JOURNAL OF ECONOMIC THEORY **53**, 261–286 (1991)

Corporate Financial Hedging with Proprietary Information*

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Received March 6, 1989; revised July 31, 1990

If a firm has information pertinent to its own dividend stream that is not made available to its shareholders, it may be in the interests of the firm and its shareholders to adopt a financial hedging policy. This is in contrast with the Modigliani–Miller Theorem, which implies that, with informational symmetry, such financial hedging is irrelevant. In certain cases, hedging policies are identified that are unanimously supported by shareholders. *Journal of Economic Literature* Classification Numbers: 021, 022, 026, 313, 521.

1. INTRODUCTION

The purpose of this paper is to demonstrate that, if a firm has information pertinent to its own dividend stream that is not made available to its shareholders, it may be in the interests of its shareholders for the firm to adopt an appropriate financial hedging policy. Moreover, though markets are incomplete, circumstances are identified in which there exist optimal hedging policies that are unanimously supported by all shareholders of the firm. Finally, even though hedging may be costly (i.e., a risk premium must be paid), these policies typically call for the firm to hedge the risk "completely."

* We acknowledge the comments and discussion provided by Karen Van Nuys, Peter Hammond, David Hirshleifer, Hayne Leland, David Starrett, and an anonymous referee.

[†] The support of Batterymarch Financial Management is gratefully acknowledged.

642/53/2-4

0022-0531/91 $3.00

Our results are in direct contrast with the Modigliani–Miller (MM) theory [20], which implies that with informational symmetry and the usual perfect market assumptions, corporate financial policy has no effect on firm value. This theory is based on the presumption that any financial strategy the firm might adopt could equally well be adopted pro rata by its shareholders, implying its irrelevance in markets without transactions costs, bankruptcy costs, taxes, principal-agent effects, and similar "imperfections." (For a general form of the MM theory, see DeMarzo [7]; for work indicating cases in which it may not apply, see Miller [19].)

In practice, publicly traded firms commonly pay a great deal of attention to their financial policies and, in particular, account for the bulk of hedging positions in futures and forward markets. While many different practical considerations might justify this behavior, it seems worthwhile to model one in particular: Firms have proprietary information. The privacy of this information may be of strategic importance in the firm's marketplace, or may merely be due to the cost of disseminating up-to-date news on the corporation's production plans and other ventures. In any case, it is impossible for shareholders to adopt for themselves financial strategies that are based on information they do not have. The firm, however, may hedge on their behalf.

Private information held by managers of firms has been shown to overturn the MM conclusions in other settings as well. In particular, Jensen and Meckling [16] discuss the implications of a firm's capital structure on managerial incentives when managers' actions are not fully observable by shareholders. Also, Ross [21] shows the firm's capital structure decision may signal information to shareholders regarding the firm's prospects. Both these lines of research are very different than the motives for hedging considered here. First, we consider a case in which managers act on behalf of shareholders, and ignore problems of managerial incentive "misalignment." Second, we assume that the actual hedging portfolio held by a firm is not directly observable by shareholders, and hence is not used to signal information (shareholders may, however, infer information from price changes in securities markets).

Our results demonstrate that with regard to a financial hedging policy, there are circumstances for which unanimous agreement by shareholders can be attained. Moreover, the optimal policies we identify typically involve complete hedging of any "spanned" risk by the firm. This is true even for cases in which hedging is costly, in that futures contracts trade at their expected value plus some non-zero risk premium. Since any individual's portfolio would not be completely hedged under such circumstances, such policies imply the firm acts as though it were "infinitely" risk averse with regard to its financial policy. This contrasts with the production decisions of the firm, which generally would be made from a more risk neutral perspective. This behavioral dichotomy raises serious doubts regarding approaches to modeling firm objectives via a concave firm "utility" function. (See also Drèze [11] on this point.)

This paper is based on the intuition that the elimination of noise in a firm's dividend stream is unanimously supported by shareholders. In order to make succinct statements, however, we must in certain cases rely on the strong assumption that dividends include noise (with respect to the information available to shareholders) that is spanned by financial markets, given the information known to the firm. Our conclusions are thus related to the intuition of the spanning literature regarding production decisions, beginning with Diamond [9] and Ekern and Wilson [13], and extended by Leland [17]. They have shown that production changes within the span of existing markets are consistently evaluated by shareholders. Alternatively, in the absence of spanning, such shareholder unanimity regarding a redistribution of the firm's dividend stream is generically impossible (see DeMarzo [8], Duffie and Shafer [12], and Geanakoplos, Magill, Quinzii, and Drèze [14]).

A key premise of the paper is that there exist situations in which the firm chooses not to inform its shareholders of financial risks that they could otherwise hedge on security markets. Since shareholders do not known how to hedge these risks, they want the firm to hedge on their behalf. We only examine situations in which the firm hedges in a way that is unanimously supported by its shareholders, and thus the decision of the firm not to inform its shareholders of these risks is not to the shareholders' detriment, both in the competitive sense, and in a social welfare sense. Nevertheless, this still begs the question of why the firm holds propriety information in the first place.

We have not found a convenient way to model natural incentives for propriety information without obscuring our conclusions with additional interaction effects. For example, one of the incentives we allude to, but do not model, is the possibility that shareholders have some conflict of interest, being perhaps shareholders of other firms that could make strategic use of the proprietary information of the firm in question. Our model is competitive, and does not address these strategic effects.

For a different motivation for proprietary information, also non-competitive and also not modeled in this paper, suppose the firm wishes to take an asset position "silently," so as not to inform the rest of the market of the potential movement up or down the supply curve for the asset. In other words, the firm acts monopolistically, realizing that its own spot market commitments, when actually purchased or sold, will move the equilibrium price for those assets against itself. The firm can hedge against those effects by taking an offsetting, or partially offsetting, futures position. In many cases, a firm may try to do this silently by dealing with different brokers and gradually assuming a position without drawing attention to its intentions. By informing shareholders, the firm reveals its intent to purchase or sell, and forces the firm to bear the full costs of its impact on the supply curve for its target assets, rather than sharing some of those costs with the other side of its futures positions. Once again, we do not attempt to build these rather complicated effects into our model.

Finally, although there is nothing in our model that would suggest this, it is costly for firms to share their production plans and future spot market commitments with their shareholders on an ongoing basis. Moreover, there would often be time lags between the date on which a firm commits to its production plan and the dates that shareholders could individually hedge these plans.

The remainder of the paper proceeds as follows: Section 2 outlines the primitive notions in the model in a simple two-period setting under uncertainty. In order to provide some contrasting background to our results, Section 3 reviews the standard MM theory in our setting. Section 4 presents the basic idea that we have to offer in the bluntest possible terms; hopefully the reader will find the assumptions in later sections more palatable. Section 5 presents our notion of unanimously supported corporate financial hedging strategies. In Section 6, which could be viewed as the principal body of our results, we present unanimously supported hedging strategies in various settings. Finally, Section 7 shows that when firms adopt such financial hedging policies, the resulting equilibrium allocations are in fact constrained Pareto optimal. Concluding remarks are made in Section 8.

