta, x\}$. When

$$s - (1 - \alpha(0, 0))\gamma\Delta < x - \Delta, \tag{E.1}$$

there is interior entry of slow traders, determined by the equation

$$s = \gamma\left[(1 - \phi(\lambda)\,\alpha(\lambda, 0))\,x - \alpha(\lambda, 0)(1 - \phi(\lambda))\,\Delta\right] + (1-\gamma)\left[(1 - \alpha_h(\lambda))(x - \Delta)\right], \tag{E.2}$$

suppressing from the notation the argument $x$ of $\lambda(x)$. Because slow traders buy from the first dealer, social welfare (as a function of gains from trade) is

$$W_b(x) = \mu\left(x - (1-\gamma)^N \Delta\right) + \lambda(x)(1-\mu)\left(x - (1-\gamma)\Delta - s\right). \tag{E.3}$$

(We note that by equation (E.1), $c \leq v - \Delta$). The expected profits of low-cost dealers are

$$\chi_b^l(x) = \left[\frac{\lambda(x)(1-\mu)}{N} + \mu(1-\gamma)^{N-1}\right]\left[\phi(\lambda(x))(x - \Delta) + \Delta\right]. \tag{E.4}$$

The expected profits of high-cost dealers are

$$\chi_b^h(x) = \frac{\lambda(x)(1-\mu)}{N}(x - \Delta). \tag{E.5}$$

**No-benchmark setting.** In the no-benchmark setting we concentrate on the reservation-price equilibrium with a reservation price (of slow traders) equal to $v$. (See the comment below Lemma 10 in Section C.3 for details.[51]) This equilibrium exists if search costs are sufficiently large. Under the parameter restrictions of the Theorem we can have two types of equilibria without the benchmark.

First, consider the case $c < v - \Delta$, in which we have an equilibrium analogous to that under the benchmark. In particular, conditional on cost realization $x > \Delta$, social surplus is

$$W_{nb}(x) = \mu\left(x - (1-\gamma)^N \Delta\right) + \lambda^\star(1-\mu)\left(x - (1-\gamma)\Delta - s\right),$$

where $\lambda^\star$ denotes the equilibrium probability of entry of slow traders (in this case a constant, not a function of $x$). Low-cost dealers make conditional expected profits

$$\chi_{nb}^l(x) = \left[\frac{\lambda^\star(1-\mu)}{N} + \mu(1-\gamma)^{N-1}\right]\left[\phi(\lambda^\star)(x - \Delta) + \Delta\right],$$

[51] As explained in Section C.3, although $N = 2$ is assumed throughout that subsection, the characterization of equilibrium with $r^\star = v$ is valid for an arbitrary $N$.

and high-cost dealers earn

$$\chi^h_{nb}(x) = \frac{\lambda^\star(1-\mu)}{N}(x-\Delta).$$

Second, consider $c \geq v - \Delta$. Now high-cost dealers cannot make positive profits, so they post a deterministic price equal to $c + \Delta$. Social surplus conditional on $x$ is given by

$$W_{nb}(x) = \mu\left(1-(1-\gamma)^N\right)x + \lambda^\star(1-\mu)(\gamma x - s).$$

Low-cost dealers have a conditional expected profit of

$$\chi^l_{nb}(x) = \left[\frac{\lambda^\star(1-\mu)}{N} + \mu(1-\gamma)^{N-1}\right]x.$$

We now turn our attention to traders. When $x > \Delta$, conditional on $x$ and entry, a slow trader has an expected profit of

$$-s + \gamma\left[(1-\phi(\lambda^\star)\alpha(\lambda^\star,\, 0))\,x - \alpha(\lambda^\star,\, 0)(1-\phi(\lambda^\star))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda^\star))\,(x-\Delta)\right].$$

When $x \leq \Delta$, the corresponding expected profit is

$$-s + \gamma(1-\alpha(\lambda^\star,\, 0))x.$$

Because slow traders do not observe $c$ when there is no benchmark, their entry decision is determined by taking an expectation with respect to the distribution of $x$. Thus, $\lambda^\star$ solves

$$s = G(v-\Delta)\mathbb{E}\left[\Lambda(\lambda^\star, x)\,|\,x > \Delta\right] + (1-G(v-\Delta))\,\mathbb{E}\left[\gamma(1-\alpha(\lambda^\star,\, 0))x\,|\,x \leq \Delta\right], \qquad \text{(E.6)}$$

where

$$\Lambda(\lambda^\star, x) = \gamma\left[(1-\phi(\lambda^\star)\alpha(\lambda^\star,\, 0))\,x - \alpha(\lambda^\star,\, 0)(1-\phi(\lambda^\star))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda^\star))\,(x-\Delta)\right].$$

