# Funding Value Adjustments*

Leif Andersen[†], Darrell Duffie,[‡] and Yang Song[§]

June 8, 2017

**Abstract**

We demonstrate that the large funding value adjustments (FVAs) being made by major dealers are equal to the debt-overhang costs to their shareholders for funding derivatives positions. In order to maximize shareholder value, dealer quotations for swaps should therefore adjust for FVAs, even though the FVAs are not themselves components of the market values of the derivatives. The current dealer practice of reducing the disclosed market values of their swap books by FVA is inconsistent with any coherent notion of fair market value, but does align incentives between swap desks and shareholders. While others have already suggested that FVA accounting suffers from coherence problems, this is the first paper to identify and characterize these problems with a structural model of a dealer's balance sheet. We also establish a pecking order for preferred swap financing strategies, characterize the valuation effects of initial margin financing (known as "MVA"), and provide a new interpretation of the standard debit value adjustment (DVA).

JEL Classification Codes: G12, G23, G24, G32

*We are grateful for comments from the referees, the associate editor, and the editor, as well as Claudio Albanese, Shalom Benaim, Damiano Brigo, Stéphane Crépey, Yuanchu Dang, Rupert Brotherton-Ratcliffe, Yann Coatanlem, Stéphane Crépey, Youssef Elouerkhaoui, Marco Francischello, Jon Gregory, Lincoln Hannah, Burton Hollifield, John Hull, David Lando, Wujiang Lou, Alexander Marini, Martin Oehmke, Andrea Pallavicini, Stephen Ryan, Taylor Spears, and Hongjun Yan.

†Bank of America Merrill Lynch.

‡Dean Witter Distinguished Professor of Finance, Graduate School of Business, Stanford University, and National Bureau of Economic Research.

§Graduate School of Business, Stanford University.

# I. Introduction

We demonstrate that the large funding value adjustments (FVAs) being made by major dealers to their swap books are actually debt-overhang costs to their shareholders. Although the current dealer practice of including FVAs as components of the disclosed market values of their swap books is not consistent with any coherent notion of fair market value, it does align incentives between a dealer's swap desk and its shareholders.

In its simplest form, we show that FVA is merely a wealth transfer from a dealer's shareholders to its legacy creditors. FVAs can therefore be an important source of friction in over-the-counter markets that require significant dealer financing. Dealers must quote prices that extract sufficient trading profits from their counterparties to overcome the associated FVA costs to their shareholders.

While others have already suggested that FVA accounting suffers from various coherence problems, this is the first paper to identify and characterize these problems with a structural model of a dealer's balance sheet.

A typical argument by dealers in support of FVA practice is that their derivatives desks do not have access to funding at secured financing rates, and instead must rely on the firm's treasury to finance their activities, often through the issuance of unsecured debt. The financial crisis of 2007-2009 increased spreads on bank debt significantly, making these financing activities much more costly to a bank-affiliated dealer's derivatives desk, which is charged for access to cash at the firm's unsecured borrowing rate. Ostensibly, the FVA is the present value of these financing costs, and is presented by dealers as an adjustment to the market values of derivatives.

As a simple motivating example, consider the purchase by a dealer of \$100 face value of one-year T-Bills. For ease of illustration, suppose the dealer commits to hold the T-bills to maturity and that risk-free interest rates are zero. Suppose the dealer purchases the T-bills at their mid-market value of \$100, and that the purchase is funded by unsecured one-year debt with a credit spread of 50 basis points. In one year, the T-bills will pay \$100 and the dealer will repay \$100.50 on its financing. The dealer's shareholders will suffer a net loss after financing costs of \$0.50. The loss will be borne by the dealer's shareholders only if the dealer survives. Thus, the dealer's shareholders suffer a loss in the initial market value of their equity of $p$, where $p$ is the dealer's risk-neutral one-year survival probability. For example, taking $p = 0.99$, the net cost to shareholder equity value is $p\ell = \$0.495$. This cost to shareholders is the FVA for this trade. The FVA is a transfer in value to legacy creditors, who now have access to an additional safe asset in default. (In the presence of default distress costs, we will show that the FVA is the sum of a transfer in value from equity to debt and an increase in default distress costs.)

If the dealer were to apply FVA accounting in this example following the same principles used in practice for swaps, then the T-bills would be assigned a fair market value equal to the mid-market value of \$100 less a funding value adjustment of \$0.495, for a net marked value of only \$99.505. By assumption, however, the T-bills have an actual market value of \$100, implying an inconsistency. In current practice, dealers do not actually apply FVAs to bonds. This inconsistency applies only to their swap books.

Table I Funding value adjustments of major dealers (millions). Source: supplementary notes of quarterly or annual financial disclosures.

| | Amount | Date Disclosed |
|---|---|---|
| Bank of America Merrill Lynch | \$497 | Q4 2014 |
| Morgan Stanley | \$468 | Q4 2014 |
| Citi | \$474 | Q4 2014 |
| HSBC | \$263 | Q4 2014 |
| Royal Bank of Canada | C\$105 | Q4 2014 |
| UBS | Fr267 | Q3 2014 |
| Crédit Suisse | Fr279 | Q3 2014 |
| BNP Paribas | €166 | Q2 2014 |
| Crédit Agricole | €167 | Q2 2014 |
| J.P. Morgan Chase | \$1,037 | Q4 2013 |
| Nomura | \$98 | Q1 2014 |
| ANZ | AUD61 | Q4 2013 |
| Bank of Ireland | €36 | Q4 2013 |
| Deutsche Bank | €364 | Q4 2012 |
| Royal Bank of Scotland | \$475 | Q4 2012 |
| Barclays | £101 | Q4 2012 |
| Lloyds Banking Group | €143 | Q4 2012 |
| Goldman Sachs | Unknown | Q4 2011 |

Under the deal terms of this T-bill example, a dealer acting on behalf of its shareholders should not be willing to enter the trade. For the trade to benefit shareholders, the dealer requires a trading profit that exceeds the FVA, and thus should quote a bid price for the T-bills that is no higher than \$99.505. In general, a dealer's quotation practice should reflect funding value adjustments, in order to align its market-making function with shareholder interests. Although the current swap valuation practice of dealers is not correct, it does achieve this alignment of incentives. As we will discuss, there are other ways to obtain this alignment that do not involve valuation inconsistencies, and which could be applied not only to swap market making, but more broadly wherever funding requirements are significant.

Funding costs have long been informally considered an input to dealer trading decisions. Until recently, however, the costs of derivatives funding have not shown up in fair-market-value accounting disclosure. In a significant change of market practice, major dealer banks recently started to formally show FVAs on their balance sheets, as indicated by Cameron (2014b) and Becker (2015). Many of the resulting write-downs of assets and corresponding adjustments to earnings are large. For example, as indicated in Table I, in 2013 J.P. Morgan Chase recorded an FVA charge of over \$1 billion to the fair value of its assets. Details on how these accounting adjustments have been made are discussed by Albanese, Andersen, and Iabichino (2015). Sometime around 2014, dealers stopped making separate disclosures of their funding value adjustments. We suspect that these adjustments are now significantly higher than those shown in Table I because of substantial increases in regulatory margin requirements for dealers that were introduced in 2016.

The move by dealers to introduce funding value adjustments probably has several causes. First, beginning in 2008, severe deviations of dealers' borrowing rates from risk-free rates resulted in funding costs that were so large that excluding them from financial statements might have been considered imprudent. (Indeed, we provide assumptions under which large FVAs should be made, although not to the asset side of the balance sheet.) Second, the finance departments of many dealers now feel confident that funding cost adjustments are observable in market transaction terms. (Our model explains why this should be the case.) Third, despite the absence of published financial accounting standards that support FVA practice, large accounting firms have signaled a willingness to accept FVA disclosures in dealers' financial statements. See, for example, KPMG (2013) and Ernst and Young (2012).

Current accounting practice also implies that FVAs generate tax savings for dealers, because their taxable incomes are lowered whenever swap values are reduced for accounting purposes by FVAs. In economic terms, however, FVAs do not actually involve a reduction in income. Like any other debt-overhang cost, an FVA is actually a transfer in value from equity shareholders to creditors, not a reduction in income.

While FVA accounting has seemed natural to many practitioners, the practice has not been without controversy. Concerns about the validity of FVA accounting have been raised, for instance by Hull and White (2012, 2016), Cameron (2013, 2014a), Becker and Sherif (2015), and Sherif (2016b). Some have pointed to questionable asset-liability asymmetries in FVA accounting, a seeming absence of accounting for the DVA effects of the associated debt issuance, and an incongruity in the way that FVA for derivatives liabilities overlap with already-reported DVA for derivatives. These issues have been discussed by Hull and White (2012, 2014, 2016), Albanese and Andersen (2014), and Albanese et al. (2015), among others. In addition, there appears to be significant variation across dealers in the manner in which dealers compute their FVA metrics, particularly with respect to measurement of the relevant unsecured borrowing rates. Recently, the Office of the Comptroller of the Currency, a U.S. banking regulator, announced the formation of a working group to examine industry practices for FVA determination. (See Sherif (2015b).)

Missing from the controversy over FVA, to this point, has been a model that is consistent with underpinning theories of asset pricing and corporate finance and that accounts for the impact of unsecured derivatives funding strategies on the market valuation of claims on a dealer's assets, most importantly equity and debt. We provide such a model, along with a number of implications for dealer quotations, swap desk incentives, and preferred financing strategies.

We show, by theory and calibrated numerical examples, that financing cost adjustments are also an important determinant of dealer bid-ask spreads. Because the financing of collateral or cash upfront payments can cause a change in capital structure that is costly to dealer shareholders, dealers maximize shareholder value by using quoting strategies that overcome this cost to their shareholders with a sufficient widening of bid-ask spreads.

As an empirical example, Wang, Wu, Yan, and Zhong (2016) estimate the impact of the 2009 "big-bang" introduction of upfront payments for credit default swaps on CDS bid-ask spreads. They write: "Intuitively, the upfront payment is an impediment to trading, and so reduces the market liquidity, leading to higher bid-ask spreads." Our model justifies this intuition. Wang et al.

(2016) indeed find that big-bang upfronts widened bid-ask spreads significantly.[1]

As another example, we consider the post-crisis violations of covered interest parity (CIP) documented by Du, Tepper, and Verdelan (2017) and Rime, Schrimpf, and Syrstad (2017). For a dealer to benefit its shareholders by arbitraging a CIP violation, our FVA calculations imply that (absent unused netting opportunities) the CIP basis must roughly exceed the dealer's credit spread.

More generally, our results are part of a growing body of work, including for example Adrian, Etula, and Muir (2014) and Brunnermeier and Pedersen (2009), that examines the impact of dealer capital structure on asset price behavior. Because over-the-counter (OTC) markets rely heavily on intermediation by dealers, FVA can play a significant role in the liquidity of OTC products whose intermediation requires substantial amounts of dealer funding.

To our knowledge, of prior related work on FVA,[2] only Burgard and Kjaer (2011) and Castagna (2013, 2014) specifically incorporate the incremental cash flows of a swap into a model of the balance sheet of a dealer. Using a reduced-form model of the event of the dealer's default, but explicitly capturing the impact of swaps on the dealer's default recovery, Burgard and Kjaer (2011) show that adding an appropriately hedged derivative has no impact on the dealer's funding costs.[3] They do not use their balance-sheet model to isolate the nature of FVA as a cost to shareholders. Indeed, contrary to our results, their approach allows swap market values to be affected by dealer funding costs.[4] In a narrower setting, Castagna (2013, 2014) calculates a marginal funding-cost impact on shareholders that is similar in spirit to our own. In the end, however, Castagna (2014) concludes that the market valuation of derivatives should include the FVA component, which is opposite to our result. The similar approach but different conclusion of Castagna arises from his implicit assumption that the valuation of a financial instrument is the value of only that component of its cash flows that is ultimately assigned to equity shareholders.[5]

The rest of this paper is organized as follows. Section II introduces a basic two-period model of the marginal effects of investments and investment financing decisions on the market valuation of the firm's debt and equity. Section III applies and extends these basic results to swap valuation, the impact of swap valuation on a dealer's equity and debt, and swap rate quotation. Here, we provide a new theoretical foundation for funding value adjustment, showing how it applies to a dealer's equity with a compensating partial adjustment to debt valuation, but with no impact on fair swap

[1] They find that "for a CDS contract with a spread level of 300 basis points, at the average level of the Libor-OIS spread in our sample, 32 basis points, the upfront payment introduced by the CDS Big Bang increases the bid-ask spread by 1.5 basis points. This is a sizeable effect as the bid-ask spread in our sample has a mean of 9.6 basis points and median of 5.3 basis points."

[2] There is a large body of applied derivatives valuation research that addresses FVA and related concepts. Key examples include Pallavicini, Perini, and Brigo (2012), Pallavicini, Perini, and Brigo (2011) and Elouerkhaoui (2016).

[3] See, for example, their equations (20)-(25).

[4] Burgard and Kjaer (2011) also construct dealer strategies that can "shield the balance sheet" from funding costs, thus eliminating or reducing inconsistencies that arise in current practice when the same swap cash flows are not valued symmetrically by their two counterparties due to funding value adjustments.