2. The Model

For ease of exposition, we will examine a simple two-period economy with a single consumption good. In the first period, financial markets are open, and agents may trade securities and shares of firms. These securities and stocks entitle the agents to receive payoffs of the consumption good contingent upon the state of the economy, which is initially unknown. In the second period, this uncertain state is realized, and agents receive payoffs according to their portfolios. We formalize this model below.

2.1. *Agents*

We assume that the state of the economy is an element ω drawn from a set Ω, where the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ is common knowledge. Let L denote the set of bounded, real-valued random variables on this space. Conditional expectation statements throughout the paper are implicitly "almost surely."

There is a finite set I of agents. A consumption plan for agent $i \in I$ is a random variable $c_i \in L$, where we interpret $c_i(\omega)$ as consumption in state ω. The preferences of agent i are described by a Von Neumann–Morgenstern utility function. That is, a consumption plan c has the utility

$$U_i(c) \equiv \mathbf{E}[u_i(c) \mid \mathcal{H}_i],$$

where u_i is strictly increasing, differentiable, and concave, and $\mathcal{H}_i \subset \mathcal{F}$ is the σ-algebra representing the information available to agent i.

Each agent $i \in I$ is initially endowed with some random consumption e_i. The agent also learns some information in the first period (which may be relevant to this endowment variable) that is represented by a signal s_i, a random variable (valued in some measurable signal space whose nature varies with the context). The set of all agents' signals is denoted s_I.

2.2. *Securities and Shareholdings*

Agents may exchange contingent consumption plans by purchasing or selling shares of firms or other securities in the first period.

There is a finite set J of firms; each firm $j \in J$ pays a random consumption dividend $D_j \in L$ in the second period. Agents are initially endowed with shares $\bar{\theta}^i_j$, $i \in I$, such that $\sum_{i \in I} \bar{\theta}^i_j = 1$ for each firm j. Agents may buy or sell these shares for some price $v_j \in L$ in the first period.

Also available for trade in this economy is a finite set F of securities. Each share of a security $f \in F$ entitles (obligates) the holder to a contingent claim (payment) represented by the random variable $Z_f \in L$. These securities are in zero net supply; agents initially hold no shares. They may purchase or sell any security f in the first period at a price of $p_f \in L$ per share.

One element of F represents a bond. Each share of the bond provides 1 unit of consumption in the second period, independent of the state. Since agents' preferences are monotonic, the bond must always trade for a positive price in equilibrium; hence we may designate the bond as numeraire in the first period.

2.3. *Portfolio Problem*

Given the above set of assets available for trade, the first period portfolio problem for agent i can be described as

$$\begin{aligned}
(P_i)\ \max_{\theta, \varphi, b, c} \quad & U_i(c), \\
\text{subject to} \quad & b + \theta v + \varphi p \leqslant \bar{\theta}^i v, \\
& c = e_i + b + \theta D + \varphi Z,
\end{aligned}$$

where b is the number of bonds purchased, D is the (column) vector of firms' dividends, v is the vector of firms' share prices, θ is the (row) vector of the agent's stock portfolio, and similarly for Z, p, and φ, which correspond to the assets in F other than the bond. Since b, θ, and φ are chosen after agent's information s_i and prices (v, p) are observed, these choices are contingent on these random variables; that is, they are $\mathscr{H}_i$-measurable, where $\mathscr{H}_i = \sigma(s_i, v, p)$ is the σ-algebra representing the information available to agent i.

The monotonicity of agents' preferences implies that the first periods budget constraint is always strictly binding, so that

$$b_i = (\bar{\theta}^i - \theta^i)v - \varphi^i p$$

at any optimal solution.

We will adopt the assumption that agents have rational expectations; in particular, they use all of the information available to them from prices. Hence, the portfolio problem is rewritten as

$$\begin{aligned} (P_i)\ \max_{\theta,\varphi} \quad & \mathbf{E}[u_i(c) \mid s_i, v, p], \\ \text{where} \quad & c = e_i + \bar{\theta}^i v + \theta(D - v) + \varphi(Z - p). \end{aligned}$$

Note that this formulation of the problem is consistent with an interpretation of the zero-net-supply securities as futures contracts. In this case, one may think of the stochastic payoff Z_f as the future "spot" price, and p_f as the contract price, determined in the first period but paid in the second. This terminology will occasionally be used for interpretation of the results.

Finally, the assumed concavity of the utility function allows us to characterize the optimal portfolio choice for agent i in terms of simple first order conditions:

LEMMA 1. *The portfolio (θ, φ) solves (P_i) if and only if*:

$$\mathbf{E}[u_i'(c_i)(D - v) \mid s_i, v, p] = 0, \tag{1}$$

$$\mathbf{E}[u_i'(c_i)(Z - p) \mid s_i, v, p] = 0. \tag{2}$$

Proof. Since c_i, D, and Z are bounded, and u_i is concave, "differentiation inside the expectation" is justified, and the usual first and second order condition for optimization apply. ▮

2.4. *Firms*

We assume that firms' production decisions have been made and that each firm $j \in J$ produces output $Y_j \in L$. In the first period, the firm privately observes a signal represented by some random variable s_j. Let s_J denote the set of consisting of all firms' signals.

Aside from its productive output, each firm may generate revenues via its financial policy. Specifically, after observing its signal s_j, firm j may choose to hold a portfolio φ^j of securities and finance its purchases with b_j bonds. That is, it may hold any security portfolio (φ^j, b_j) subject to the first period budget balancing constraint

$$b_j + \varphi^j p = 0. \tag{3}$$

Given such a portfolio, the dividend of the firm is the sum of its productive and financial revenues. That is,

$$D_j = Y_j + \varphi^j Z + b_j.$$

Since the firm chooses its financial strategy after observing its signal, a *hedging strategy* for firm j is a portfolio (φ^j, b_j) that is $\sigma(s_j, v, p)$-measurable and bounded. For notational emphasis on the fact that φ^j depends on the information revealed by s_j in addition to the public information revealed by (v, p), we will often write $\varphi^j(s_j)$ for the portfolio outcome given the signal s_j. Thus, using the budget equation to solve for bond purchases,

$$D_j = Y_j + \varphi^j(s_j)(Z - p). \tag{4}$$

2.5. *Equilibrium*

An equilibrium for this economy will be specified with respect to an exogeneously given production and hedging strategy on the part of firms. Thus, the economy is described by

$$\langle(\Omega, \mathscr{F}, \mathscr{P}), (u_i, e_i, \bar{\theta}^i, s_i), (Y_j, s_j, \varphi^j), Z\rangle,$$

where φ^j is the hedging strategy of firm j.