**A few steps to simplify the problem.** First, we notice that the comparison of welfare can only be more favorable for the no-benchmark case when we condition on $c \leq v - \Delta$. As for the profits of dealers, we have $\bar{c} \leq v - \Delta$ by assumption. Thus from now on, we assume without loss of generality that $\bar{c} \leq v - \Delta$. This simplifies the formulas in the no-benchmark setting. Unconditional expected welfare can now be written as

$$W_{nb} = \mu\left(X - (1-\gamma)^N\Delta\right) + \lambda^\star(1-\mu)\left(X - (1-\gamma)\Delta - s\right). \qquad \text{(E.7)}$$

The expected profit of low-cost dealers is

$$\chi^l_{nb} = \left[\frac{\lambda^\star(1-\mu)}{N} + \mu(1-\gamma)^{N-1}\right]\left[\phi\left(\lambda^\star\right)(X-\Delta) + \Delta\right]. \qquad \text{(E.8)}$$

The expected profit of high-cost dealers is

$$\chi^h_{nb} = \frac{\lambda^\star(1-\mu)}{N}(X-\Delta). \qquad \text{(E.9)}$$

Finally, the key equation (E.6) determining entry in the no-benchmark setting simplifies to

$$s = \gamma\left[(1-\phi(\lambda^\star)\alpha(\lambda^\star,0))\,X - \alpha(\lambda^\star,0)(1-\phi(\lambda^\star))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda^\star))\,(X-\Delta)\right]. \tag{E.10}$$

Note that, by equations (E.2) and (E.10), $\lambda(X) = \lambda^\star$.

The crucial (and most tedious) step in the proof is to show the following lemma, which generalizes Lemma 3.

**Lemma 14.** *The function $\lambda(x)$ is convex (for $x$ in the range permitted by the assumptions of the Theorem).*

Before we show the proof of Lemma 14, we analyze its consequences. It is easy to observe that if $\lambda(x)$ is convex, then also $W_b(x)$, $\chi_b^l(x)$ and $\chi_b^h(x)$ are convex.[52] Hence, we can apply Jensen's Inequality to these three functions (just as in the proofs of Theorem 1 and Theorem 2). The observation made above that $\lambda(X) = \lambda^\star$, and direct inspection of formulas (E.3) and (E.7), (E.4) and (E.8), (E.5) and (E.9), complete the proof.

*Proof of Lemma 14.* The full proof is long and tedious, and thus some of the details are omitted.

We rewrite equation (E.2), suppressing from the notation the dependence of $\lambda$ on $x$, as

$$s \equiv \gamma\left[(1-\alpha_l(\lambda))\,x - (1-\tilde{\alpha}_l(\lambda))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda))\,x - (1-\alpha_h(\lambda))\Delta\right], \tag{E.11}$$

where

$$\alpha_l(\lambda) = \phi(\lambda)\alpha(\lambda,0)$$

and

$$\tilde{\alpha}_l(\lambda) = 1 - \alpha(\lambda,0)(1-\phi(\lambda)).$$

A real-valued function $f(x)$ is strictly increasing and convex if and only if $f^{-1}(x)$ is strictly increasing and concave. Thus, to show that the solution $\lambda(x)$ of equation (E.11) is convex, it is enough to prove that the function $\lambda \mapsto x(\lambda)$, defined by

$$x(\lambda) = \frac{s + (\gamma(1-\tilde{\alpha}_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda)))\,\Delta}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))},$$

is concave. We have

$$\begin{aligned} x(\lambda) &= \frac{s}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} + \Delta + \frac{\alpha_l(\lambda) - \tilde{\alpha}_l(\lambda)}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))}\gamma\Delta \\ &= \frac{s-\gamma\Delta}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} + \Delta + \frac{1+\alpha_l(\lambda) - \tilde{\alpha}_l(\lambda)}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))}\gamma\Delta. \end{aligned}$$

Because a sum of concave functions is concave, and due to $s \geq \gamma\Delta$, a sufficient condition for concavity of $x(\lambda)$ is that

$$\frac{1}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} \tag{E.12}$$

[52] In case of $\chi_b^l(x)$ this requires some calculation that we omit.

and

$$\frac{1+\alpha_l(\lambda)-\tilde{\alpha}_l(\lambda)}{\gamma(1-\alpha_l(\lambda))+(1-\gamma)(1-\alpha_h(\lambda))} \tag{E.13}$$

are both concave in $\lambda$. We show formally the concavity of the first of these functions and omit a similar proof of concavity of the second one.