[5] For example, Castagna (2014) states, at page 14, that "The results just shown confirm also that the practice of including the funding valuation adjustment (FVA) in the valuation (i.e.: internal pricing) process of a contract is fully justified: this thesis was supported in Castagna [7] (arguing against the opposite view in Hull&White [12] and [14]) but not proved analytically."

valuation. We treat swaps with and without upfront payments, as well as the impact of initial and variation margin. In Section IV, we illustrate the magnitudes and directional responses of FVAs and DVAs that may be anticipated in practical settings of plain-vanilla interest-rate swaps, based on a reduced-form analogue of a structural multi-period version of the model that is developed in an appendix. Section V summarizes our key results and discusses some key implications. Proofs and other extensions are found in appendices.

## II. Shareholder Financing Costs

This section characterizes the effect on a firm's shareholders and creditors of financing an investment, or a package of financial transactions. The results include explicit calculations of the impact of an investment on market valuations. In this section, we focus on debt financing, which is the basis for FVA. Appendix A provides the analogous explicit calculations for the impact to shareholder value of equity financing and financing with existing balance sheet cash, as well as a pecking order of preferred financing methods. These results recapitulate relatively standard concepts of asset pricing and corporate finance in a somewhat novel form that is useful for explaining the role of FVA and for solving the valuation and price quotation problems faced by a swap dealer.

### *A. Representation of Fair Valuations*

Our most basic setting is a market at time 0 for claims to uncertain cash flows at time 1. For simplicity, we assume that the set of possible states of the world at time 1 is finite. All of our results apply in the general case of infinitely many states of the world under standard technical continuity conditions.[6] The proofs of our results, given in Appendix B, cover both finite-state and infinite-state cases. Without loss of generality, each state has some given strictly positive probability. All investors in our model have the same information.

In order to characterize the fair valuation[7] of financial instruments that may appear on the balance sheet of a dealer, we fix the set $\mathcal{L}$ of payoffs at time 1 to which a fair value at time

[6] For general one-period models with the potential for infinitely many states or infinitely many traded instruments, we can fix an arbitrary probability space $(\Omega, \mathcal{F}, P)$. In addition to the given assumptions, sufficient additional regularity is obtained by assuming that the set $\mathcal{L}$ of payoffs to which a valuation is assigned is a linear subspace of the set $L^1(P)$ of random variables with finite expectation having the property that $\mathcal{L} - L^1(P)_+$ is closed in $L^1(P)$. The existence of a bounded stochastic discount factor $\nu$ then follows from Yan's Separation Theorem. See, for example, Schachermayer (1992). Dalang, Morton, and Willinger (1990) extends this representation result in the obvious way, without need for finite-state or continuity assumptions, to settings with a finite number of intermediate trading periods and with a finite number of primitive traded financial instruments.

[7] Some of the controversy about FVA arises in part from tension over how to measure fair values. For example, international accounting standard IFRS 13 refers to the use of "exit prices," meaning roughly the price that the firm would receive when selling (if a net asset) or transferring (if a net liability) a derivatives portfolio to a new counterparty in an orderly transaction. This approach to fair valuation raises some additional consistency issues that we do not address. Both U.S. accounting standards (in particular FASB 157 and 159) and international accounting standards (IFRS 13) require that traded OTC derivatives be disclosed at their fair value, rather than by ordinary accrual (or cost) accounting. We merely take fair valuation as a given concept subject only to the two coherency axioms stated above (linearity in payoffs and increasing in payoffs), which are rather compelling for any approach to measuring fair market value.

zero is assigned by some given "fair-market-value" function $V : \mathcal{L} \to \mathbb{R}$. We impose only minimal coherency assumptions on fair-market-value assignments, namely that $V(\,\cdot\,)$ is linear[8] and increasing in payoffs. That is, *(i)* the value of a portfolio of different cash flows is the sum of the values of the elements of the portfolio, and *(ii)* if payoff $X$ is greater than or equal to payoff $Y$ in every state of the world, and if $X > Y$ in some states of the world, then $V(X) > V(Y)$.

Under these two coherency assumptions, Stiemke's Lemma implies that there is stochastic discount factor, that is, a strictly positive random variable $\nu$ with the property that the value of any payoff $Y$ is $V(Y) = E(\nu Y)$. We take one of the payoffs to be that of a risk-free bond. The associated risk-free discount is $\delta = E(\nu)$, implying a risk-free gross rate of return of $R = \delta^{-1}$. It follows that fair valuations, henceforth called "valuations" or simply "values," can be assigned according to "risk-neutral" expectation. That is, we can define risk-neutral expectation $E^*$ by letting $E^*(Y) = E(\nu Y)R$, so that the value of any payoff $Y$ can be represented as $V(Y) = E(\nu Y) = \delta E^*(Y)$. The associated risk-neutral probability measure $P^*$ is defined by $P^*(B) = E^*(1_B)$ for any event $B$, with indicator $1_B$. Because $\nu$ is not necessarily uniquely determined, the risk-neutral probability measure $P^*$ is not necessarily unique.

Although this seems familiar from the standard setup of an arbitrage-free asset pricing model, we do not actually assume the absence of arbitrage in the usual sense. We have merely given a representation of how fair valuations are assigned by $V(\,\cdot\,)$. Fair valuations need not coincide in all cases with the prices at which instruments are actually traded in an over-the-counter market. In fact, we will show that a dealer should refuse to trade some types of financial instruments unless it can buy them at prices strictly below fair valuation or sell them at prices strictly above fair valuation. The ability of dealers to execute trades at "arbitrage" prices that reflect non-zero bid-ask spreads arises from the imperfect nature of financial markets, particularly over-the-counter swap markets, in which search costs and other frictions frequently give dealers a trading advantage over non-dealers. As we shall explain, bid-ask spreads are needed to cover more than a dealer's overhead and trading expenses (which we ignore here). We will show the amounts by which a swap dealer may need to widen its a bid-ask spread so as to overcome a variant of debt overhang, representing the cost to the dealer's shareholders of financing the cash needed to enter swap positions.

### B. *Preliminaries on the Valuation of Corporate Assets, Liabilities, and other Claims*

We consider a firm whose assets and liabilities have payoffs at time 1 (before additional trades are considered) given by random variables $A$ and $L$, respectively. The firm defaults in the event $D = \{A < L\}$. At default, liquidation or reorganization may lead to distress costs. The asset value remaining after default, net of distress costs, is $\kappa A$, for some recovery parameter $\kappa \in (0, 1]$. The values of the firm's equity and debt are therefore $\delta E^*[(A - L)^+]$ and $\delta E^*(\kappa A 1_D + L 1_{D^c})$, respectively, where $D^c = \{A \geq L\}$ is the event of no default.

We now consider a potential new investment by the firm, such as a swap, whose time-1 payoff

[8] That is, $\mathcal{L}$ is a linear space and for any two payoffs $X$ and $Y$ and any scalars $a$ and $b$, the value of the portfolio payoff $aX + bY$ is $V(aX + bY) = aV(X) + bV(Y)$.

$Y$ to the firm may be positive in some states and negative in other states. Our convention is to treat the positive part $Y^+ = \max(Y, 0)$ as an asset and the negative part $Y^- = \max(-Y, 0)$ as a contingent liability. The positive part $Y^+$ is measured net of any losses due to counterparty default. If the contingent liability $Y^-$ is fully secured (that is, if $A > Y^-$), then it has a value to the firm of $-\delta E^*(Y^-)$, so that the total value of the financial instrument is $\delta E^*(Y)$.

If the contingent liability $Y^-$ is not fully secured, we must specify how the associated counterparty recovers on its claim in case the firm defaults. We assume throughout that the firm's unsecured liabilities are *pari passu* with each other, so that the various claimants' default recoveries are pro rata with their claim sizes. In practice, the unsecured portions of a firm's swap contingent liabilities are normally *pari passu* with its unsecured senior debt claims. If the firm acquires a new financial instrument with cash flow $Y$ whose liability component $Y^-$ is the firm's only other unsecured default claim, then the value to the firm of this claim is $\delta E^*(\mathcal{C})$, where $\mathcal{C}$ is net actual cash flow to the firm, given by

$$\mathcal{C} = 1_{\{A+Y \geq L\}}\, Y + 1_{\{A+Y<L\}} Y^+ - 1_{\{A+Y < L\}}\, \rho\kappa A, \tag{1}$$

where

$$\rho = \frac{Y^-}{L + Y^-}$$

is the pro-rata share of this contingent liability.

In order to later treat collateralized swap positions, we will also need to consider cases in which the contingent liabilities include both secured and unsecured components. For this purpose, we allow for the case of a financial position whose cash flows to be paid to the firm at time 1, before considering the effect of the firm's own default, have a decomposition of the form $Y = Y_1 + Y_2$, where the first contingent liability $Y_1^-$ is secured and the second contingent liability $Y_2^-$ is unsecured and *pari passu* in default with other unsecured creditor claims. In this case, the firm's valuation of the associated net time-1 cash flow is $\delta E^*(\mathcal{C})$, where $\mathcal{C}$ is the net actual cash flow at time 1, given by

$$\mathcal{C} = Y_1 + 1_{\{A+Y \geq L\}} Y_2 + 1_{\{A+Y<L\}} Y_2^+ - 1_{\{A+Y < L\}} \kappa(A + Y_1)\rho, \tag{2}$$

where

$$\rho = \frac{Y_2^-}{L + Y_2^-}$$

is the pro-rata share of the unsecured liability $Y_2^-$. (Here, we have assumed for simplicity that adding the given position has no impact on the proportional default recovery coefficient $\kappa$.) In order to guarantee that the contingent liability $Y_1^-$ is truly secured, we assume that $A + Y_1 \geq 0$.

For a position that has net actual cash flows at time 0 of $c_0$ and at time 1 of $c_1$, the total valuation is of course $c_0 + \delta E^*(c_1)$. In the next subsection, we examine the preferences of the firm's shareholders for how the initial cash flow $c_0$ is financed, meaning transformed into time-1 cash flows by issuing new debt or new equity.

*C. The Marginal Value to Shareholders of a Debt-Financed Investment*

The firm contemplates entering some quantity $q$ of an investment, such as a package of financial instruments with one or more counterparties. In this subsection, we are mainly concerned with the impact of entering this investment on the firm's shareholders. Before considering the effect of the firm's default, the per-unit payoff of the package at time 1 is given by some random variable $Y$, which may have a negative outcome with positive probability. The net cash-flow to the firm at time 1 for a position of size $q$ is therefore $qY$. We allow that $Y$ may be of the form $Y = Y_1 + Y_2$, where $Y_1^-$ is secured and $Y_2^-$ is unsecured.

As shown in Appendix B, the following calculations also apply without change to an infinite-state setting provided that, with respect to $P^*$, the random variables $A$, $L$, $Y_1$, and $Y_2$ have finite expectations, and provided that $A$ and $L$ have a continuous joint probability density.

The investment cost for $q$ units of the new position is some given amount $U(q)$. We do not require that the firm's investment cost is equal to the fair value $q\delta E^*(Y)$ of the future net cash flows generated by the investment, allowing the potential for a non-zero dealer trading profit of $q\delta E^*(Y) - U(q)$. The marginal investment cost, $u \equiv \lim_{q\downarrow 0} U(q)/q$ is assumed to be well-defined. We allow $U(q)$ to have either sign. If $U(q)$ is positive, the initial investment cost must be financed at time 0. If $U(q)$ is negative, the firm may invest the cash received, $-U(q)$, or use it to retire debt or equity. We assume that the firm faces a competitive capital market for new debt and equity issuances. That is, those competing to offer equity or debt financing to the firm break even by paying the fair value of any claim issued to them by the firm.

We now calculate the marginal value of the investment for the firm's shareholders, assuming debt financing. Appendix A provides the analogous explicit calculations for equity and cash financing.

Throughout the remainder, "marginal value" means the first derivative of the fair value of the claim under consideration, per unit of the claim. Except for cases in which the size of the investment is large relative to the firm's entire balance sheet, this first-order valuation approach accurately characterizes the benefit of the investment, and provides intuitively natural and simple analytical results. Appendix B shows how the second-order valuation effect (in the sense of the Taylor series) explicitly reflects the asset-substitution benefit to shareholders of adding risk.[9]

For an investment of $q$ units, let $s(q)$ be the credit spread on the new debt that must be issued to finance the cost $U(q)$ of the new position. If $U(q)$ is negative, the associated cash proceeds to the firm are used to retire debt by purchasing it on the capital market. We assume throughout that capital markets price the dealer's debt and equity at fair value. This rules out additional sources of gain or loss such as "liquidity" spreads in the market for the dealer's unsecured debt.

Because we assume that the new creditors who finance the cost $U(q)$ are pari passu with all of the other unsecured senior creditors of the firm (including the unsecured counterparty of the new position), the credit spread $s(q)$ is determined by both the legacy balance sheet and the new

[9]The potential for a strictly positive gain to shareholders from the purchase of risky assets, even at an investment cost that is equal to or somewhat above the fair market value $\delta E^*(Y)$, is commonly known as "asset substitution," as characterized by Jensen and Meckling (1976) and Myers (1977).

position. A detailed calculation of $s(q)$ is provided in Appendix B.

Although $s(q)$ depends in general on the decomposition of $Y$ into the sum $Y_1 + Y_2$ of its secured and unsecured components, we also show in Appendix B that the limiting spread $\lim_{q \downarrow 0} s(q)$ is

$$S = \frac{E^*(\phi)R}{1 - E^*(\phi)},$$

reflecting the proportional default loss to creditors of

$$\phi = \frac{L - \kappa A}{L} 1_D. \tag{3}$$

We also show in Appendix B that $S$ is invariant to the decomposition of $Y$ into its secured and unsecured components. In the case that $L$ is deterministic, $S$ is identical to the credit spread of the firm's legacy debt.