An equilibrium for this economy is given by a set of portfolios and prices

$$\langle(\theta^i, \varphi^i), v, p\rangle$$

such that

1. Agents optimize conditional on their information:

   (θ^i, φ^i) is $\sigma(s_i, v, p)$-measurable and solves (P_i) given signal s_i and prices v and p;

2. Markets clear:

$$\sum_{i \in I} \theta^i = \sum_{i \in I} \bar{\theta}^i$$

$$\sum_{i \in I} \varphi^i + \sum_{j \in J} \varphi^j = 0;$$

3. Prices are demand-measurable:

$$(v, p) \text{ is } \sigma\left(s_I \cup \left\{\sum_{j \in J} \varphi^j\right\}\right)\text{-measurable.}$$

The last condition, demand-measurable prices, is simply a weak condition necessary to rule out "perverse" equilibria in which prices reveal information not contained in agents' and firms' demands. The crucial assumption is that agents should not be able to infer the portfolio choice of each firm by observing security prices.

Although this paper is not concerned with proving the existence of equilibria, our results apply only in equilibrium, so some comments on existence are in order. Under dimensionality restrictions, Allen [2] demonstrates the existence of fully revealing rational expectations equilibria, that is, equilibria in which prices reveal all private information. (This is true, in particular, with a finite number of states of the world.) A fully revealing equilibrium is fairly unnatural, and in any case trivializes the results in this paper. (We have therefore left the cardinality of Ω unrestricted.)

In order to demonstrate partially revealing equilibria, on the other hand, it has so far been found necessary to (i) make severe restrictions on utility functions, as in Ausubel [5], or (ii) introduce noise, for example in the form of un-modeled supply perturbations or garbled observations of prices, as in Allen [3], Anderson and Sonnenschein [4], Admati [1], and other papers, or (iii) allow a relaxation, as in MacAllister [18], of the rational expectations assumption that prices are given by a function of the underlying state of the world that is later revealed to be correct. While these alternative routes to partially revealing equilibrium with asymmetric information may eventually be replaced by a more general set of assumptions or definition of equilibrium, the results in this paper apply in any of these sorts of models, so long as individual agent optimality is characterized by the usual first order conditions.

As a final remark on the question of existence, our results are of interest even if individual agents are symmetrically informed, so long as they have incomplete information about the signals and choices of the firm. Since the firms' choices are taken as given in the model, we can therefore claim the existence of equilibrium with symmetrically informed agents under standard technical regularity conditions by referring to the available results on random Walrasian equilibria. (The object here is merely a $\sigma(s_I \cup \{\sum_{j\in J} \varphi^j\})$-measurable selection from the Walrasian correspondence defined on Ω, which provides an equilibrium state-by-state.) Yannelis [24], for example, obtains and surveys selection results of this variety.

3. Modigliani–Miller Theorem

The MM Theorem states that, in a world of perfect information, the financial policies of firms have no effect on equilibrium prices and allocations. In the context of this paper, however, firms may make financial decisions based on information not available to shareholders, violating the MM hypothesis. Any financial decision of the firm that is observed by agents, however, has no effect on the resulting equilibria. That is, if we define

$$\sigma_P \equiv \bigcap_{i\in I} \sigma(s_i, v, p),$$

so that σ_P represents the information commonly known to all agents in equilibrium, the following version of the MM Theorem holds:

Theorem 2. *Suppose that, given hedging policies φ^j, the economy has an equilibrium $\langle(\theta^i, \varphi^i), v, p\rangle$. Then, if firms adopt new hedging strategies $\hat{\varphi}^j$ such that $\hat{\varphi}^j - \varphi^j$ is σ_P-measurable, the economy has a new equilibrium with identical prices, consumption plans, and firm ownership, and with the new security portfolios*

$$\hat{\varphi}_i = \varphi^i - \sum_{j\in J} \theta^i_j(\hat{\varphi}^j - \varphi^j).$$

Proof. Obviously, since firm ownership has not changed, the market for firms' shares still clears. Also, since $\sum_i \varphi^i + \sum_j \varphi^j = 0$ in the initial equilibrium,

$$\sum_{i\in I} \hat{\varphi}^i = \sum_{i\in I} \varphi^i - \sum_{j\in J} (\hat{\varphi}^j - \varphi^j) = -\sum_{j\in J} \hat{\varphi}^j,$$

so that securities markets still clear. Next, the new consumption is given by

$$\begin{aligned}\hat{c}_i &= e_i + \bar{\theta}^i v + \theta^i(\hat{D} - v) + \hat{\varphi}^i(Z - p)\\ &= e_i + \bar{\theta}^i v + \theta^i(\hat{D} - v) + \left[\varphi^i - \sum_{j\in J} \theta^i_j(\hat{\varphi}^j - \varphi^j)\right](Z - p).\end{aligned}$$

But $\hat{D}_j = D_j + (\hat{\varphi}^j - \varphi^j)(Z - p)$, so that

$$\theta^i(\hat{D} - v) = \theta^i(D - v) + \sum_{j \in J} \theta^i_j(\hat{\varphi}^j - \varphi^j)(Z - p).$$

Therefore,

$$\hat{c}_i = e_i + \bar{\theta}^i v + \theta^i(D - v) + \varphi^i(Z - p) = c_i,$$

and consumption plans are unchanged. The optimality condition (2) is therefore unchanged. Since $\sigma_P \subset \sigma(s_i, v, p)$, the new portfolios are feasible for the agents and

$$\mathbf{E}[u_i'(c_i)(\hat{D} - v) \mid s_i, v, p] = \mathbf{E}[u_i'(c_i)(D - v) \mid s_i, v, p] + (\hat{\varphi}^j - \varphi^j)\,\mathbf{E}[u_i'(c_i)(Z - p) \mid s_i, v, p] = 0,$$

so condition (1) is also satisfied and agent optimization is guaranteed. ∎

Of course, we are most interested here in cases for which this theorem is not directly applicable; that is, cases in which the firms' hedging policies depend upon information that is not publicly known. Theorem 2 does imply, however, that results regarding any particular hedging policy actually apply to a whole equivalence class of policies, those generated by adding to a given policy any publicly known policy.

4. An Example: Cancellable Risk

This section presents a special case clearly illustrating the possibility of corporate hedging policies that are unanimously supported by shareholders.