Because the function (E.12) is twice continuously differentiable, concavity is implied by the second derivative being non-positive. The second derivative of the function (E.12) is non-positive if and only if (omitting for simplicity the notational dependence on $\lambda$)

$$[\gamma\alpha_l''+(1-\gamma)\alpha_h'']\left(\gamma(1-\alpha_l)+(1-\gamma)(1-\alpha_h)\right)+2\left(\gamma\alpha_l'+(1-\gamma)\alpha_h'\right)^2\leq 0.$$

Expanding this inequality, we can obtain a sufficient condition by requiring that each of the terms multiplied by $\gamma^2$, $\gamma(1-\gamma)$, and $(1-\gamma)^2$, accordingly, is non-positive:

$$\alpha_l''(1-\alpha_l)+2\left(\alpha_l'\right)^2\leq 0, \tag{E.14}$$

$$\alpha_h''(1-\alpha_h)+2\left(\alpha_h'\right)^2\leq 0, \tag{E.15}$$

$$\alpha_l''(1-\alpha_h)+\alpha_h''(1-\alpha_l)+4\alpha_l'\alpha_h'\leq 0. \tag{E.16}$$

Inequalities (E.14) and (E.15) are proven in exactly the same way as inequality (A.4) in the proof of Lemma 3. We show how to prove inequality (E.16). Using the definitions of functions $\alpha_l$ and $\alpha_h$, and after a tedious calculation of the first and second derivatives, we can express the inequality equivalently as

$$2\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\left(\lambda+\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)\right)^2}\,dz\right)\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\left(\lambda+\beta(1-\gamma)^{N-1}z^{N-1}\right)^2}\,dz\right)$$

$$\leq\underbrace{\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\left(\lambda+\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)\right)^3}\,dz\right)}_{a}\underbrace{\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\lambda+\beta(1-\gamma)^{N-1}z^{N-1}}\,dz\right)}_{b}$$

$$+\underbrace{\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\left(\lambda+\beta(1-\gamma)^{N-1}z^{N-1}\right)^3}\,dz\right)}_{c}\underbrace{\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\lambda+\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}\,dz\right)}_{d}.$$

By Hölder's Inequality applied twice to the two integrals on the left-hand side, we get

$$2\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\left(\lambda+\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)\right)^2}\,dz\right)\left(\int_0^1\frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\left(\lambda+\beta(1-\gamma)^{N-1}z^{N-1}\right)^2}\,dz\right)$$

$$\leq 2\sqrt{ad}\sqrt{bc}.$$

Thus, we have to prove that for any positive constants $a$, $b$, $c$ and $d$, we have

$$2\sqrt{ad}\sqrt{bc}\leq ab+cd.$$

This follows immediately from the fact that $(ab - cd)^2 \geq 0$.

# F Separating the Entry-Promoting Role of a Benchmark

As argued in our discussion of Theorem 1, introducing a benchmark encourages entry through two channels: *(i)* signaling when gains from trade are high and *(ii)* increasing the slow traders' share of gains from trade by reducing the informational advantage of dealers concerning the cost of the asset. In order to distinguish between these two effects, we study in this section (only) an intermediate "costly-benchmark-observation" setting in which traders observe the benchmark only upon making their first contact with a dealer (after making the entry decision but before accepting or rejecting an offer). Essentially, this means that slow traders must pay the search cost $s$ to learn the outcome of the benchmark. This artificial costly-benchmark-observation setting allows us to characterize in the next proposition the specific entry screening effect *(ii)* of benchmarks, while keeping the other entry effect *(i)* "switched off."

**Proposition 12.** *A reservation-price equilibrium always exists (and is payoff-unique) in the costly-benchmark-observation setting. Moreover, under the condition that* $(1-\bar{\alpha})X < s < X$, *the equilibrium in the costly-benchmark-observation setting has a strictly higher expected social surplus than that of the reservation-price equilibrium without the benchmark.*

The proposition states that channel *(ii)*, reducing information asymmetry between dealers and traders, always works in favor of introducing a benchmark. By providing slow traders with information about the market-wide cost of the asset to dealers, the presence of a benchmark increases traders' expected payoffs off the equilibrium path, thus encouraging their entry and raising total social surplus on the equilibrium path.

The next result states that role *(i)* of a benchmark, signaling when there are high gains from trade, is also relevant.