The contractual new debt payback at time 1 is $(R + s(q))U(q)$. Shareholders receive the residual $A + qY - L - U(q)(R + s(q))$, unless this amount is negative, in which case the firm defaults and shareholders get nothing. The marginal increase in the value of the firm's equity, per unit investment, is therefore

$$G = \left. \frac{\partial E^*[\delta(A + qY - L - U(q)(R + s(q)))^+]}{\partial q} \right|_{q=0}, \tag{4}$$

provided of course that this derivative is well defined. The appendix includes a proof of the next result, and of all results to follow.

PROPOSITION 1: *THE MARGINAL VALUE TO SHAREHOLDERS OF DEBT FINANCING. The marginal gain $G$ in equity value is well defined and given by*

$$G = p^*\pi - \delta \operatorname{cov}^*(1_D, Y) - \Phi, \tag{5}$$

*where*

*$p^* = P^*(D)$ is the risk-neutral survival probability of the bank.*

*$\pi = \delta E^*(Y) - u$ is the marginal profit on the trade.*

*$\Phi = p^* \delta u S$ is known as the funding value adjustment (FVA).*

The term $\operatorname{cov}^*(1_D, Y)$ in equation (5) reflects the cost to shareholders of investing in an asset whose payoff is positively correlated with the firm's default, given that shareholders give up all payoffs in the event to default to creditors. The last term, the funding value adjustment $\Phi$, is the present value to shareholders of their share of the net financing costs, $uS$. Shareholders pay these financing costs if and only if the firm survives.

Proposition 1 reflects a well known principle of corporate finance known as "debt overhang," by which even an investment whose upfront cost $u$ is strictly below the value $\delta E^*(Y)$ of its payoff may sometimes be declined by a firm because the payoffs accrue excessively to creditors rather than shareholders.[10] In Sections III and IV, debt overhang will play a significant role in our analysis of the impact of financing costs on dealer swap quotation.

Appendix A provides the explicit marginal valuations to equity shareholders associated with equity financing and with cash financing. Under a non-degeneracy condition, we show a strict pecking order. Cash financing, when possible, is strictly preferred by shareholders over debt financing, which is in turn strictly preferred over equity financing. Other financing strategies could be considered. For instance, the firm could sell non-cash assets or could arrange a combination of equity, cash, and debt funding. Song (2016) extends to the case of repo financing. Dealer industry metrics are rarely based on these alternative strategies, and we shall not consider them further here.

Under a linear combination of different financing methods, our technical assumptions imply that valuation is continuously differentiable in the quantity of each of the types of financing. This implies that a linear combination of financing strategies generates the corresponding linear combination of the respective marginal shareholder values.

## III. How Funding Costs Affect Swap Valuation

We now apply the basic theory of the previous section to a dealer's swap transactions. Interest rate swaps, a primary example in practice and the focus of our numerical examples in Section IV, make up the majority of a typical dealer's derivatives inventory, representing tens of trillions of dollars of total notional positions for each of the largest dealers.

Our main objective here is to calculate the impact of FVA on swap valuation and on swap rates that a dealer would quote in order for its shareholders to break even, after considering FVA. An additional marginal contribution of this section is a novel implication of debit value adjustments (DVAs) for shareholder break-even swap rate quotation.

In this section, we consider an unsecured swap transaction. Appendix C extends to the cases of *(i)* an unsecured swap transaction packaged with an inter-dealer hedge, and *(ii)* a swap secured by initial margin. The funding value adjustment associated with initial margin is known in industry practice as a "margin value adjustment" (MVA), rather than an FVA.

Appendix D generalizes the basic one-period model of this section to a two-period model that allows for the financing of intermediate-date coupons and variation margin payments, and also allows for default at the intermediate date.

In the one-period setting considered here, a swap is a contract promising some underlying floating payment $X > 0$ in exchange for some fixed payment $K$. We take $K$ as given for now, and assume that the dealer pays fixed and receives floating, for a net contractual receivable at time 1 of $X - K$, before considering the effect of counterparty default. Results for the reverse case, in which

[10] See Myers (1977).

the dealer receives fixed and pays floating, are obvious by analogy.

In the infinite-state case, the following calculations apply if $A$, $L$, and $X$ have finite risk-neutral expectations and a continuous joint risk-neutral density function.

### *A. Valuing Unsecured Swaps with Upfronts*

In this subsection, the swap is assumed to be fully unsecured. That is, the swap is not covered by collateral. For simplicity, we suppose that there are no pre-existing positions between the swap client and the dealer. Otherwise, the results would be complicated by the effect of netting the new swap cash flows against those of the dealer's legacy positions with the same client. This more general case is analyzed in Appendix F.

We let $B$ denote the event of the client's default. At the client's default, the dealer recovers a fraction $\beta$, possibly random, of any remaining contractual amount due to the dealer, $(X - K)^+$. In the event that $X < K$ and the dealer defaults, the unsecured swap client recovers a pro-rata share of the dealer's estate, *pari passu* with the dealer's unsecured creditors.

A swap position of size $q$ requires the dealer to make an upfront payment of $U(q)$. Given our pecking order for dealer financing preferences, a positive payment is preferably funded by excess balance-sheet cash, and a negative payment is preferably used to retire equity. In practice, however, dealers' swap trading units are typically cash-constrained and are not in a position to freely retire equity. Consistent with industry practice, we therefore assume that a positive financing requirement amount is funded by issuing debt. Likewise, any net positive cash flow to the dealer is used to retire debt.

Our resulting definition of FVA is therefore "symmetric," in the sense that cash inflows and outflows are assumed to be financed or to reduce financings, respectively, at a spread of $S$. For the case of cash inflows, this implicitly assumes that there is always some short-term unsecured debt to roll over whose total amount can be reduced by swap cash inflows. This is a simplifying abstraction of a practical setting in which much of the surplus funds created temporarily by derivatives trading would more likely be "parked" in short-term low-risk assets. A corresponding definition of "asymmetric funding value adjustment" (AFVA) is provided by Albanese and Andersen (2014). Asymmetric funding strategies of this and other types are captured in a straightforward, albeit more complicated, way within our modeling framework by assuming that cash inflows are financed with unsecured debt and cash outflows are financed at the risk-free rate. The basic thrust of our conclusions, however, is not changed when substituting FVA with AFVA. In the simple one-period model of this section, the AFVA is merely the positive part of the FVA.

In the absence of a dealer default, the payment flowing to the dealer at time 1, per unit notional position, is

$$Y = y(K) \equiv X - K - \gamma(X - K)^+, \tag{6}$$

where $\gamma = (1 - \beta)1_B$ is the fractional counterparty default loss.

In order to establish the fair value of the swap, we must consider the potential default of the dealer. With $q$ units of the swap traded, we can use (1) to express the effective time-1 payoff of the swap to the dealer as

$$\mathcal{C}(q) = q(X-K) - q\gamma(X-K)^{+} + (1-\kappa\rho(q))\, q\,(X-K)^{-}\, 1_{\mathcal{D}(q)},$$

where, given debt financing, the asset-to-debt payoff ratio is

$$\rho(q) = \frac{A}{L + U(q)\,(R+s(q)) + q\,(X-K)^{-}}.$$

and where

$$\mathcal{D}(q) = \{A - L + qY - U(q)(R+s(q)) < 0\}$$

is the dealer's default event after considering the new position. Our basic valuation framework of the previous section implies that the fair value of the swap payoff is $\mathcal{V}(q) = \delta E^{*}\left(\mathcal{C}(q)\right)$.

The proof of the following proposition, provided in Appendix B, shows that the marginal value $v = \partial \mathcal{V}(q)/\partial q|_{q=0}$ of the swap payoff at time 1, after financing the upfront, does not depend on the financing strategy. This invariance of the marginal value to the financing method can be thought of as a consequence of the Modigliani-Miller Theorem.[11] Nevertheless, the value $\mathcal{V}(q)$ of a non-trivial position of size $q > 0$ in general depends non-trivially on the financing method, because the incremental distress costs depend on the financing method.

PROPOSITION 2: *FAIR MARKET VALUE OF AN UNSECURED SWAP. Whether the dealer finances a swap by issuing debt, issuing equity, or using existing cash on its balance sheet, the marginal value of the swap payoff is well defined and given by*

$$v = \delta E^{*}(X-K) - \delta E^{*}\left(\gamma(X-K)^{+}\right) + \delta E^{*}\left(\phi\,(X-K)^{-}\right), \tag{7}$$

*where* $\phi = 1_D(L - \kappa A)/L$.

The swap value (7) includes two adjustments of the default-free value, $\delta E^{*}(X-K)$, for default risk. The first of these, a reduction of $\delta E^{*}\left(\gamma(X-K)^{+}\right)$, is the credit valuation adjustment (CVA) for the contingent asset component of the swap. The second adjustment, $\delta E^{*}\left(\phi\,(X-K)^{-}\right)$, is the debit valuation adjustment (DVA) for the contingent liability. These adjustments, CVA and DVA, are now reasonably well established in finance theory and generally accepted principles for fair-value accounting.[12]

The marginal valuation $v$ may easily be verified to be equal in magnitude, and opposite in sign, to the total marginal value of the package of transactions to the dealer's swap counterparty.[13]

[11] See Modigliani and Miller (1958).

[12] For DVA and CVA analysis, see, for example, Sorensen and Bollier (1994), Duffie and Huang (1996), and Gregory (2015). Spears (2017) discusses the history of DVA and CVA adjustments.

[13] For this, we can note that the dealer's CVA (DVA) is the counterparty's DVA (CVA). While sometimes questioned, the inclusion of self-default benefits through the DVA term is critical to ensure symmetry in the dealer and counterparty valuations.

If there are no default distress costs ($\kappa = 1$), we may view $v$ as the choice of upfront payment $u$ that would make a total claimant on the dealer's balance sheet (debt plus equity) indifferent to entering the swap transaction. Whenever trading decisions are made, however, we assume that the dealer's preferences are determined by shareholder value maximization. We therefore focus on the upfront payment $v^*$ for the swap that would leave shareholders indifferent to the swap transaction.

Given the debt financing of the position, the marginal value of the position to the dealer's shareholders is given by Equation (5). Assuming that the dealer's survival probability is non-zero, we have

$$v^* = \frac{E^*(1_{D^c} Y)}{(R+S)P^*(D^c)}. \tag{8}$$

If the dealer's default indicator $1_D$ and the swap cash flow $Y$ are uncorrelated (under $P^*$), then

$$v^* = (v - d)\,\frac{R}{R+S}, \tag{9}$$

where $d = \delta E^*\left(\phi\,(X-K)^-\right)$ is the DVA of the swap transaction. In this simple case, from the viewpoint of shareholder value maximization, the dealer's breakeven upfront price $v^*$ for entering the swap is an adjustment of the fair market value $v$ that:

*(i)* removes the DVA term $d$ from $v$.

*(ii)* substitutes the dealer's unsecured discount rate $R+S$ for the risk-free rate $R$.

The first of these adjustments does not depend on the funding strategy and reflects the lack of any shareholder benefit from paying the swap counterparty less than the contractually promised amount when the dealer defaults (because the equity holder receives nothing at default). The second adjustment is for the funding cost to shareholders, who must pay the credit spread $S$ to the new creditors without gaining any marginal benefit from the right to default on the new debt. (This is a negative adjustment if the upfront is negative.)

When ignoring distress costs (by taking $\kappa = 1$), the difference between the shareholder break-even value $v^*$ and the total value $v$ to all dealer claimants (debt plus equity) amounts to a wealth transfer by the dealer's equity shareholders to the dealer's creditors. This wealth transfer is triggered both by the swap cash flow itself (through the DVA) and also by the financing strategy used by the dealer to fund the upfront. The net shareholder cost $v^* - v$ of entering the swap is not entirely transferred to other stakeholders if the dealer has distress costs at default. In general, the net gain to the dealer's legacy creditors is calculated in Appendix B.

### *B. Dealer Quotation and FVA for Unsecured Swaps*

Assuming that the dealer maximizes shareholder value, it would rationally not trade the swap unless the upfront payment to the dealer is at least $v^*$. If the dealer manages to execute the trade at this level, the firm as a whole would make a trading profit of $v - v^*$. This profit can have either sign. Although the DVA effect always lowers[14] $v^*$ relative to $v$, the funding-cost component can either increase $v^*$ relative to $v$ (which occurs if $v < d$), or decrease it (whenever $v > d$). Loosely speaking, the funding component increases shareholder value for swaps that are predominantly liabilities (have a high fixed rate $K$ relative to $E^*(X)$) and decreases shareholder value for swaps that are predominantly assets (have a low $K$ relative to $E^*(X)$).