Suppose the set F of securities can be partitioned into two sets F_1 and F_2 such that the corresponding payoff vectors Z_1 and Z_2 are independent. Further, suppose that subset F_1 represents risks that are held in a random amount by each firm, yet "cancel out" across firms. That is, suppose

1. $Y_j = G_j + s_j Z_1$, for real-valued s_j, for each firm $j \in J$,
2. (G, s) and Z_1 are independent, and,
3. $\sum_{j \in J} s_j = 0$.[1]

Finally, suppose that Z_1 and any agent's endowment e_i are independent. If firm j adopts the hedging strategy $\varphi^j_1(s_j) = -s_j$, then $D_j = G_j + s_j p$ is

[1] For this cancellation assumption, one might make formal arguments in a different model with infinitely many firms using the law of large numbers.

independent of Z_1. If agents choose portfolios with $\varphi_1^i = 0$, their consumption plans will also be independent of Z_1. It is easy to check that such an equilibrium exists with, from Eq. (2), $p_1 = \mathbf{E}[Z_1]$.

Now consider an alternative hedging policy $\hat{\varphi}_1^j$ for firm j, generating the new output

$$\hat{D}_j = D_j + (\hat{\varphi}_1^j - \varphi_1^j)(Z_1 - p_1).$$

Since $\hat{D}_j$ is a mean-preserving spread of D_j, a risk averse shareholder is worse off with $\hat{\varphi}^j$, as shown by Rothschild and Stiglitz [22]. Hence, the original hedging strategy is unanimously supported by the firm's shareholders.

5. Shareholder Unanimity

Since we are interested in determining whether the shareholders of a firm unanimously approve of a given hedging strategy it has undertaken, we must first examine the preferences held by shareholders over such strategies. Intuitively, a shareholder of a firm agrees with a particular hedging strategy if, given control of the firm, that shareholder would choose to adopt the same policy. Formally,

Definition 1. The hedging policy φ^j is optimal for shareholder i if, taking prices as given, it solves the following problem:

$$(P_{ij}) \max_{\theta, \varphi, \varphi^j(\cdot)} \quad \mathbf{E}[u_i(c) \mid s_i, v, p],$$

$$\text{where} \quad c = e_i + \bar{\theta}^i v + \theta(D - v) + \varphi(Z - p),$$

$$D_j = Y_j + \varphi^j(s_j)(Z - p).$$

This definition allows shareholder i to choose the firm's policy *after* observing s_i, making this notion of optimality stronger than if the policy is chosen ex-ante.

Suppose shareholder i holds a non-zero share of firm j. Then allowing i to choose j's hedging strategy allows i to hedge her own portfolio using both her own information and the firm's. This suggests that this optimality condition may be characterized by adjusting the first order condition (2) of the shareholder's hedging problem in order to account for the firm's information. This is verified in the following theorem:

Theorem 3. *Given an equilibrium with $\theta_j^i \neq 0$, the hedging policy φ^j of firm j is optimal for shareholder i if and only if*

$$\mathbf{E}[u_i'(c_i)(Z - p) \mid s_j, s_i, v, p] = 0. \tag{5}$$

Proof. Clearly, Eq. (5) is implied by the first order condition for the optimality of φ^j in (P_{ij}) when $\theta^i_j \neq 0$. Thus, we must demonstrate sufficiency. Unfortunately, the problem (P_{ij}) is not convex (the constraints involve a product of portfolios), so a first-order approach does not apply. Consider, however, the alternative problem

$$\begin{aligned} (P'_{ij})\ \max_{\theta, \varphi(\cdot)} \quad & \mathbf{E}[u_i(c) \mid s_i, v, p], \\ \text{where} \quad & c = e_i + \bar{\theta}^i v + \theta(D - v) + \varphi(s_j)(Z - p), \\ & D_j = Y_j. \end{aligned}$$

It is easy to check that the feasible consumption plans under (P'_{ij}) contain those under (P_{ij}). Thus, if a consumption plan solves (P'_{ij}), it must also solve (P_{ij}) if feasible. Problem (P'_{ij}) is convex, so optimality is implied by the first order conditions

$$\mathbf{E}[u'_i(c_i)(D - v) \mid s_i, v, p] = 0,$$
$$\mathbf{E}[u'_i(c_i)(Z - p) \mid s_j, s_i, v, p] = 0.$$

Since the first condition is already guaranteed in equilibrium, the second is sufficient for optimality of the firm's hedging policy, which completes the theorem. ▌

This result motivates the following definition, to be used shortly.

DEFINITION 2. The hedging policy of firm j is *unanimously supported* by its shareholders if Eq. (5) holds in equilibrium for each shareholder i of the firm.

Indeed, in the proofs that follow, we will often verify the even stronger condition

$$\mathbf{E}[u'_i(c_i)(Z - p) \mid s] = 0, \tag{6}$$

which implies Eq. (5) since $\sigma(s_j, s_i, v, p) \subset \sigma(s)$, where $s = s_I \cup s_J$.

6. OPTIMAL HEDGING POLICIES

Given the specification of the model so far, agents would typically have an incentive to make their portfolios contingent on all available information. For example, each component of s might yield additional information about a given agent's endowment. Since we wish to investigate motives for hedging based on the firm's proprietary knowledge of its own production risks, we must explicitly rule out these secondary effects. In order to do so, we first introduce a notion of "non-informativeness."

Recall that random variables A and B are conditionally independent relative to random variable C if, for any measurable subsets S_A and S_B of the respective ranges of A and B,

$$\mathscr{P}(A \in S_A, B \in S_B \mid C) = \mathscr{P}(A \in S_A \mid C)\,\mathscr{P}(B \in S_B \mid C).$$

Further, such conditional independence (and integrability) implies that

$$\mathbf{E}[A \mid B, C] = \mathbf{E}[A \mid C],$$

which reflects the intuition that B provides no additional information about A, once C is known. See Chung [6] for further reference.

This idea of conditional independence can be interpreted most simply in the case of random variables with a joint normal distribution. In this case, A and B are conditionally independent relative to C if and only if

$$\text{cov}_C(A, B) \equiv \mathbf{E}[AB \mid C] - \mathbf{E}[A \mid C]\,\mathbf{E}[B \mid C] = 0;$$

that is, if and only if A and B are conditionally uncorrelated, given C.[2]

We will use the notion of conditional independence to make assumptions that rule out secondary motives for trading on information that is unrelated to production risks. For instance, suppose that s_j is informative about e_i or Z to agent i. Then i has an incentive to adapt the hedging strategy of firm j so as to hedge i's own endowment. Consider also the case that s_h is informative about e_i or Z to agent i, for some agent $h \neq i$. Again, agent i would have an incentive to use the firm's financial policy to hedge i's own endowment, since the information s_j of the firm might, through prices, yield better information about s_h. These effects motivate the following:

Assumption A. For each agent i, s and (e_i, Z) are conditionally independent relative to s_i.

Note that, in the case of joint normality of the random variables, this condition is equivalent to the statement that for all agents i and any agent or firm h,

$$\text{cov}_{s_i}(s_h, e_i) = \text{cov}_{s_i}(s_h, Z) = 0.$$

Thus, conditional on i's own information, the signals of other agents and firms are not informative about agent i's endowment or the payoffs of the futures contracts.