**Proposition 13.** *There exists* $\underline{s} < X$ *such that for any search cost* $s \in (\underline{s}, v - \underline{c})$ *the expected social surplus is strictly higher in the benchmark case than in the costly-benchmark-observation case.*

## F.1 Proof of Proposition 12

Using the same arguments used in the derivation of equilibrium from Proposition 1 we can show that in the costly-benchmark case there exists a reservation-price equilibrium, and that equilibrium payoffs are unique. Fixing the probability of entry at $\lambda$ (and noting that it is independent of $c$), we compute the reservation price

$$r_c^{cb} = \min\left\{v,\, c + \frac{1}{1-\alpha(\lambda)}s\right\}.$$

A slow trader buys from the first contacted dealer if $c \leq v$. The profit of a slow trader who enters, conditional on $c$, can be shown to be

$$\pi_c^{cb}(\lambda) = \max\left\{v - \frac{1}{1-\alpha(\lambda)}s - c,\, -s + (1-\alpha(\lambda))(v-c)\right\}$$

if $c \leq v$, and $-s$ if $c > v$. When $s \geq X$, there can be no entry in equilibrium. If the equilibrium probability of entry $\lambda^{cb}$ is interior, then it must be determined by the indifference condition, analogous to (3.8) and (3.10), given by

$$\mathbb{E}\pi_c^{cb}(\lambda^{cb}) = 0. \tag{F.1}$$

The solution to that equation exists and is unique if $X \geq s \geq (1-\bar{\alpha})X + \phi$, where

$$\phi = \frac{\bar{\alpha}}{1-\bar{\alpha}} \int_{\underline{c}}^{v - \frac{s}{1-\bar{\alpha}}} \left[(1-\bar{\alpha})(v-c) - s\right] dG(c) \geq 0.$$

When $s < (1-\bar{\alpha})X + \phi$, we must have entry with probability one.

To show that surplus is higher in the costly-benchmark case than in the no-benchmark case, it is enough to show that entry is higher. Because the function max is convex, we can apply Jensen's Inequality to conclude that, for all $\lambda$,

$$\mathbb{E}\pi_c^{cb}(\lambda) \geq -s + (1-\alpha(\lambda))X = \pi^{nb}(\lambda),$$

that is, the expected profit is always higher in the costly-benchmark setting (and is strictly higher provided that $(1-\bar{\alpha})X < s < X$). It follows that equilibrium entry of slow traders must also be higher (from equations (3.10) and (F.1)).

## F.2 Proof of Proposition 13

By Theorem 1 we know that when $s$ is higher than $(1-\psi)X$, surplus under the benchmark is higher than in the reservation-price equilibrium of the no-benchmark case.[53] It is easy to observe that the difference in surpluses is bounded away from zero as a function of $s$ (under the assumption that $v - \underline{c} > s > (1-\psi)X$). Given Proposition 12, it suffices to show that the surplus of the costly-benchmark case converges to the surplus of the no-benchmark case as $s$ goes to $X$ (when $s \geq X$, they coincide). It is enough to prove that $\lambda^{cb}$, the solution of equation (F.1), converges to $\lambda^\star$, the solution of equation (3.10), as $s \to X$. Because the solution of equation (F.1) is continuous in $s$ and equal to 0 at $s = X$, $\lambda^{cb}$ converges to 0, and so does $\lambda^\star$.

[53] Even if the latter equilibrium does not exist, the comparison between surpluses is valid, and that is all we need for the proof.