From the viewpoint of shareholder value maximization, bank quotation practices have traditionally adjusted appropriately for the DVA effect, but have not correctly accounted for the funding-cost effect. That is, rather than quoting $v^*$ as suggested by the shareholder breakeven upfront payment (8), banks have instead conventionally quoted

$$v - d = \delta E^*(X - K) - \delta E^*(\gamma(X - K)^+), \tag{10}$$

which is the fair-market value of a default-free swap less the CVA, but removing the DVA adjustment that is now an accepted element of the fair value accounting for swaps reflected in (7). If the swap is executed at this conventional level $v-d$, then (5) implies that shareholders experience a marginal gain in value of

$$g(v - d) = g(\delta E^*(Y)) = -\delta \operatorname{cov}(1_D, Y) - \Phi, \tag{11}$$

where

$$\Phi = p^* \delta(\delta E^*(Y))S. \tag{12}$$

Here again, $\Phi$ is the debt-based funding valuation adjustment (FVA), recently introduced by dealers as adjustments to their reported accounting incomes.[15] The FVA $\Phi$ may be interpreted as a transfer of wealth away from dealer's shareholders due to the adverse impact of funding costs. This conceptual basis for FVA, however, is not commonly recognized within the dealer community. The dealer's shareholders are not compensated for this wealth transfer unless they can obtain an offsetting wealth transfer from their swap counterparties through a widening of the effective bid-ask spread.

In order to make this point more transparent, we Taylor-expand the expression (9) for the shareholder valuation $v^*$ of the swap position, for a small credit spread $S$ and for a survival probability $p^*$ close to 1. We see that

$$v^* = (v - d)\,\frac{1}{1 + S/R} \approx (v - d)\left(1 - \frac{S}{R}\right) \approx v - d - \Phi. \tag{13}$$

Thus, the current practice by dealers of making a downward FVA adjustment to their mark-to-market swap valuations, although not consistent from a valuation viewpoint, causes a valuation bias that better aligns the interests of the dealer's traders with the dealer's shareholders.

[14]This is true unless the swap is a pure asset with no DVA at all, that is, unless $K$ is so low that $X - K > 0$.

[15] There are some variations in industry practice. For instance, while (12) is a common definition of FVA, some dealers ignore the terms associated with counterparty default and instead use $\delta(\delta E^*(X - K))S$ as their measure of FVA. Some dealers replace their own credit spread $S$ in this formulaic adjustment with an estimated average of major-dealer credit spreads.

In order to trade with a dealer that quotes swaps in a manner reflecting these shareholder incentives, the client swap counterparties must be willing to "donate" the sum of the DVA $d$ and the FVA $\Phi$. In practice, this "donation" would be implemented through an effective widening of the dealer's bid-ask spread, manifested either in the upfront $u$ or in the swap rate $K$, or both. In Section IV, we provide a numerical example that illustrates the magnitude of the minimal compensating bid-ask widening. We argue that the magnitudes are economically significant.

It follows from our results that the most creditworthy dealers, those with the lowest credit spread $S$ and therefore the lowest FVAs, usually have a head start over less well capitalized dealers in finding swap clients willing to enter trades at terms that are beneficial to the dealer's shareholders. Even the best capitalized dealer, however, must attract clients that are sufficiently anxious to trade (given their own hedging or speculative motives) that they are willing to give up some value to the dealer. This concession can be buried into the bid-ask spread quoted by the dealer.

A dealer sometimes finds itself in a position to enter a swap that *lowers* its aggregate margin requirement, because the new swap hedges or offsets a legacy position with the same counterparty. In this case, the margin that is released by the trade is a source of profit to the dealer's shareholders in the form of a reduction in FVA, as shown in Appendix F. This gives the dealer an advantage over other dealers (even some dealers with lower credit spreads) in "winning" the trade. This benefit, a negative FVA, is known in practice as a "funding benefit adjustment" (FBA).

Appendix C extends the results of this section to treat hedged swaps and swaps that are secured with variation margin and, potentially, initial margin. In industry terminology, the additional funding value adjustment associated with the financing of initial margin is called a "margin value adjustment" (MVA) rather than a "funding value adjustment."

## IV. Valuation Adjustments for Long-Term Swaps

This section illustrates the numerical implications of our model for valuation adjustments in some practical settings. After setting up a general reduced-form swap valuation framework that parallels the structural model of the previous sections and Appendix D, we provide a numerical illustration of the magnitude of valuation adjustments for plain-vanilla interest rate swaps, showing that funding valuation adjustments are economically important in practice, and also indicating relative responses to the term structure of interest rates and to the fixed coupon rate of swaps.

For this purpose, we begin with a relatively standard continuous-time setting that allows us to appeal to standard reduced-form models of default timing and recovery. Our reduced-form model is otherwise conceptually faithful to the solution for funding value adjustments in our structural model. In order to capture the effects of interim coupon and variation margin payments in a manner consistent with the spirit of the structural model, Appendix D generalizes the basic one-period model of Section III to a two-period model that allows for the financing of coupon and intermediate-date margin payments, and also allows for default at the intermediate date.

### A. Reduced-Form Valuation Framework

Our continuous-time framework is based on standard technical assumptions given in Appendix E. The model begins with a default-risk-free short-rate process $r = \{r_t : t \geq 0\}$, implying that the risk-free discount at time $t$ for risk-free cash flows at time $T$ is $E_t^*(\delta_{t,T})$, where $\delta_{t,T} = e^{-\int_t^T r(s)\, ds}$.

Before considering the effect of incremental cash flows associated with a new position, the derivatives dealer defaults at a stopping time $\tau_D$ whose conditional mean arrival rate at time $t$ is $\lambda_D(t)$. The fractional loss to the creditor claim associated with default at time $t$ is $\ell_D(t)$. That is, an unsecured claim of size $C$ on the dealer's estate at default is paid $(1 - \ell_D(\tau_D))C$, for some proportional loss process $\ell_D$ taking outcomes in $[0, 1]$. This implies that the dealer's short-term credit spread at time $t$ is $S_t = \lambda_D(t)\ell_D(t)$. That is, each unit of the dealer's short-term unsecured debt can be continually renewed, or "rolled over," by making continual floating-rate interest payments at the adjusting rate $r_t + S_t$, as justified in Appendix E.

Similarly, a given client swap counterparty has default risk characterized by a default time $\tau_C$ whose conditional mean arrival rate at time $t$ is $\lambda_C(t)$, and by a proportional loss given default at time $t$ of $\ell_C(t)$.

We will characterize various valuation adjustments for an unsecured swap between the dealer and the client. Johannes and Sundaresan (2007) have modeled the important valuation distinction between unsecured and collateralized swaps. Our objective here, instead, is to calculate the swap FVA, MVA,[16] CVA, and DVA.

This swap has maturity date $T$ and contractually promises the dealer, before considering the effect of counterparty default, net payments $C_1, \ldots, C_N$ at some respective increasing sequence $\{t_1, \ldots, t_N = T\}$ of coupon exchange times. For notational simplicity, for any time $t \leq T$, we let $\eta(t) \in \{0, 1, ..., N\}$ denote the index of the associated coupon period. That is, $t \in \left(t_{\eta(t)-1}, t_{\eta(t)}\right]$, taking $t_{-1} = -\infty$.

The market value at time $t < T$ of a default-free version of the swap is, by definition,

$$V_t = E_t^* \left( \sum_{j\, =\, \eta(t)+1}^{N} \delta_{t,t_j} C_j \right).$$

By direct analogy with the structural multi-period model of Appendix D, the CVA and DVA are, respectively,

$$\Pi_c = E^* \left(1_{\{T > \tau_C,\, \tau_D > \tau_C\}} \delta_{0,\tau_C} \ell_C V(\tau_C)^+\right) = \int_0^T E^* \left(\delta_{0,t} \kappa_t \lambda_C(t) \ell_C V_t^+\right)\, dt, \tag{14}$$

where $\kappa_t = e^{\int_0^t -[\lambda_D(s) + \lambda_C(s)]\, ds}$, and

$$\Pi_d = E^* \left(1_{\{T > \tau_D,\, \tau_C > \tau_D\}} \delta_{0,\tau_D} \ell_D V(\tau_D)^-\right) = \int_0^T E^* \left(\delta_{0,t} \kappa_t \lambda_D(t) \ell_D V_t^-\right)\, dt. \tag{15}$$

[16] As we discuss in Appendix C.C, MVA applies if the dealer hedges the unsecured client swap with an inter-dealer swap that requires the dealer to post initial margin.

By direct analogy with the marginal valuation of the swap that we provided for our discrete-time structural model, the market value of the swap is

$$v \equiv V_0 - \Pi_c + \Pi_d. \tag{16}$$

To repeat, this is the total value of the swap cash flows to both equity and debt claimants. By implication of the structural model, there is no funding value adjustment assignable to this (total) swap market value.

In order to compute funding value adjustments to shareholder value, we suppose that the dealer can enter small notional positions of the swap at a per unit upfront payment of $u$. Just as for our structural model, we do not require that this upfront payment $u$ is the equal to the initial value $v$ of the swap. Because this is merely a reduced-form model as opposed to a structural model, there is no point in making a distinction here between the marginal value and the per-unit value of a position of a given non-zero size.

In order to compute the FVA, we suppose that the dealer issues short-term unsecured debt to finance any pre-default swap-related payments, including the upfront payment and any interim coupon payments. Any swap-related receivables to the dealer are likewise used to retire outstanding short-term unsecured debt. The FVA of swap position can in this case be defined by direct analogy with that of the multi-period model of Appendix D by

$$\Phi(u) = E^* \left( u \int_0^\tau S_t \, dt - \sum_{i=0}^{\eta(\tau)-1} \delta_{0,t_i} \, C_i \int_{t_i}^\tau S_t \, dt \right),$$

where $\tau = \min(\tau_C, \tau_D, T)$. Given our default-time assumptions, this reduces to

$$\Phi(u) = E^* \left( u \int_0^T \kappa_t \, S_t \, dt - \sum_{i=0}^{N-1} \delta_{0,t_i} C_i \int_{t_i}^T \kappa_t S_t \, dt \right). \tag{17}$$

If the dealer hedges the unsecured swap with a fully collateralized inter-dealer swap that requires the dealer to post variation margin and intial margin, as will soon be the case under Dodd-Frank and MiFID regulations, there is also a margin value adjustment (MVA), which can be computed by analogy with the multi-period structural model of Appendix D as

$$\Psi = E^* \left( \int_0^T \kappa_t I_t \delta_t S_t \, dt \right), \tag{18}$$

where $I_t$ is the initial margin at time $t$. For the special case in which the unsecured swap is executed at an upfront equal to the default-free market value $V_0$, direct algebraic calculations, as in Appendix D, yield the FVA

$$\Phi(V_0) = E^* \left( \int_0^T \kappa_t V_t \delta_t S_t \, dt \right). \tag{19}$$

By analogy with (13), the upfront $v^*$ that would leave shareholders indifferent to the swap transactions is approximated as

$$v^* \approx V_0 - \Pi_c - \Phi(V_0) - \Psi = v - \Pi_d - \Phi(V_0) - \Psi. \tag{20}$$

### B. *Illustrative Numerical Example of XVAs*

We now give illustrative magnitudes of FVAs, MVAs, DVAs, and CVAs based on a simple parametric term-structure model. While the term-structure models used by major dealers are generally more sophisticated than our illustrative model, we believe that the magnitudes of these "XVAs" that we calculate give realistic indications of their relative economic importance in practice, and help us understand how they vary with swap rates, credit risk, and the slope of the term structure.

For this purpose, we consider an unsecured 10-year, semi-annual-coupon, plain-vanilla interest rate swap with a notional size of \$100 million. The underlying floating rate is six-month LIBOR. For our example, this floating rate is the simple six-month (money-market) interest rate associated with a hypothetical borrower whose six-month credit spread over the risk-free six-month simple interest rate is taken to be some constant $\epsilon$. At base case, we take $\epsilon$ to be 30 basis points. For the initial term structure of risk-free interest rates, we calibrate to the risk-free discount term structure given by

$$p(0,t) = E^*(\delta_{0,t}) = e^{-(0.005+0.001t)t}, \tag{21}$$

roughly corresponding to market conditions in January 2016. That is, the continuously compounding yield curve starts at 50 basis points and slopes upward at a rate of 10 basis points per year.

Until default, the net coupon $C_i$ paid to the dealer at the $i$-th coupon date $t_i$ is the current six-month LIBOR floating rate less some fixed coupon rate $K$. (We will consider various fixed coupon rates.) In addition to this payer swap, we will also provide results for the corresponding receiver swap, by which the dealer receives the fixed rate $K$ net of the six-month LIBOR floating rate. A default-free swap whose market value $V_0$ is zero corresponds to a fixed coupon rate of $K = 1.783\%$.

The risk-free short-rate process $r$, which we treat as the short rate underlying the overnight index swap (OIS) swap term structure, is determined by a one-factor Hull-White term-structure model[17] calibrated consistently with (21). That is, the short-rate process $r$ satisfies $r_t = -d\log(p(0,t))/dt + z_t$, where $p(0,t)$ is given by (21) and $z_t$ satisfies the stochastic differential equation

$$dz_t = (\theta_t - \alpha z_t)\,dt + \sigma\,dW_t, \quad z_0 = 0, \tag{22}$$

where $\alpha$ and $\sigma$ are constants, $W$ is a standard Brownian motion under the valuation measure $P^*$, and

$$\theta_t = \int_0^t \sigma^2 e^{2\alpha(u-t)}\,du, \tag{23}$$

We set $\alpha = 0.05\%$ and $\sigma = 0.70\%$, which approximate the implied volatility levels of long-dated

[17]See Hull and White (1993).