[2] Though we do not use this assumption of joint normality for our results, we often restate our independence conditions using it to provide a more natural interpretation.

We also choose to rule out situations in which other agents have information about a firm j's production risks in addition to that known by firm j itself. In that case, agents might again have a purely informational demand to control other firms' hedging policies, since the information of those other firms could, together with prices, be informative about the risks of firm j. Finally, we also suppose that the firms are not "disadvantaged" in their hedging decisions due to poor information about futures payoffs. Thus, we make the following assumption:

Assumption B. For each firm j, s and (Y, Z) are conditionally independent relative to s_j.

Thus, the information of other agents and firms is not informative to firm j about its own production risk and hedging possibilities.

6.1. CARA Utility

Suppose all agents' preferences can be represented by utility functions with constant absolute risk aversion (CARA). This implies that

$$u_i(c) = -e^{-r_i c}$$

for some positive constant r_i.

Next, suppose that the risks faced by firms are "decomposable" in the following sense:

Definition 3. The production payoff of firm j is *decomposable* if

$$Y_j = G_j + \alpha^j(s_j) Z + M_j,$$

such that for each agent i, (s, M) and (e_i, Z, G) are conditionally independent relative to s_i; that is, s and M are not informative in each agents' decision to hedge G.

Under the assumption of joint normality, decomposability requires that s and G be conditionally uncorrelated given the agent's information, and similarly that M and (e_i, Z, G) be conditionally uncorrelated given the agent's information. Thus, decomposability implies that the firm's private information does not pertain to the component G of production, but only to a component of production that is "spanned" by the existing securities, $\alpha^j Z$, together with a third component M_j. This third, unspanned component, however, is uncorrelated with the other factors affecting the agent's consumption (i.e., e_i, Z, and G) given the agent's own information.

Decomposability substantially strengthens our initial assumptions, but still admits certain interesting cases:

EXAMPLE 1. Suppose firms have private information regarding the magnitudes of payments to be made in, say, various foreign currencies, and suppose currency futures are available. In this case, we take s_j to be a row vector of the same dimension as Z, the currency spot prices, and suppose that $Y_j = G_j + s_j Z$. More generally, if the futures contracts only approximately hedge this risk, we might have

$$Y_j = G_j + \alpha^j Z + \gamma^j \varepsilon_j,$$

with $s_j \equiv (\alpha^j, \gamma^j)$. This production Y_j is decomposable if, for each agent i, (s, ε) and (e_i, Z, G) are conditionally independent relative to s_i. Under the additional assumption of joint normality, this condition becomes

$$\mathrm{cov}_{s_i}(s_h, G_j) = 0,$$
$$\mathrm{cov}_{s_i}(e_i, \varepsilon_k) = \mathrm{cov}_{s_i}(Z_f, \varepsilon_k) = \mathrm{cov}_{s_i}(G_j, \varepsilon_k) = 0$$

for any i, h, j, k, and f.

Again, decomposability essentially implies that the private information of the firm only pertains to components of the dividend stream that are either spanned by the futures market, or are otherwise independent of variables of interest to the agents. If the firm completely hedges the spanned risk by adopting the futures position $\varphi^j = -\alpha^j$, its dividend stream is

$$D_j = G_j + \alpha^j p + M_j = G_j + \hat{M}_j, \tag{7}$$

where the residual $\hat{M}_j$ is viewed by each shareholder as an independent "wealth shock." Under the assumption of CARA utility, such a wealth shock has no effect on shareholders' "risk preferences," and hence the agents do not wish to make their portfolios contingent on this shock. This intuition leads to the following theorem regarding optimal hedging strategies:

THEOREM 4. *Suppose agents have CARA utility and firms' production payoffs are decomposable (Definition 3). Then, in equilibrium, shareholders unanimously support the hedging policy* $\varphi^j \equiv -\alpha^j$.

Proof. It is enough to verify that Eq. (5) holds for each agent i. Indeed, we will demonstrate the stronger condition

$$\mathbf{E}[u_i'(c_i)(Z - p) \mid s] = 0.$$

Decomposability implies $D = G + \hat{M}$ from Eq. (7), so that $c_i = e_i + \bar{\theta}^i v + \theta^i(G + \hat{M} - v) + \varphi^i(Z - p)$. Thus,

$$u_i'(c_i) = r_i \exp[-r_i(e_i + (\bar{\theta}^i - \theta^i)v + \varphi^i(Z - p) + \theta^i G + \theta^i \hat{M})],$$

and (6) can be rewritten

$$\mathbf{E}[r_i \exp[-r_i(e_i + (\bar{\theta}^i - \theta^i)v + \varphi^i(Z - p) + \theta^i G + \theta^i \hat{M})](Z - p) \mid s] = 0.$$

From the definition of decomposability, $\hat{M}$ and (G, Z, e_i) are conditionally independent relative to s_i, so that we may factor the above as the product of $\mathbf{E}[r_i \exp[-r_i \theta^i \hat{M}] \mid s]$ and

$$\mathbf{E}[\exp[-r_i(e_i + (\bar{\theta}^i - \theta^i)v + \varphi^i(Z - p) + \theta^i G)](Z - p) \mid s],$$

so that (6) holds if and only if

$$\mathbf{E}[\exp[-r_i(e_i + (\bar{\theta}^i - \theta^i)v + \varphi^i(Z - p) + \theta^i G)](Z - p) \mid s] = 0.$$

Since s is not informative about (e_i, Z, G) given s_i (by decomposability), this is equivalent to

$$\mathbf{E}[\exp[-r_i(e_i + (\bar{\theta}^i - \theta^i)v + \varphi^i(Z - p) + \theta^i G)](Z - p) \mid s_i] = 0.$$

Multiplying by $\mathbf{E}\ [r_i \exp(-r_i \theta^i \hat{M}] \mid s_i]$ yields the equivalent equality

$$\mathbf{E}[u_i'(c_i)(Z - p) \mid s_i] = 0,$$

which is clearly implied by the equilibrium condition for the agent's optimization problem,

$$\mathbf{E}[u_i'(c_i)(Z - p) \mid s_i, v, p] = 0.$$

Thus, Eq. (6) is satisfied for all shareholders, and unanimous support follows. ∎

6.2. "*Quadratic*" *Utility*

In this section we consider economies in which all agents have quadratic utility in a neighborhood of equilibrium consumption c_i, in the sense that, after a suitable linear transformation of u_i,

$$u_i'(c_i) = a_i - c_i, \tag{8}$$

for some positive constant a_i.

In this case it is natural to define a "full hedging" policy as the conditional L^2 projection of the risk onto the space spanned by the securities.