# G Glossary of symbols

| Symbol | Definition | Remarks |
|---|---|---|
| *Primitive parameters* | | |
| $N$ | number of dealers | $N \geq 2$ |
| $c$ | common cost component of dealers | |
| $G$ | cdf of common dealer cost $c$ | with support $[\underline{c}, \bar{c}]$ |
| $\Delta$ | cost differential between high-cost and low-cost dealers | idiosyncratic cost $\epsilon_i$ is 0 or $\Delta$ |
| $\gamma$ | ex-ante probability of a low-cost dealer | $P(\epsilon_i = 0) = \gamma$ |
| $\mu$ | fraction of fast traders | $\mu \in (0, 1)$ |
| $s$ | search cost of slow traders | $s > 0$ |
| $v$ | traders' asset valuation | a constant |
| $\Gamma$ | probability of at least 2 low-cost dealers | Section 4 only |
| *Derived quantities* | | |
| $\lambda$ | slow-trader entry probability | An additional subscript '$c$' indicates that the quantity depends on $c$ (in the benchmark case); the superscript $^\star$ denotes the quantity in equilibrium. |
| $r$ | reservation price of slow traders | |
| $\theta$ | probability that a slow trader buys from a high-cost dealer | |
| $F_c^i$ | cdf of dealer offers, $i \in \{l, h\}$ | lower limit: $\underline{p}_c^i$; upper limit: $\bar{p}_c^i$ |
| $q(\lambda, \theta)$ | $\frac{N\mu}{N\mu + \frac{1-\theta^N(1-\gamma)^N}{1-\theta(1-\gamma)}\lambda(1-\mu)}$ | probability that a contacting trader is fast |
| $\alpha(\lambda, \theta)$ | $\int_0^1 \left(1 + \frac{q(\lambda,\theta)\left(1-(1-\gamma)^{N-1}\right)}{1-q(\lambda,\theta)(1-(1-\gamma)^{N-1})}\Phi(z)\right)^{-1} dz$ | values between 0 and 1, strictly increasing in both arguments |
| $\Phi(z)$ | $\frac{\sum_{k=1}^{N-1}\binom{N-1}{k} z^k \gamma^k (1-\gamma)^{N-1-k}}{1-(1-\gamma)^{N-1}}$ | strictly increasing polynomial, $\Phi(0) = 0$, $\Phi(1) = 1$. |
| $\hat{\alpha}$ | $\alpha(1, 1)$ | upper bound on $\alpha(\lambda, \theta)$; strictly below 1 |
| $\kappa$ | $\frac{(1-\gamma)^{N-1}(\mu+\lambda^\star(1-\mu))}{\mu(1-\gamma)^{N-1}+\lambda^\star(1-\mu)\frac{1-(1-\gamma)^N}{N\gamma}}$ | Special cases: $\frac{(1-\gamma)^{N-1}}{\mu(1-\gamma)^{N-1}+(1-\mu)\frac{1-(1-\gamma)^N}{N\gamma}}$ when $\lambda^\star = 1$, $\frac{2(1-\gamma)(\mu+\lambda^\star(1-\mu))}{\lambda^\star(2-\gamma)(1-\mu)+2\mu(1-\gamma)}$ when $N = 2$. |
| *Special case of homogeneous dealers' cost* $(\gamma = 1)$ | | |
| $q(\lambda)$ | $\frac{N\mu}{N\mu+\lambda(1-\mu)}$ | equal to $q(\lambda, 0)$ |
| $\alpha(\lambda)$ | $\int_0^1 \left(1 + \frac{N\mu}{\lambda(1-\mu)} z^{N-1}\right)^{-1} dz$ | equal to $\alpha(\lambda, 0)$ |
| $\bar{\alpha}$ | $\alpha(1)$ | upper bound on $\alpha(\lambda)$; strictly below 1 |
| $X$ | $G(v)\,[v - \mathbb{E}\,[c \mid c \leq v]]$ | expected gain from trade |
| $\varphi(\lambda)$ | $\frac{\lambda(1-\mu)}{N\mu+\lambda(1-\mu)}$ | - |

| Symbols used only in appendices | | |
|---|---|---|
| $\zeta$ | probability that a low-cost dealer quotes an offer above the reservation price of slow traders | An additional subscript '$c$' indicates that the quantity depends on $c$ (in the benchmark case); the superscript $\star$ denotes the quantity in equilibrium. |
| $X_\Delta$ | $G(v-\Delta)\mathbb{E}\left(v-c-\Delta\right\| c \leq v-\Delta)$ | Expected gain from trade when only high-cost dealers are present |
| $\vartheta(\zeta)$ | $\frac{(1-\gamma)^{N-1}}{\mu(1-\gamma\zeta)^{N-1}+(1-\mu)\frac{1-(1-\gamma\zeta)^N}{N\gamma\zeta}}$ | $\vartheta(1)=\kappa$ |
| $\tilde{\alpha}(\zeta)$ | $\int_0^1 \left(1+\frac{1-(1-\gamma\zeta)^{N-1}}{(1-\gamma)^{N-1}}\mu\vartheta(\zeta)\Phi(z;\,\zeta)\right)^{-1} dz$ | $\tilde{\alpha}(1)=\alpha(1,\,1)$ |
| $\Phi(z;\,\zeta)$ | $\frac{\sum_{k=1}^{N-1}\binom{N-1}{k}z^k(\gamma\zeta)^k(1-\gamma\zeta)^{N-1-k}}{1-(1-\gamma\zeta)^{N-1}}$ | strictly increasing polynomial, $\Phi(0;\,\zeta)=0$, $\Phi(1;\,\zeta)=1$ |
| $\alpha_h(\lambda)$ | $\int_0^1 \left(1+\frac{q(\lambda,0)(1-\gamma)^{N-1}}{1-q(\lambda,0)}z^{N-1}\right)^{-1} dz$ | $\alpha_h(0)=0$ |
| $\phi(\lambda)$ | $\frac{1-q(\lambda,0)}{1-(1-(1-\gamma)^{N-1})q(\lambda,0)}$ | $\phi(0)=0$ |