Table II This table shows XVAs, in thousands of dollars, for a 10-year plain-vanilla interest-rate swap of notional size \$100 million. Each cell of the table uses the format $x_P \,|\, x_R$ to show the XVA $x_P$ of the dealer's payer version of the swap on the left and the corresponding XVA $x_R$ of the dealer's receiver version of the same swap on the right. Shown in parentheses are the running-spread equivalents of the associated XVAs, in basis points, meaning the adjustments to the fixed swap rate $K$ that substitute that compensate for eliminating the upfront payments. The columns of the table correspond to the fixed coupon rate of the swap. The rows correspond, respectively, to the funding value adjustment (FVA) given by $\Phi(V_0)$ of (19), the margin value adjustment (MVA) given by $\Psi$ of (18), the credit value adjustment (CVA) given by $\Pi_c$ of (14), and the debit value adjustment (DVA) given by $\Pi_d$ of (15).

| | $K = 1.0\%$ | $K = 1.783\%$ | $K = 2.5\%$ |
|---|---|---|---|
| **FVA** | $428 \,\vert\, -428$ | $116 \,\vert\, -116$ | $-171 \,\vert\, 171$ |
| | $(4.6 \,\vert\, -4.6)$ | $(1.2 \,\vert\, -1.2)$ | $(-1.8 \,\vert\, 1.8)$ |
| **MVA** | $116 \,\vert\, 116$ | $116 \,\vert\, 116$ | $116 \,\vert\, 116$ |
| | $(1.2 \,\vert\, 1.2)$ | $(1.2 \,\vert\, 1.2)$ | $(1.2 \,\vert\, 1.2)$ |
| **CVA** | $942 \,\vert\, 85$ | $479 \,\vert\, 247$ | $236 \,\vert\, 577$ |
| | $(10.0 \,\vert\, 0.9)$ | $(5.1 \,\vert\, 2.6)$ | $(2.5 \,\vert\, 6.1)$ |
| **DVA** | $42 \,\vert\, 471$ | $124 \,\vert\, 240$ | $289 \,\vert\, 118$ |
| | $(0.5 \,\vert\, 5.0)$ | $(1.3 \,\vert\, 2.5)$ | $(3.1 \,\vert\, 1.3)$ |

Bermudan LIBOR swaptions as of January 2016.

We assume that the swap counterparty has a constant default intensity $\lambda_C$ of 4% and a constant fractional loss given default $\ell_C$ of 50%. The dealer has a constant default intensity of $\lambda_D$ of 2% and a constant fractional loss given default $\ell_D$ of 50%. This implies a constant short credit spread $S$ for the dealer of 1%, and for the counterparty of 2%.

We assume that the dealer hedges the unsecured swap with a fully collateralized inter-dealer swap that requires initial margin. When calculating the MVA, the initial margin $I_t$ is set at the level required by BCBS/IOSCO (BCBS (2013)), that is, at the 99th percentile of the 2-week change in market value $V_t$ of the default-free version of the swap, excluding any jumps associated with coupon payments. A detailed analysis of the computation of the CVA, DVA, FVA, and MVA, based on the formulae provided in the previous section, is found in Appendix E.

Table II shows these "XVAs." For the FVA, we report $\Phi(V_0)$, meaning the FVA associated with an upfront equal to the default-free market value $V_0$.

### *C. The Magnitudes of the XVAs and Their Impacts on Dealer Quotation*

To interpret the results in Table II, we focus at first on the payer swap at a fixed coupon rate of 1.783%, and consider how the shareholder value $v^*$ of (20) differs from the market value $v$ of (16).

The fair value of the swap is obtained by subtracting the CVA net of the DVA from the value $V_0$ of a default-risk-free swap (which in this example is zero), for a total reduction of $\$479,000 - \$124,000 = \$355,000$, which is economically equivalent to $5.1 - 1.3 = 3.8$ basis points in running-coupon terms. Relative to this fair value, the funding value adjustment $\Phi(V_0)$ for the swap represents a cost to shareholders of approximately \$116,000, which is economically equivalent in terms of its cost to shareholders to an increase of approximately 1.2 basis points in the fixed coupon rate paid by the dealer. The margin value adjustment $\Psi$ (assuming that the dealer is actually subject to initial margin) represents an additional $\$116,000$ cost to shareholders. (The approximate numerical equivalence of FVA and MVA in this example is merely coincidental.) Finally, the DVA benefit of approximately $\$124,000$ is of no value to shareholders, so the impact of the swap trade on the value of the dealer's equity is less than the fair market value $v$ of the swap by approximately $\$116,000 + \$116,000 + \$124,000 = \$356,000$ (which is economically equivalent to an impact on shareholders of $1.2 + 1.2 + 1.3 = 3.7$ basis points running).

From a quotation perspective, the "par" coupon rate $K$, that making the swap have a zero market value, is approximately $178.3 - 3.8 = 174.5$ basis points. However, as we just noted, entering the swap at these "fair-market" terms represents a swap-rate disadvantage to the dealer's shareholders of 3.7 basis points. That is, the dealer's swap desk, if acting on behalf of shareholders, should be willing to enter the swap only if the fixed rate paid by the dealer is no greater than 170.8 basis points.

As for the receiver version of this swap, the dealer's shareholders benefit only if they receive an upfront that is increased above the initial fair market value of the swap by the sum of the FVA, MVA, and DVA, which is \$240,000, or a running-spread equivalent of 2.5 basis points of notional. (In this case, the FVA is negative, but this funding benefit to shareholders is more than offset by the total of the MVA and DVA.) Equivalently, the shareholder breakeven receiver swap rate is 2.5 basis points above the fair-market rate of $178.3 + 2.6 - 2.5 \simeq 178.4$ basis points. That is, the swap desk should not enter as a receiver at a zero upfront payment unless it can receive a swap rate of at least $178.4 + 2.5 = 180.9$ basis points. If quoting both sides of the swap so as to ensure that shareholders break even, this represents a bid-ask spread of approximately $180.9 - 170.8 = 11.1$ basis points, an enormous widening of the spread relative to current unsecured dealer-to-client bid-ask spreads of under 0.2 basis points.

This example, however, is extreme relative to typical XVA impacts on dealer shareholders and on bid-ask spreads. Until recently, dealers have not been providing initial margin. Removing the MVA impacts would reduce the bid-ask spread by 2.4 basis points, leaving a bid-ask spread widening effect of 8.7 basis points, corresponding to the impact on shareholders of FVA and DVA.

Furthermore, the FVA and DVA impacts of new swaps are frequently *beneficial* to the dealer, through netting effects relative to legacy swap positions. This netting benefit is shown in a structural version of our model found in Appendix F. In practice, according to OCC (2015), on average across the largest U.S. swaps dealers as of the end of the third quarter of 2015, netting reduced the gross positive fair value of swaps by 87%. There is no available breakdown, however, of the impact of netting on dealer-to-client swaps versus inter-dealer swaps, and no breakdown of the effects of netting cash flows across counterparties (which reduces FVA impacts on shareholders) and netting within counterparty positions (which reduces both FVA and DVA impacts on shareholders).

If, for example, the dealer has 25% more payers than receivers, implying a reduction from gross to net notional positions of 8/9, then the average FVA effect on the total book of all swaps is a loss to shareholders of only about 1/9 of the impact of a stand-alone payer, per unit of total gross notional. In our example, the FVA effects for standalone payers and receivers are the same at all of the coupon rates that we considered. This 1.2 basis point spread compensation to dealer shareholders is then reduced by netting to about 0.13 basis points running of the gross notional, or, equivalently, a market value impact on shareholders of about $12,900 per $100 million notional.

Any MVA effects also benefit from netting, by the degree to which the dealer's initial margin payments are concentrated among its different CCPs and dealer counterparties. For the case of credit default swaps, the degree to which initial dealer margin to CCPs and other dealers is reduced by netting is examined empirically by Duffie, Scheicher, and Vuillemey (2015).

As opposed to the case of FVA, the impact of netting on shareholder DVA costs do not net across counterparties. However, DVA impacts do net across offsetting positions with the *same* counterparty. For example, if the dealer has a DVA that is reduced through counterparty-level netting by an average factor of two, then the adverse DVA effect on shareholders (relative to market value) is also reduced by a factor of two. In our example of interest-rate swaps entered at a fixed rate of 178.3 basis points, the adverse DVA effects on shareholders per $100 million notional, of $124,000 for payers and $240,000 for receivers, would then each be cut in half.

These illustrative netting effects would imply an average net bid-ask running spread effect of FVA and DVA of $2 \times 0.13 = 0.26$ basis points and $(1.3 + 2.5)/2 = 1.9$ basis points respectively, for a total average widening of the bid-ask spread necessary to compensate shareholders of about 2.2 basis points. Again, these numerical effects of netting are purely illustrative. Nevertheless, they portray the importance of netting in reducing the adverse impacts on shareholders of FVA and DVA, and they illustrate the still large residual adverse impacts on shareholders, relative to typical inter-dealer bid-ask spreads. As we have emphasized, if the dealer aligns the incentives of its swap trading desk appropriately, the shareholder costs are passed through to clients in the form of wider bid-ask spreads.

Naturally, as shown in Table II, FVA decreases with the fixed coupon rate $K$. At a coupon rate $K$ of 1.0%, this FVA impact is nearly four times bigger than for a coupon rate of 1.783%. That is, the higher is the fixed rate, the lower is the value to the dealer, resulting in lower upfront financing costs to shareholders. For a sufficiently high coupon rate, the FVA becomes negative, corresponding to a net funding *benefit* to the dealer. Even though the swap has almost no upfront at a fixed rate of $K = 1.783\%$, its has a positive FVA because of the upward-sloping yield curve. That is, the swap is projected to increase in market value over time, as the net coupons flowing to the dealer are expected (under the valuation measure $P^*$) to increase over time. In our model setting, the MVA is invariant to the fixed coupon rate $K$.

As of January 2016, the bid-offer spread on a 10-year par-coupon plain-vanilla LIBOR swap has been around 0.1 bps to 0.2 bps, or about \$10,000 to \$20,000 in dollar terms. As one can see, the impacts of FVA, DVA, and MVA on equity breakeven swap rates are much larger than these typical bid-offer spreads. The fact that dealers now pay close attention to "XVA optimization," as reported by Sherif (2016a), is therefore not surprising.

## V. Concluding Discussion

We now conclude by briefly recapitulating our main results and then discussing additional implications and new research directions.

### *A. Summary of Main Results*

Based on a neoclassical structural model of the balance sheet of a dealer, we show that the quantity known in practice as the "funding value adjustment" is essentially the cost to the dealer's shareholders for financing up-front counterparty cash payments and collateral requirements. This cost to shareholders (which can be negative for swaps that generate positive cash flows to the dealer) is at least partially offset by a change in the value of dealer creditor claims. The total of these value effects on shareholders and creditors is a change in the value of the dealer's frictional financial distress costs.

Our modeling approach is to *(i)* provide a marginal valuation theory for debt and equity benefits associated with financing new investments, *(ii)* derive a pecking order for shareholder financing preferences, *(iii)* apply our framework to the impact on equity and debt values of the unsecured debt financing of swap upfront payments and initial and variation margin cash flows, *(iv)* analyze the impact of shareholder preferences on dealer swap quotations, *(v)* extend by analogy our simple discrete-time structural model to a reduced-form continuous-time term-structure setting, and *(vi)* for a parametric example of the continuous-time model calibrated to recent interest-rate derivatives, obtain illustrative magnitudes of XVAs and their running-spread equivalents in various examples.

We show that the FVA and its close cousin the margin value adjustment (MVA) can be viewed as debt-overhang costs to shareholders that can easily discourage dealers from entering swaps, even on terms that may add a positive market value to the dealer's balance sheet. On average across the swap book, a dealer's shareholders must be compensated for FVAs, MVAs, and debit value adjustments (DVAs) by counterparty "donations" in the form of swap pricing terms that imply trading losses to swap clients. In particular, for a dealer's shareholders to avoid a loss when their firm enters a new swap position, the swap terms must imply a gain the market value of the dealer's swap positions that is at least as large as the sum of the incremental FVA, MVA, and DVA of the swap. In some cases, however, this sum can be negative, implying a gain to shareholders above and beyond the P&L on the trade.

For example, consider the stand-alone \$100 million notional interest-rate payer swap of our illustrative numerical example, at a fixed coupon rate of 1.78%. For a term structure of interest rates and swap-rate volatility like those for the US dollar swap market in January 2016, entering this unsecured swap is beneficial to the dealer's shareholders only if the swap terms imply a trading gain to the dealer's balance sheet of $356,000, in roughly equal parts for FVA, MVA (if initial margin is applicable), and DVA. In practice, as we have discussed in the previous section and modeled formally in Appendix F, netting the swap cash flows against those of legacy swaps would typically reduce this required threshold gain significantly, and in proportion to the degree of netting, case by case.

Although the implications of DVA for swap market values are widely treated in the research literature and in practice, as far as we are aware they are for the first time shown in this paper to have a significant incremental impact on shareholder breakeven valuation and breakeven swap quotation.

Others[18] have already noted (although without a supporting structural model) that treating FVA as an adjustment to the market value of a dealer's swaps causes various logical contradictions. A common informal argument has been that adjusting the market value of a swap for funding costs is a violation of the Modigliani-Miller (MM) Theorem. We confirm that this is the case in the absence of frictional distress costs and provided that valuation impacts are measured in a marginal sense. (Otherwise, MM theory does not precisely apply.[19]) Another inconsistency, emphasized by Burgard and Kjaer (2011), is that an FVA adjustment to swap values violates the simple symmetry condition by which (in the absence of frictional default distress costs) the value to a dealer for entering a swap must be equal and opposite to the value of the swap to the counterparty. These same inconsistencies apply to margin value adjustments (MVAs). In particular, unless there are frictional financial distress costs, it would be impossible for two dealers entering a swap with each other to *both* suffer a loss in the market value of their swap books for the associated margin financing costs, given that the total of the cash flows on the new swap to the two dealers is clearly zero. Further, Hull and White (2012) and Burgard and Kjaer (2011) point out that funding cost adjustments to swap values can imply windfall profits to counterparties or creditors.