DEFINITION 4. The *full hedging policy* of firm j is given by

$$\varphi^j(s_j)' = -\mathbf{E}[(Z-p)(Z-p)' \mid s_j]^{-1}\mathbf{E}[Y_j(Z-p) \mid s_j].$$

Given this definition, we have the following result regarding shareholder unanimity:

THEOREM 5. *Suppose agents have quadratic utility in a neighborhood of equilibrium consumption; that is,* (8) *holds. Then shareholders unanimously support full hedging policies in equilibrium.*

Proof. First, we demonstrate condition (6) for all shareholders:

$$\mathbf{E}[u_i'(c_i)(Z-p) \mid s] = 0.$$

Since $c_i = e_i + \bar{\theta}^i v + \theta^i(D-v) + \varphi^i(Z-p)$, Eq. (6) can be written, using (8), as

$$\mathbf{E}([a_i - e_i - (\bar{\theta}^i - \theta^i)v - \varphi^i(Z-p)](Z-p) \mid s) - \sum_{j \in J} \theta_j^i \mathbf{E}[D_j(Z-p) \mid s] = 0. \tag{9}$$

By Assumption A, s is not informative about (e_i, Z) given s_i, so that

$$\begin{aligned}&\mathbf{E}([a_i - e_i - (\bar{\theta}^i - \theta^i)v - \varphi^i(Z-p)](Z-p) \mid s)\\ &\quad = \mathbf{E}([a_i - e_i - (\bar{\theta}^i - \theta^i)v - \varphi^i(Z-p)](Z-p) \mid s_i).\end{aligned}$$

Further, the full hedging policy of the firms implies that

$$\begin{aligned}\mathbf{E}[D_j(Z-p) \mid s] &= \mathbf{E}([Y_j + \varphi^j(Z-p)](Z-p) \mid s)\\ &= \mathbf{E}[Y_j(Z-p) \mid s] + \mathbf{E}[(Z-p)(Z-p)' \mid s]\,\varphi^{j'}\\ &= \mathbf{E}[Y_j(Z-p) \mid s] + \mathbf{E}[(Z-p)(Z-p)' \mid s_j]\,\varphi^{j'}\\ &= \mathbf{E}[Y_j(Z-p) \mid s] - \mathbf{E}[Y_j(Z-p) \mid s_j]\\ &= 0,\end{aligned}$$

where the last equations follow since s is not informative about (Y_j, Z) given s_j, by Assumption B. Thus, both terms in the sum (9) are s_i-measurable, so that (6) is equivalent to

$$\mathbf{E}[u_i'(c_i)(Z-p) \mid s_i] = 0,$$

which is implied by the equilibrium condition (2) for shareholders. Hence unanimity holds. ▌

As noted in Section 3, this unanimity result extends to any hedging policy of the firm that differs from the full hedging policy φ^j in a manner that is publicly known. In order to demonstrate that this characterization is necessary as well as sufficient for unanimity, and additional assumption is required:

Assumption C. For each realization of the agents' information s, the conditional distribution of Z is non-degenerate; that is, the matrix $\mathbf{E}[ZZ' \mid s]$ is positive definite for all s.

Essentially, this assumption requires that in no state of the world can any agent know the payoff of a portfolio $\varphi \neq 0$ with certainty. This allows the following characterization:

COROLLARY 6. *Under the conditions of Theorem* 5 *and Assumption* C, *consider an alternative hedging policy* $\hat{\varphi}^j$ *for firm* j *that differs from the full hedging policy* φ^j. *In equilibrium, this hedging policy is unanimously supported by the shareholders of firm* j *if and only if, for each shareholder* i *with* $\theta^i_j \neq 0$, $\hat{\varphi}^j - \varphi^j$ *is measurable with respect to* $\sigma(s_i, v, p)$, *the information of shareholder* i.

Proof. If firm j adopts an alternative hedging policy $\hat{\varphi}^j$, we have

$$\begin{aligned}&\mathbf{E}[u_i'(c_i)(Z-p) \mid s_i, v, p] - \mathbf{E}[u_i'(c_i)(Z-p) \mid s_j, s_i, v, p]\\ &\quad = \theta^i_j \mathbf{E}[(Z-p)(Z-p)' \mid s_i](\hat{\varphi}^j - \varphi^j - \mathbf{E}[\hat{\varphi}^j - \varphi^j \mid s_i, v, p])'.\end{aligned}$$

Thus, in equilibrium, this policy is optimal for shareholder i if and only if

$$\theta^i_j \mathbf{E}[(Z-p)(Z-p)' \mid s_i](\hat{\varphi}^j - \varphi^j - \mathbf{E}[\hat{\varphi}^j - \varphi^j \mid s_i, v, p])' = 0. \tag{10}$$

By hypothesis, $\theta^i_j \neq 0$, and from Assumption C, $\mathbf{E}[(Z-p)(Z-p)' \mid s_i]$ is positive definite for any s_i. Hence (10) is equivalent to

$$\hat{\varphi}^j - \varphi^j = \mathbf{E}[\hat{\varphi}^j - \varphi^j \mid s_i, v, p],$$

which is precisely the condition that $\hat{\varphi}^j - \varphi^j$ be $\sigma(s_i, v, p)$-measurable. ∎

6.3. *Partial Disclosure*

The previous sections have required substantial restrictions on the form of shareholders' preferences in order to demonstrate unanimously supported hedging policies. One might expect, however, that in the case where production includes a component of risk that is completely spanned by the securities markets, shareholders would prefer the firm to hedge these risks. This intuition may fail since, even if the firm were to hedge such risks, it would still hold proprietary information as to the market value of its portfolio. Since this market value affects shareholder wealth, unless agents' risk preferences are independent of wealth (for example, CARA utility), or unless the wealth effects are identical across shareholders (as is the case with quadratic utility), shareholders disagree about the firm's optimal response to such information.

In this section we consider cases in which the firm may communicate a limited amount of information to its shareholders costlessly. Obviously, if the firm can communicate its signal s_j, there is no role for a firm's hedging policy since shareholders are perfectly informed and can adjust their own portfolios to suit their needs, as was shown in the MM results of Section 3. If information is proprietary or costly to disseminate, there may be some partial disclosure that recovers unanimity for certain financial hedging strategies of benefit to shareholders. As the above discussion suggests, it may be particularly convenient for the firm to reveal the total market value of the hedged risk.

DEFINITION 5. The private information s_j of firm j is *reducible* with respect to a hedging strategy φ^j if, given the associated dividends

$$D_j = Y_j + \varphi^j(s_j)(Z - p),$$

for any agent i, s and (e_i, Z, D) are conditionally independent relative to s_i; that is, the signal s is not informative for each agent's hedging decision.