### *B. Market-Making Incentives and Other Strategic Implications of FVAs*

Although the common practice of FVA and MVA adjustments to swap market values is inappropriate, it may have arisen from the understandable incentive of large bank holding companies to discourage their swap desks from entering positions that require significant cash financing, given that these are (as we show) a drag on shareholder returns. These funding costs became obvious only after the financial crisis caused significant increases in dealer credit spreads. If accounting practices eventually change so as to correctly reflect the true nature of FVA, some other form of incentives for swap traders should presumably be substituted. For example, the variable component of swap traders' compensation could be based on their trading P&L, *less* an estimate of the incremental

[18] We have already cited Hull and White (2012), Cameron (2013), Cameron (2014a), Becker and Sherif (2015), Hull and White (2014), Albanese and Andersen (2014), and Albanese, Andersen, and Iabichino (2015).

[19] The MM principle is that in the absence of distress costs the dealer's total balance-sheet cash flows and thus total market value are invariant to its capital structure. This is not enough on its own to treat the valuation effects of swap financing, given that adding a swap changes the dealer's total cash flows.

impact of their trading on the firm's FVA, MVA, and DVA.

As we have noted, the clients of dealers must, on average, pay extra, above and beyond the fair market values of their swap positions, in order to give dealers sufficient incentives to enter swaps with them. Swap clients are often willing to do so because they have motives to enter swaps, such as hedging, that dominate these XVA-related trading losses. To the extent that these XVA "donation effects" are positive, which is the case on average, there is a significant business advantage to relatively highly capitalized dealers. The losses that clients must incur in order to compensate dealer shareholders for FVA, MVA, and DVA are all roughly proportional to the dealer's credit spread. (Appendix E provides numerical support for the near linearity of these XVAs over a wide range of dealer credit spreads.) Thus, if Bank A has a credit spread that is half of that of Bank B, then the shareholders of Bank A can break even with a widening of bid-ask spreads for FVA, MVA, and DVA that is only about half of the corresponding widening of bid-ask spreads that Bank B must quote to its customers. For the average case in which FVA, MVA, and DVA sum to a negative impact on dealer shareholders, this would obviously cause buy-side firms to prefer to trade with Bank A over Bank B, other things equal. Our illustrative numerical example showed this advantage to Bank A to be quite significant in economic terms. This XVA advantage to Bank A in attracting more clients is further magnified by the increased degree of netting that would be expected with a larger number of swap positions, thus further reducing the XVA-related component of bid-ask spreads quoted by Bank A, with a positive feedback effect. For special cases in which there is a significant funding *benefit* associated with an incremental position, the dealer with the *higher* credit spread would be expected to benefit most from the position, and to bid more aggressively for the trade. This explains recent aggressive bidding by dealers for cross-currency swaps, because of their typically high funding benefits to dealers, as explained by Wood (2016).

The effect of legacy swap positions for the matching of a buyside firm to a dealer on a new swap trade, however, can swamp any credit spread advantage of one dealer over another. The dealer whose netting (and credit spread) result in the lowest *incremental* sum of FVA, MVA, and DVA is the dealer that is most efficiently positioned to get the trade. Search costs and OTC market opaqueness, however, can prevent this most advantaged dealer from actually winning the trade. Even when there are no legacy swap positions with a given client, the dealer may quote for the effect of XVA costs to shareholders on the basis of expected future netting effects with that client.

The accounting disclosures of dealers such as J.P. Morgan (2014) state that FVA adjustments originate primarily from unsecured derivatives positions with non-financial corporate clients. Dealer-to-dealer transactions normally have had little FVA, as they typically exploit a variation-margin mechanism that, as suggested by Piterbarg (2010), provides the effect of "built-in" financing. Starting in late 2016, however, inter-dealer derivatives positions will be required by U.S. regulators to incorporate initial margin, in order to mitigate the risk of missing payments during the closeout period that would follow a dealer's default, as explained by BCBS (2013). European regulators have delayed implementation of this rule. Initial margin need not be re-pledgable by either party.

The "trapped" portion of initial margin will need to be financed by dealers.[20] According to ISDA (2013), these new regulations will lock up trillions of additional dollars worth of posted margin. For example, Duffie, Scheicher, and Vuillemey (2015) estimate that new inter-dealer margin requirements will increase the aggregate amount of collateral needed in the CDS market by about 70%, before considering other effects such as central clearing and compression trading.

While accounting value adjustments to swap books for initial margin funding (MVAs) have not yet been systematically implemented by dealers, one may expect these adjustments for initial margin financing to ultimately have a significant adverse impact on the shareholder returns of major dealer banks, unless swap businesses are reduced dramatically or further collateral economies are achieved. As we have shown, these MVA adjustments should not be made to swap books, as is current practice, but rather to dealer equity value, but the net impact on reported shareholder returns would be significant and negative by either approach. This implies sharply increased incentives for central clearing and new forms of compression trading. Some of these MVA-related costs to dealer shareholders will end up being passed through to swap clients, as we have explained, in the form of wider bid-ask spreads. This will likely dampen the overall demand for swaps.

It is no surprise that some major dealers have initiated "XVA optimization" programs.[21] Some dealers may find it necessary to significantly reduce their swap intermediation businesses. One major dealer, Deutsche Bank, has already eliminated the bulk of its single-name CDS intermediation business, although the precise motive for this decision was not specifically reported to be motivated by XVA costs. In 2016, another major dealer, Barclays, sold its substantial "non-core" swap portfolio to J.P. Morgan.[22] Our model shows that this novation trade can be motivated by the fact that the associated funding costs to J.P. Morgan's shareholders are lower than those to Barclay's shareholders, given that J.P. Morgan's credit spreads are significantly lower.[23] If FVA were to be treated instead, as suggested by current dealer accounting, as an adjustment to the value of the derivatives themselves, the novation of this swap portfolio to JP Morgan cannot be motivated by any such gain to Barclays' shareholders, who cannot avoid a mark down in the value of their swaps merely by selling the swaps at a reduced market value. Alternatively, and also consistent with our model, there may be cases in which the novation generates better netting for one dealer's shareholders than the other's, and thus a net gain in both FVA and DVA, when summed across the two dealers.[24]

Our structural model of dealer funding costs also has implications for other areas of asset

[20] An international accord reported by Financial Stability Board (2013) mandates the central clearing of standardized swaps, subject to rules and exemptions that vary by jurisdiction, will also have an impact on collateral demand. The advent of regulations governing initial margin will soon further reduce systemic risk, as explained by BCBS (2013).

[21] See Sherif (2016a).

[22] See Morris (2016) and Parsons (2016).

[23] As explained by Sherif (2016a), and consistent with our model, "For banks trying to estimate other banks' FVA costs, SG CIB's Lascar describes a rule-of-thumb method that involves using their five-year credit default swap (CDS) spread. The bank would take its own FVA, divide it by its own CDS spread, and then multiply the result by the other bank's CDS spread."

[24] In this case, however, the novation can also be motivated in part by the associated reduction through netting in deadweight expected financial distress costs.

pricing. For example, based on an extension of our model that allows for the alternative of repo financing of derivatives hedging positions, Song (2016) shows that some supposed "no-arbitrage" pricing relationships frequently break down to an economically important degree in the presence funding costs to derivatives dealers' shareholders for carrying and hedging dealing inventory. In particular, Song (2016) shows that put-call parity must be adjusted significantly for longer-dated options in order to obtain reasonable synthetic pricing for equity dividend strips. He shows that a failure to do so may have lead to a potentially important bias in prior research on the term structure of S&P 500 equity risk premia.

### *C. Adjustments for Use of Regulatory Capital*

In addition to their FVA and MVA adjustments, some banks have recently begun to make further valuation adjustments, so as to factor the effects of regulatory capital requirements into their accounting valuations. As discussed by Sherif (2015a) and Sherif (2016a), a "capital value adjustment" known as "KVA" is purportedly a markdown of the market value of the dealer's swaps associated with the amount of capital needed to support derivatives trading, whether to meet economic risk management requirements or regulatory capital rules. In practice, KVAs are not based on any sort of coherent model. Our basic theory in Section II does indeed imply that when swap or other positions calls for additional equity capital, there is an associated cost to shareholders, which we calculate. This is not, however, an adjustment to the value of the positions themselves, but rather to the value of equity and debt clams on the dealer. Our calculations, however, consider only the amount of equity needed for financing cash or collateral, and not any additional equity that is required to meet specific regulations, such as those associated with Risk Weighted Assets or the Supplementary Leverage Ratio. We have also ignored the incremental costs to shareholders for swap or other new positions associated with meeting the Liquidity Coverage Ratio rule (which may trigger the need to finance additional High Quality Liquid Assets), the Net Stable Funding Ratio, and stress tests (such as CCARs). These rules imply incremental costs to legacy shareholders, and thus have implications for dealer quotation and trader compensation analogous to, but structurally different from, those that we have analyzed in this paper. We leave these KVA and other related implications to future work.

### *D. Final Remarks*

Also left for future research are models determining optimal intermediation strategies, from the viewpoint of dealer shareholder value maximization, given the implications that we have shown for a divergence between fair market values of new positions (in the form of "P&L") and the associated changes in the equity value of the dealer's firm. For example, it is interesting to note that two banks are able to execute trades with each other at prices that can improve the shareholder values of both firms, especially in the context of MVA. Margin lending strategies, as explained by Albanese, Andersen, and Iabichino (2015), can give dealers access to comparatively cheap funding, and provide efficient collateralized funding for lower-rated banks. We believe this is also a topic that will increase in recognized importance.

In general, the management of various "XVA costs" to bank shareholders will test the ability of financial market participants to adapt to a new reality in which a variety of previously under-appreciated financing and regulatory costs to dealer shareholders must be managed in order for robust over-the-counter market intermediation by regulated dealers to remain viable. A potential market adaptation for those OTC financial instruments that are broadly and frequently traded is the introduction of all-to-all trading, for example on an exchange operator's central limit order book.

# REFERENCES


Adrian, Tobias, Erkko Etula, and Tyler Muir, 2014, Financial intermediaries and the cross-section of asset returns, *Journal of Finance* 89, 2557–2596.

Albanese, Claudio, and Leif Andersen, 2014, Accounting for OTC derivatives: Funding adjustments and the re-hypothecation option. Available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2482955`, Working paper, Cass School of Business.

Albanese, Claudio, Leif Andersen, and Stefano Iabichino, 2015, FVA accounting, risk management, and collateral trading, *Risk,* February, 64–69. Available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2517301`.

BCBS, 2013, Margin requirement for non-centrally cleared derivatives, Basel Committee on Banking Supervision and Board of the International Organization of Securities Commissions. Available at `http://www.bis.org/publ/bcbs261.pdf`.

Becker, Lukas, 2015, BAML and Morgan Stanley take FVA losses, *Risk,* February, Available at `http://www.risk.net/risk--magazine/news/2390522/baml--takes--usd497--million--fva--loss`.

Becker, Lukas, 2017, Margin rule mismatch spawns new vm funding cost for buy side, *Risk,* March, Available at `http://www.risk.net/derivatives/4507761/margin--rule--mismatch--spawns--new--vm--funding--cost--for--buy--side`.

Becker, Lukas, and Nazneen Sherif, 2015, The black art of FVA, Part III: A $4 billion mistake?, *Risk,* April, 12–16. Available at `http://www.risk.net/risk--magazine/feature/2402050/the--black--art--of--fva--part--iii--a--usd4--billion--mistake`.

Breeden, Douglas, and Robet Litzenberder, 1978, Prices of state-contingent claims implicit in option prices, *Journal of Business* 51, 621–651.

Brunnermeier, Markus, and Lasse Pedersen, 2009, Market liquidity and funding liquidity, *Review of Financial Studies* 22, 2201–2238.

Burgard, Christoph, and Mats Kjaer, 2011, In the balance, *Risk,* November, 72–75. Available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1785262`.

Cameron, Matt, 2013, The black art of FVA: Banks spark double-counting fears, *Risk,* April, 15–18. Available at `http://www.risk.net/risk--magazine/feature/2257428/the--black--art--of--fva--banks--spark--doublecounting--fears`.

Cameron, Matt, 2014a, The black art of FVA, Part II: Conditioning chaos, *Risk,* April, 14–19. Available at `http://www.risk.net/risk--magazine/feature/2335564/the--black--art--of--fva--part--ii--conditioning--chaos`.

Cameron, Matt, 2014b, J.P. Morgan takes $1.5 billion funding valuation adjustment loss, *Risk,* February, available at http://www.risk.net/risk--magazine/news/2322843/jp--morgan--takes--usd15--billion--fva--loss.

Castagna, Antonio, 2013, Funding valuation adjustment (FVA) and theory of the firm: A theoretical justification of the inclusion of funding costs in the evaluation of financial contracts, Research note, Iason, Ltd., available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2278595.

Castagna, Antonio, 2014, Towards a theory of internal valuation and transfer pricing of products in a bank: Funding, credit risk and economic capital, Research note, Iason, Ltd., available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2392772.

Dalang, Robert, Andrew Morton, and Walter Willinger, 1990, Equivalent martingale measures and no-arbitrage in stochastic securities market models, *Stochastics and Stochastics Reports* 29, 185–201.

Du, Wenxin, Alexander Tepper, and Adrien Verdelan, 2017, Deviations from covered interest rate parity, Working paper, Board of Governors of the Federal Reserve System.

Duffie, Darrell, 2001, *Dynamic Asset Pricing Theory*, third edition (Princeton University Press, Princeton).

Duffie, Darrell, and Ming Huang, 1996, Swap rates and credit quality, *The Journal of Finance* 51, 921–949.

Duffie, Darrell, Martin Scheicher, and Guillaume Vuillemey, 2015, Central clearing and collateral demand, *Journal of Financial Economics* 116, 237–256.

Elouerkhaoui, Youssef, 2016, The funding invariance principle, *Risk,* February, Available at http://www.risk.net/risk--magazine/technical--paper/2444761/the--funding--invariance--principle.

Ernst and Young, 2012, Reflecting credit and funding adjustments in fair value: A survey. Available at http://www.ey.com/GL/en/Newsroom/News-releases/News_Impact-of-regulatory-and-accounting-changes-critical-to-viability-of-banks-OTC-derivatives-

Financial Stability Board, 2013, OTC derivatives market reforms, Fifth progress report on implementation. Available at http://www.financialstabilityboard.org/2013/04/r_130415/.

Gregory, Jon, 2015, *The XVA Challenge: Counterparty Credit Risk, Funding, Collateral, and Capital, Third Edition* (John Wiley & Sons, New York).

Hillion, Pierre, 2016, Derivatives and funding value adjustments: A simple corporate finance approach, Teaching Note, INSEAD, Fontainebleaux, December.

Hull, John, and Alan White, 1993, One factor interest rate models and the valuation of interest rate derivative securities, *Journal of Financial and Quantitative Analysis* 28, 235–254.

Hull, John, and Alan White, 2012, The FVA debate, *Risk,* July, 83–85. Available at http://www.risk.net/risk--magazine/analysis/2188684/risk--25--the--fva--debate.

Hull, John, and Alan White, 2014, Valuing derivatives: Funding value adjustments and fair value, *Financial Analysts Journal* 70, 46–56.

Hull, John, and Alan White, 2016, XVAs: A gap between theory and practice, *Risk* April, available at http://www.risk.net/risk-magazine/opinion/2454400/xvas-a-gap-between-theory-and-practice.

ISDA, 2013, Response to second BIS/IOSCO consultation on "Margin requirements for non-centrally cleared derivatives, International Swaps and Derivatives Association, Available at http://www2.isda.org/attachment/NTQwMg==/ISDA%20Response%20to%20BCBS%20242.pdf.

Jensen, Michael, and William Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of Financial Economics* 3, 305–360.

Johannes, Michael, and Suresh Sundaresan, 2007, The impact of collateralization on swap rates, *The Journal of Finance* 62, 383–410.

J.P. Morgan, 2014, Consolidated financial statements. Available at http://investor.shareholder.com/jpmorganchase/annual.cfm.

KPMG, 2013, FVA: Putting funding into the equation. Available at https://www.kpmg.com/UK/en/IssuesAndInsights/ArticlesPublications/Documents/PDF/Market%20Sector/Financial%20Services/funding-value-adjustments-june13.pdf.

Mengle, David, 2010, The importance of close-out netting, ISDA Research Notes. Available at http://www.isda.org/researchnotes/pdf/Netting-ISDAResearchNotes-1-2010.pdf.

Modigliani, Franco, and Merton Miller, 1958, The cost of capital, corporation finance and the theory of investment, *The American Economic Review* 48, 261–297.

Morris, Stephen, 2016, Barclays agrees to offload derivatives contracts to J. P. Morgan, *Bloomberg*, available at http://www.bloomberg.com/news/articles/2016-02-03/barclays-agrees-to-offload-derivatives-contracts-to-jpmorgan.

Myers, Stewart, 1977, Determinants of corporate borrowing, *Journal of Financial Economics* 5, 147–175.

OCC, 2015, OCC's quarterly report on bank trading and derivatives activities, Office of the Comptroller of the Currency, Washington, DC. Available at http://www.occ.treas.gov/topics/capital-markets/financial-markets/trading/derivatives/dq315.pdf.

Pallavicini, Andrea, Daniele Perini, and Damiano Brigo, 2011, Funding valuation adjustment: A consistent framework including CVA, DVA, collateral, netting rules and re-hypothecation, Working paper, Imperial College London, Department of Mathematics, available at http://arxiv.org/pdf/1112.1521.pdf.

Pallavicini, Andrea, Daniele Perini, and Damiano Brigo, 2012, Funding, collateral and hedging: Uncovereing the mechanics and the subtleties of funding valuation adjustments, Working paper, Department of Mathematics King's College, London, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2161528.

Parsons, Joe, 2016, Barclays offloads non-core derivatives to JP Morgan, *The Trade.* Available at http://www.thetradenews.com/Sell-side/Barclays-offloads-non-core-derivatives-to-JP-Morgan/.

Piterbarg, Vladimir, 2010, Funding beyond discounting: Collateral agreements and derivatives pricing, *Risk,* February, 97–102. Available at http://www.risk.net/digital_assets/895/042--048_AR0310_Cuttingedge.pdf.

Protter, Philip, 2005, *Stochastic Integration and Differential Equations*, second edition (Springer-Verlag, New York).

Rime, Dagfinn, Andreas Schrimpf, and Olav Syrstad, 2017, Segmented money markets and covered interest parity arbitrage, Working paper, Bank for International Settlements.

Schachermayer, W., 1992, A Hilbert space proof of the fundamental theorem of asset pricing in finite discrete time, *Insurance Mathematics and Economics* 11, 249–257.

Sherif, Nazneen, 2015a, KVA: Banks wrestle with the cost of capital, *Risk* March, available at http://www.risk.net/risk-magazine/feature/2395869/kva-banks-wrestle-with-the-cost-of-capital.

Sherif, Nazneen, 2015b, U.S. regulator asks banks for FVA information, *Risk,* August, 5. Available at http://www.risk.net/risk--magazine/news/2419232/us--regulator--presses--banks--for--fva--information.

Sherif, Nazneen, 2016a, Banks launch drive to crush outsized XVAs, *Risk* February, available at http://www.risk.net/risk-magazine/comment/2448322/banks-launch-drive-to-crush-outsized-xvas.

Sherif, Nazneen, 2016b, FVA sceptics lose ground in valuation debate, *Risk* .

Song, Yang, 2016, Dealer funding costs: implications for the term structure of dividend risk premia. Available at http://ssrn.com/abstract=2732133, Working paper, Stanford University.

Sorensen, Eric, and Thierry Bollier, 1994, Pricing swap default risk, *Financial Analysts Journal* 50, 23–33.

Spears, Taylor, 2017, The price of an uncertain promise: CVA, fair value, and the cultures of counterparty risk modeling, Working paper, University of Edinburgh.

Wang, Xingjie, Yangru Wu, Hongjun Yan, and Zhandong Zhong, 2016, Funding liquidity shocks in a natural experiment: Evidence from the CDS Big Bang, Working paper, Southern University of Science and Technology, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2730877.

Wood, Duncan, 2016, How FVA saved the cross-currency swap, *Risk* January, available at http://www.risk.net/risk-magazine/opinion/2440243/how-fva-saved-the-cross-currency-swap.

# Appendix A A Pecking Order of Financing Choices

Section II considers the primary case of debt financing. Here, we consider the alternatives of equity financing and of financing with cash from the firm's balance sheet.

First, in the context of the model setup of Section II, we consider the funding of an investment paying $Y$ by issuing equity. Because investors in a competitive market for newly issued equity break even on their purchase of shares, the incremental effect on the valuation of the legacy shareholders' equity is $\delta E^*[(A+qY-L)^+] - \delta E^*[(A-L)^+] - U(q)$. A calculation shown in Appendix B implies that the marginal value to the legacy shareholders of entering the position is in this case

$$G^0 = \delta E^*(1_{D^c} Y) - u. \tag{24}$$

The calculation (24) of $G^0$ reflects the fact that legacy shareholders must give up the entire valuation of the incremental cash flows that arise from the investment when the firm defaults.

As another alternative financing choice, if the firm is able to, and does, finance the position by using cash from its balance sheet, the initial equity valuation is $\delta E^*[(A - U(q)R + qY - L)^+]$. The marginal value of entering the position to the shareholders is shown in Appendix B to be

$$G_0 = \delta E^*(1_{D^c} Y) - uP^*(D^c). \tag{25}$$

Details underlying the calculations shown for $G^0$ and $G_0$ are omitted for brevity because they are similar to that shown in Appendix B for the calculation of $G$ (the debt financing case) in the proof of Proposition 1.

PROPOSITION 3: *A PECKING ORDER OF FINANCING PREFERENCES. Suppose that the firm's probability of default is not zero and that the marginal investment cost $u$ is strictly positive. The marginal value $G_0$ to the firm's existing shareholders of financing the investment with existing cash is strictly higher than the marginal value $G$ under debt financing, which in turn is strictly higher than the marginal value $G^0$ under equity financing. That is, $G^0 < G < G_0$.*

To prove this result, we will show that $G^0 \leq G \leq G_0$, and that the inequalities are strict if the dealer's default probability is positive. By the fact that

$$G = \delta E^*[1_{D^c}(Y - u(R+S))] = \delta E^*(1_{D^c} Y) - \delta u(R+S)E^*(1_{D^c}),$$

we have $G_0 \geq G$. Moreover, $G_0 > G$ if the credit spread $S$ is strictly positive.

By the fact that $G^0 = \delta E^*(1_{D^c} Y) - u$, it suffices to show that $u \geq \delta u(R+S)E^*(\mathbf{1}_{D^c})$ in order to see that $G \geq G^0$. We recall that $S = RE^*(\phi)/(1 - E^*(\phi))$ and $\phi = 1_D(L - \kappa A)/L$. Thus,

$$1 - E^*(\phi) \geq P^*(D^c),$$

which is equivalent to $1 \geq \delta(R+S)P^*(D^c)$. Again, the inequality is strict if $S$ is positive.

For the case in which the investment cost $u$ is strictly negative, the strict pecking order shown in Proposition 3 is reversed. If $u = 0$, meaning there is no up-front cash flow to finance, then $G^0 = G = G_0$.

# Appendix B Proofs and Calculations for Sections II and III

This appendix supplies proofs of Propositions 1, 3, and 2, and a supplementary calculation of the marginal valuation of the swap transaction package to legacy creditors in Appendix C.C. For generality, we consider two cases: *(i)* there are finite states of the world, *(ii)* there are infinitely many states of the world with a continuous joint density function of $A$ and $L$. In either case, we assume that $A$, $L$, and some given random payoff $Y$ have finite expectations with respect to $P^*$.

## A Proof of Proposition 1

Because we have assumed a competitive capital market with complete information, creditors offering the new debt break even. That is, the market credit spread $s(q)$ on the new debt, which is issued to finance the cost $U(q)$ of the new position, solves

$$U(q) = \delta E^* \left[ \mathbf{1}_{\mathcal{D}^c(q)} U(q)(R+s(q)) + \mathbf{1}_{\mathcal{D}(q)} \frac{\kappa(A+qY_1+qY_2^+)}{L+U(q)(R+s(q))+qY_2^-} U(q)(R+s(q)) \right],$$

where we recall $\mathcal{D}^c(q)$ is the dealer's survival event $\{A+qY-L-U(q)(R+s(q)) \geq 0\}$. By letting $q$ go to zero, one can easily see from the equation that $\lim_{q\to 0} s(q)$ exists, and that $\lim_{q\to 0} s(q) = S = RE^*(\phi)/(1-E^*(\phi))$, where $\phi = \mathbf{1}_D(L-\kappa A)/L$.

If the dealer finances the position by issuing new debt, the marginal value of the asset purchase to shareholders is defined by

$$G = \left. \frac{\partial E^*[\delta(A+qY-L-U(q)(R+s(q)))^+]}{\partial q} \right|_{q=0}.$$

We intend to show that the derivative exists and is given by

$$G = \delta E^*[\mathbf{1}_{D^c}(Y-u(R+S))].$$

By definition,

$$\begin{aligned} G &= \lim_{q\to 0} \delta \frac{E^*[1_{\mathcal{D}^c(q)}(A+qY-L-U(q)(R+s(q)))] - E^*[1_{D^c}(A-L)]}{q} \\ &= \lim_{q\to 0} \delta \frac{E^*[1_{\mathcal{D}^c(q)}(qY-U(q)(R+s(q)))] + E^*[(1_{\mathcal{D}^c(q)} - 1_{D^c})(A-L)]}{q}. \end{aligned}$$

We know

$$\lim_{q\to 0}\delta\frac{E^*[1_{\mathcal{D}^c(q)}(qY-U(q)(R+s(q)))]}{q} = \lim_{q\to 0}\delta E^*[\mathbf{1}_{\mathcal{D}^c(q)}(Y-U(q)/q(R+s(q)))]$$
$$= \delta E^*[1_{D^c}(Y-u(R+S))],$$

where the last equality is due to that $\lim_{q\to 0} U(q)/q$ and $\lim_{q\to 0}(R+s(q))$ exist, and that $A$, $L$, and $Y$ have finite expectations, allowing interchangeability of the limit and expectation. We only need to show

$$\lim_{q\to 0}\delta\frac{E^*[|(1_{\mathcal{D}^c(q)}-1_{D^c})(A-L)|]}{q} = 0. \tag{26}$$

There are two cases to be considered:

*(i)* If the set of possible states of the world is finite, then there exists a $q_0$ such that for any $q < |q_0|$, $\mathbf{1}_{\mathcal{D}^c(q)} - \mathbf{1}_{D^c} = 0$. Thus, (26) is immediate.