If we impose the additional assumption of joint normality, then reducibility holds if s and D_j are conditionally uncorrelated given the agent's own information. While this condition is unlikely to be satisfied in general, it will hold in the case of "spanned" risk if the shareholders are informed of the market value of the risk:

EXAMPLE 2. Consider again the foreign currency risk case in Example 1, with $Y_j = G_j + s_j Z$. Consider the hedging strategy $\varphi^j = -s_j$, which yields $D_j = G_j + s_j p$. Reducibility of s_j with respect to φ^j is satisfied if the shareholders of the firm know the magnitude $s_j p \equiv -b_j$; that is, if shareholders know the total market value of the foreign change risk, but not necessarily the particular currency commitments of the firm.

The following theorem is immediate:

THEOREM 7. *Suppose the private information of firms is reducible with respect to hedging policies (φ^j). Then, in equilibrium, shareholders unanimously support these hedging policies.*

Proof. By hypothesis, s is not informative about (e_i, Z, D) given s_i. Since c_i is a function of (e_i, Z, D), so is $u_i'(c_i)(Z-p)$, implying that

$$\mathbf{E}[u_i'(c_i)(Z-p)\,|\,s] = \mathbf{E}[u_i'(c_i)(Z-p)\,|\,s_i].$$

Hence, the unanimity condition (6) is equivalent to the shareholder's equilibrium optimality condition (2), and the theorem holds. ▌

Again, Theorem 2 implies that alternative policies which differ from φ^j in a publicly known fashion are also unanimously supported in equilibrium. With the further assumption of non-degeneracy of Z, we show that this characterization is complete:

COROLLARY 8. *Under the conditions of Theorem 7 and Assumption* C, *consider an alternative hedging policy $\hat{\varphi}^j$ for firm j. In equilibrium, this hedging policy is unanimously supported by the shareholders of firm j if and only if, for each shareholder i with $\theta_j^i \neq 0$, $\hat{\varphi}^j - \varphi^j$ is measurable with respect to $\sigma(s_i, v, p)$, the information of shareholder i.*

Proof. Suppose firm j instead adopts a policy $\hat{\varphi}^j$. The resulting dividend is

$$\hat{D}_j = Y_j + \varphi^j(Z-p) + (\hat{\varphi}^j - \varphi^j)(Z-p) = D_j + (\hat{\varphi}^j - \varphi^j)(Z-p),$$

and the equilibrium consumption of shareholder i is

$$c_i = e_i + \bar{\theta}^i v + \theta^i(D-v) + \varphi^i(Z-p) + \theta_j^i(\hat{\varphi}^j - \varphi^j)(Z-p).$$

If shareholder i could instruct the firm to adopt the hedging policy φ^j, and adjust her own portfolio φ^i to be, instead.

$$\tilde{\varphi}^i \equiv \varphi^i + \theta_j^i \mathbf{E}[\hat{\varphi}^j - \varphi^j\,|\,s_i, v, p],$$

then her new consumption $\tilde{c}_i$ would satisfy

$$c_i = \tilde{c}_i + \theta_j^i[\hat{\varphi}^j - \varphi^j - \mathbf{E}(\hat{\varphi}^j - \varphi^j\,|\,s_i, v, p)](Z-p).$$

This implies that $\mathbf{E}[c_i\,|\,\tilde{c}_i, s_i, v, p] = \tilde{c}_i$, so that, by the strict version of Jensen's Inequality, unless $\tilde{c}_i = c_i$, $\mathbf{E}[u(c_i)\,|\,s_i, v, p] < \mathbf{E}[u(\tilde{c}_i)\,|\,s_i, v, p]$. The condition $\tilde{c}_i = c_i$ is, however, equivalent to

$$\theta_j^i[\hat{\varphi}^j - \varphi^j - \mathbf{E}(\hat{\varphi}^j - \varphi^j\,|\,s_i, v, p)](Z-p) = 0,$$

which, upon post-multiplying by $Z-p$ and taking the expectation with respect to s, becomes exactly Eq. (10). Hence, the remainder of the proof follows as in that of Corollary 6. ▌

A simple application of this result applies to cases in which a firm adopts "random" or "noisy" hedging policies; that is, chooses a portfolio φ^j based on a signal s_j that is independent of the other variables in the model. Our result implies that shareholders unanimously support the elimination of such noisy portfolio choices by the firm.

7. Constrained Optimality

In this section we analyze efficiency properties of equilibria in which firms adopt "optimal" hedging policies. Since markets are incomplete, one cannot hope to achieve full Pareto optimality via market allocations. There are, nevertheless, useful second-best notions of constrained optimality. For example, in the spirit of Diamond [9], we may suppose that a social planner has available all of the information known to agents in the first period and has the ability to choose any *market feasible* allocation; that is, any allocation achievable through an appropriate distribution of firms' shares and securities.

Definition 6 (Second-Best Optimality). An allocation (θ, φ, b) is *second-best Pareto optimal* if it solves the following optimization problem for some positive constants (λ_i):

$$(P_2)\ \max_{\theta, \varphi, b} \quad \mathbf{E}\left[\sum_{i \in I} \lambda_i u_i(c_i) \mid s\right],$$

$$\text{where} \quad c_i = e_i + b_i + \theta^i Y + \varphi^i Z,\ i \in I,$$

$$\text{subject to} \quad \sum_i \theta^i = \sum_i \bar{\theta}^i,$$

$$\sum_i \varphi^i = 0, \sum_i b_i = 0.$$

Theorem 9. *A feasible allocation is second-best Pareto optimal if and only if the following conditions hold for some prices (v, p) and for all agents $i \in I$:*

$$\mathbf{E}[u_i'(c_i)(Y - v) \mid s] = 0, \tag{11}$$

$$\mathbf{E}[u_i'(c_i)(Z - p) \mid s] = 0. \tag{12}$$

Proof. Since problem (P_2) is convex, it can be characterized by the first order conditions:

$$\mathbf{E}[\lambda_i u_i'(c_i) \mid s] = \beta_b,$$

$$\mathbf{E}[\lambda_i u_i'(c_i) Y \mid s] = \beta_\theta,$$

$$\mathbf{E}[\lambda_i u_i'(c_i) Z \mid s] = \beta_\varphi,$$

where $(\beta_b, \beta_\theta, \beta_\varphi)$ are the Lagrange multipliers for the corresponding constraints, with $\beta_b > 0$. This can be reduced to the above Eqs. (11) and (12) upon substitution of v for β_θ/β_b and p for β_φ/β_b. ▮

Having characterized the conditions for a second-best allocation, we now demonstrate that such an allocation is achieved when firms' private information is reducible with respect to their hedging policies. This result is somewhat related to the work of Diamond [9] and others establishing the constrained optimality of stock market equilibria when production possibilities are spanned by existing markets, since the reducibility assumption is, in some sense, a "spanning" assumption about the nature of the private information.

THEOREM 10. *Suppose the private information of firms is reducible with respect to hedging strategies* (φ^j), *as defined in Section* 6.3. *Then, in an equilibrium in which firms adopt these strategies, the equilibrium allocation is second-best Pareto optimal.*

Proof. From Theorem 7, condition (12) is satisfied in equilibrium, and we only need to check condition (11). As was argued in the earlier proof, however, c_i is a function of (e_i, Z, D), so that s is not informative about $u_i'(c_i)(D-v)$ given s_i. This implies that

$$\mathbf{E}[u_i'(c_i)(D-v) \mid s] = \mathbf{E}[u_i'(c_i)(D-v) \mid s_i] = 0,$$

where the equality follows from the agents' optimization condition (1). Next, since $Y_j = D_j - \varphi^j(Z-p)$,

$$\begin{aligned}
&\mathbf{E}[u_i'(c_i)(Y_j - v_j \mid s]\\
&\quad = \mathbf{E}[u_i'(c_i)(D_j - \varphi^j(Z-p) - v_j) \mid s]\\
&\quad = \mathbf{E}[u_i'(c_i)(D_j - v_j) \mid s] - \varphi^j \mathbf{E}[u_i'(c_i)(Z-p) \mid s]\\
&\quad = 0.
\end{aligned}$$

Hence, both conditions are satisfied and the allocation is indeed second-best Pareto optimal. ▮

8. CONCLUDING REMARKS

In this section we comment on several possible extensions and generalizations of the results in the paper:

Weaker Criteria than Unanimity. This paper has adopted the strongest possible notion of shareholder agreement with a particular financial hedging policy: shareholder unanimity. Obviously, such a strict criterion calls for strict assumptions on the nature of the firm's risks and private information in order to obtain clear results. One could weaken this criterion in a variety of ways. For example, one could compare the current policy only with the alternative of doing nothing, and then ask whether it is preferred by a majority of the shareholdes (weighted by their holdings). This would allow one to make statements about a much broader class of production/information structures, though it would likely involve a concomitant loss in precision.

Firm Control and Objectives. Thus far, this paper has made no explicit mention of the objectives of the firm with regard to its financial policy. Under the assumption that control of the firm ultimately rests with its shareholders, the unanimity criterion developed here is rather compelling. This argument would also hold if firms were controlled by managers whose compensation depends linearly on the performance of the firm. If managers' compensation is a non-linear function of output, however, managers' and shareholders' interests would likely be in conflict, and we would not expect managers whose actions are only partially observable to implement the "optimal" hedging strategies discussed here.

Competitive Value Maximization. An alternative approach to resolving the firm's decision problem is to suppose that firms attempt to maximize their market value. With complete markets, this is done by supposing that firms make plans taking market prices as given. With incomplete markets, however, prices do not exist for all contingent commodities, so in order to maximize value, firms must conjecture prices. If we assume that firms act competitively with respect to prices, the most natural approach is to suppose that firms conjecture some "state" prices that are consistent with existing market prices. That is, firm j conjectures positive state prices $q^j \in L$ such that

$$\mathbf{E}[q^j(D-v) \mid \sigma_c] = 0,$$
$$\mathbf{E}[q^j(Z-p) \mid \sigma_c] = 0,$$

where $\sigma_c \equiv \bigcap_k \sigma(s_k)$ is the information commonly known to all agents and firms. If firm j assumes its share price is generated according to such state prices, a value-maximizing portfolio choice is one that maximizes

$$\mathbf{E}[q^j[Y_j + \varphi^j(Z-p)] \mid s_j, v, p].$$

Thus, a value-maximizing equilibrium may be characterized by the condition

$$\mathbf{E}[q^j(Z-p) \mid s_j, v, p] = 0$$

for each firm. Clearly, this condition cannot be expected to hold if arbitrary conjectures by firms are allowed, since they may then posit an arbitrary relationship between the state prices q^j and their information s_j. If agents are symmetrically informed, however, so that $\sigma(s_i) = \sigma_c$, and if the conjectures q^j are restricted to be measurable with respect to the consumption profile $(c_i)_{i \in I}$, then in the case of reducible information described in Section 6.3, the given hedging policies are indeed value maximizing.

Value Maximization with Quadratic Utility. In the case of quadratic utility, a stronger statement can be made regarding value maximization since it is possible to calculate directly the general equilibrium share values corresponding to a particular financial policy. In particular, if shareholders are symmetrically informed so that $\sigma(s_i) = \sigma_c$, then the equilibrium value of firm j is given by the formula, $v_j = \mathbf{E}[u'(\sum_i c_i)\, D_j \mid \sigma_c]$, where u is a quadratic utility function with $u'(c) = \sum_i a_i - c$. Since any financial policy by the firm does not change aggregate consumption but only redistributes it, a hedging policy maximizes the *general equilibrium share value* of the firm if and only if

$$\mathbf{E}\left[u'\left(\sum_i c_i\right)(Z - p) \mid s_j, v, p\right] = 0. \tag{13}$$

However, if firms adopt the full hedging strategies of Section 6.2, (13) can be shown to hold, exactly as in the proof of Theorem 5, so that firms are value maximizing in this strong sense.

Hedging with Common Stock. The analysis of the preceding sections could be generalized to allow firms to hedge by trading shares of other firms as well as securities. In this case,

$$D_j = Y_j + \theta^j(D - v) + \varphi^j(Z - p),$$

so that dividends must be simultaneously determined. If we define θ^J to be the matrix with row j equal to θ^j, then dividends can be calculated as follows, assuming the matrix $I - \theta^J$ is non-singular:

$$D = v + (I - \theta^J)^{-1}(Y - v) + \varphi^j(Z - p).$$

It can then be shown that the "unanimity" condition of Section 5 can be generalized to include the condition that

$$\mathbf{E}[u_i'(c_i)(D - v) \mid s_j, s_i, v, p] = 0$$

for each agent $i \in I$. The basic analysis of the paper can then be conducted in a similar fashion.

Interaction with Production. Throughout this paper, the production decisions of the firms have been taken as given. A natural extension would be to endogenize these production decisions and explore the interaction that might result when both production and financial policies are determined simultaneously. Such an extension might permit a generalization of the "spanning" literature, in a similar manner to that by Leland [17], and thus allow firms to evaluate unambiguously a broader class of production alternatives.

Multi-period, Multi-good Economies. Another obvious extension of the model presented in this paper would be to add multiple commodities and multiple time periods. Expanding the commodity space should in no way change the basic analysis of the paper, once asset payoffs are converted to a common numeraire, taking relative prices as given. Extending the time horizon of the model would also not affect the basic results, though it would introduce the possibility of intertemporal dividend smoothing, in addition to the intratemporal smoothing considered here. Also, the information revealed by the current period dividends of the firm may be quite important, as suggested by the "partial disclosure" results of Section 6.3.

In conclusion, it seems that the approach taken here yields insight into an aspect of corporate financial policy that has received relatively little attention in the debate and analysis that has arisen since Modigliani and Miller's original challenge.

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