*(ii)* If there are infinitely many states of the world, under which $A$ and $L$ have a joint continuous density function, then we know

$$\lim_{q\to 0} P^*(\mathcal{D}^c(q)) = P^*(D^c).$$

It is easy to see that

$$1_{\mathcal{D}^c(q)} - 1_{D^c} = 1_{\mathcal{D}^c(q)\cap D} - 1_{\mathcal{D}(q)\cap D^c},$$

and that $|A-L| \le q|Y-(r+s(q))U(q)/q|$ on the events $D^c(q)\cap D$ and $D(q)\cap D^c$. Thus,

$$\lim_{q\to 0}\delta\frac{E^*[|(1_{\mathcal{D}^c(q)}-1_{D^c})(A-L)|]}{q} \le \lim_{q\to 0}\delta\frac{E^*[|1_{\mathcal{D}^c(q)\cap D}(A-L)|]+E^*[|1_{\mathcal{D}(q)\cap D^c}(A-L)|]}{q}$$
$$= \lim_{q\to 0}\delta E^*[|(1_{\mathcal{D}^c(q)\cap D}+1_{\mathcal{D}(q)\cap D^c})(Y-U(q)/q(R+s(q)))|].$$

By the Lebesgue Dominated Converge Theorem,

$$\lim_{q\to 0} E^*[|(1_{\mathcal{D}^c(q)\cap D}+1_{\mathcal{D}(q)\cap D^c})Y|] = E\left[\lim_{q\to 0}|(1_{\mathcal{D}^c(q)\cap D}+1_{\mathcal{D}(q)\cap D^c})Y|\right] = 0.$$

Since $\lim_{q\to 0} U(q)/q$ and $\lim_{q\to 0}(r+s(q))$ exist, we have

$$\lim_{q\to 0} E^*\left[(1_{\mathcal{D}^c(q)\cap D}+1_{\mathcal{D}(q)\cap D^c})\frac{U(q)}{q}(R+s(q))\right] = \lim_{q\to 0} E^*[(1_{\mathcal{D}^c(q)\cap D}+1_{\mathcal{D}(q)\cap D^c})]\frac{U(q)}{q}(R+s(q)) = 0.$$

Thus,

$$\lim_{q\to 0}\delta\frac{E^*[|(1_{\mathcal{D}^c(q)}-1_{D^c})(A-L)|]}{q} = 0,$$

and we have shown that

$$G = \delta E^*[\mathbf{1}_{D^c}(Y-u(R+S))].$$

## B Marginal Valuation for Legacy Creditors

We also characterize the marginal valuation of the new position to the dealer's legacy creditors. Recall that $Y = Y_1 + Y_2$, where $Y_1^-$ is secured and $Y_2^-$ is unsecured. For an investment of $q$ units, the dealer's assets at time 1 are

$$\mathcal{A}(q) = A + qY_2^+ + qY_1,$$

and the dealer's total liabilities due at time 1 are

$$\mathcal{L}(q) = L + qY_2^- + U(q)(R + s(q)).$$

Thus, the marginal value of the transaction package to the existing creditors is defined by

$$H = \left.\frac{\partial \delta E^* \left[(1 - \mathbf{1}_{\mathcal{D}(q)})L + \mathbf{1}_{\mathcal{D}(q)} \frac{\kappa \mathcal{A}(q)}{\mathcal{L}(q)} L\right]}{\partial q}\right|_{q=0},$$

where we recall $\mathcal{D}(q)$ is the dealer's default event with the new position. Thus,

$$\begin{aligned} H &= \lim_{q \to 0} \delta \frac{E^* \left[(1 - \mathbf{1}_{\mathcal{D}(q)})L + \mathbf{1}_{\mathcal{D}(q)} \frac{\kappa \mathcal{A}(q)}{\mathcal{L}(q)} L\right] - E^*[(1 - \mathbf{1}_D)L + \mathbf{1}_D \kappa A]}{q} \\ &= \lim_{q \to 0} \delta \frac{E^*[\mathbf{1}_{\mathcal{D}(q)}(\mathcal{A}(q) - A)] - (1 - \kappa)E^*[\mathbf{1}_{\mathcal{D}(q)}\mathcal{A}(q) - \mathbf{1}_D A] - E^* \left[\mathbf{1}_{\mathcal{D}(q)} \frac{\mathcal{L}(q) - L}{\mathcal{L}(q)} \kappa \mathcal{A}(q)\right]}{q}, \end{aligned}$$

where the last equality is due to that $\lim_{q \to 0} E^*[(\mathbf{1}_{\mathcal{D}^c(q)} - \mathbf{1}_{D^c})(A - L)]/q = 0$, as we have shown. For simplicity, we write

$$\psi \equiv \lim_{q \to 0} \frac{E^* \left(\mathcal{A}(q)\mathbf{1}_{\mathcal{D}(q)}\right) - E^* \left(A \mathbf{1}_D\right)}{q} = E^*[\mathbf{1}_D(Y_2^+ + Y_1)] + \lim_{q \to 0} \frac{E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]}{q},$$

where we write $\gamma = (1 - \beta)\mathbf{1}_B$. There are two cases to be discussed:

*(i)* In the finite-space case, $\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D = 0$ for sufficiently small $q$. Thus, $\lim_{q \to 0} E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]/q = 0$, and

$$\psi = E^*[\mathbf{1}_D(Y_2^+ + Y_1)].$$

*(ii)* In the infinite-state space case,

$$\psi = E^*[\mathbf{1}_D(Y_2^+ + Y_1)] + J,$$

with $J \equiv \lim_{q \to 0} E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]/q$. The existence of $J$ is guaranteed by the fact that $A$ and $L$ have a continuous joint density.

Thus, the marginal value of the package to the existing creditors is

$$
\begin{aligned}
H &= \delta E^*[\mathbf{1}_D(Y_2^+ + Y_1)] - \delta E^*\left[\mathbf{1}_D \frac{\kappa A}{L}(Y_2^- + u(R+S))\right] - \delta(1-\kappa)\psi \\
&= \delta E^*[\mathbf{1}_D(Y - uR)] + \delta E^*(\phi Y_2^-) + \delta E^*(\mathbf{1}_{D^c} uS) - \delta(1-\kappa)\psi,
\end{aligned}
$$

where the last equality is due to that $E^*(\mathbf{1}_D uR) - E^*[\mathbf{1}_D u(R+S)\kappa A/L] = E^*(\mathbf{1}_{D^c} uS)$. In the special case that the deadweight frictional loss is zero (that is, if $\kappa = 1$), then

$$
H = \delta E^*[\mathbf{1}_D(Y - uR)] + \delta E^*(\phi Y_2^-) + \delta E^*(\mathbf{1}_{D^c} uS).
$$

## *C The Asset-Substitution Effect*

Our first-order analysis captures the effect of debt overhang, but not the effect of asset substitution, by which there may be some offsetting shareholder benefit associated with adding more volatile cash flows, given the option-like effect of the limited liability of equity positions. Here, we provide supporting calculations for the second-order, asset-substitution, impact on shareholder value. By the usual Taylor-series argument, the second-order asset-substitution effect is always dominated by the first-order debt-overhang effect for sufficiently small incremental positions.

Here, in order to get simple explicit expressions, we treat only settings in which $(A, L, Y)$ has a continuous joint density function. Cases with finitely many states of world are therefore omitted. The existence of the derivative $s'(q)$ of the credit spread with respect to the new position size $q$ is guaranteed by the joint continuous density function for $(A, L, Y)$.) We assume that $U(q)$ is differentiable, and that $\lim_{q\to 0} U'(q) = u$.

We first calculate the marginal shareholder value

$$
G(q) \equiv \delta \frac{\partial E^*[(A + hY - L - U(h)(R + s(h)))^+]}{\partial h}\bigg|_{h=q}.
$$

By definition,

$$
\begin{aligned}
G(q) &= \lim_{h\to q} \delta \frac{E^*\left[(A - L + hY - U(h)(R+s(h))\mathbf{1}_{\mathcal{D}^c(h)}\right] - E^*\left[(A - L + qY - U(q)(R + s(q)))\mathbf{1}_{\mathcal{D}^c(q)}\right]}{h-q} \\
&= \lim_{h\to q} \delta \frac{E^*\left[((h-q)Y - (U(h) - U(q))R)\mathbf{1}_{\mathcal{D}^c(h)}\right]}{h-q} - \lim_{h\to q} \delta \frac{E^*\left[(U(h)s(h) - U(q)s(q))\mathbf{1}_{\mathcal{D}^c(h)}\right]}{h-q} \\
&+ \lim_{h\to q} \delta \frac{E^*\left[(A - L + qY - U(q)(R+s(q)))\left(\mathbf{1}_{\mathcal{D}^c(h)} - \mathbf{1}_{\mathcal{D}^c(q)}\right)\right]}{h-q} \\
&= \delta E^*[(Y - U'(q)R)\mathbf{1}_{\mathcal{D}^c(q)}] - \delta E^*[\mathbf{1}_{\mathcal{D}^c(q)}(U(q)s'(q) + s(q)U'(q))] + \Pi(q),
\end{aligned}
$$

where

$$
\Pi(q) = \lim_{h\to q} \delta \frac{E^*[(A - L + qY - U(q)(R + s(q)))(\mathbf{1}_{\mathcal{D}^c(h)} - \mathbf{1}_{\mathcal{D}^c(q)})]}{h-q}.
$$

By arguments similar to those above, $\Pi(q) \equiv 0$. Thus,

$$G(q) = \delta E^*[(Y - U'(q)R)\mathbf{1}_{\mathcal{D}^c(q)}] - \delta E^*[\mathbf{1}_{\mathcal{D}^c(q)}(U(q)s'(q) + s(q)U'(q))].$$

We have shown that

$$G(0) = \delta E^*[\mathbf{1}_{D^c}(Y - uR)] - \delta E^*[u\mathbf{1}_{D^c}S].$$

The second derivative of shareholder value with respect to position size $q$ is

$$\begin{aligned}
g &= \lim_{q\to 0} \frac{G(q) - G(0)}{q} \\
&= \lim_{q\to 0} \delta \frac{E^*[\mathbf{1}_{\mathcal{D}^c(q)}(Y - u(R + s(q)))] - E^*[\mathbf{1}_{D^c}(Y - u(R + S))]}{q} - \delta E^*[\mathbf{1}_{D^c} us'(0)] \\
&= \lim_{q\to 0} \delta \frac{E^*\left[\left(\mathbf{1}_{\mathcal{D}^c(q)} - \mathbf{1}_{D^c}\right)(Y - u(R + S))\right]}{q} - 2\delta E^*[\mathbf{1}_{D^c} us'(0)] \\
&= \delta E^*[(Y - u(R + S))^2 f(L \,|\, L, Y)] - 2\delta us'(0)p^*, \qquad (27)
\end{aligned}$$

where $p^* = P^*(D^c)$ is the risk-neutral survival probability and $f(x \,|\, L, Y)$ denotes the risk-neutral probability density at $x$ of $A$ conditional on $(L, Y)$, and

$$s'(0) = \frac{R\kappa}{(1 - E^*(\phi))^2}\, E^*\left[\mathbf{1}_D\left(\frac{Au(R + S) - YL}{L^2}\right) - \frac{L - \kappa E^*(A)}{\kappa L}(Y - u(R + S))f(L \,|\, L, Y)\right].$$

For the rest of this section, we restrict attention for sake of simplicity to the case in which $L$ is a constant and $Y$ is independent (under $P^*$) of $A$. We then have

$$g = \delta f(L) E^*[(Y - u(R + S))^2] - 2p^*\delta us'(0), \qquad (28)$$

and we can write

$$s'(0) = \frac{R\kappa(a - b)}{(1 - E^*(\phi))^2},$$

where

$$a = \frac{u(R + S)E^*(\mathbf{1}_D A) - E^*(Y)LP^*(D)}{L^2}$$

and

$$b = \frac{L - \kappa E^*(A)}{\kappa L} f(L)\left(E^*(Y) - u(R + S)\right).$$

In general, $s'(0)$ can be positive, negative, or zero. However, for the case of a trade that is "breakeven" after debt servicing costs, in that $\delta E^*(Y) - \delta u(R + S) = 0$, we have

$$s'(0) = \frac{R\kappa E^*(Y)E^*[\mathbf{1}_D(A - L)]}{(1 - E^*(\phi))^2} < 0.$$

We can use the result of Breeden and Litzenberder (1978) that $\delta f(L)$ is equal to the "gamma" (second derivative) $\mathcal{E}''(L)$ of the equity value function $\mathcal{E}(\,\cdot\,)$, treated as the function mapping the strike price $L$ to the equity value $\delta E^*[(A-L)^+]$. Thus,

$$g = \mathcal{E}''(L) m_2 - 2p^* \delta u s'(0), \tag{29}$$

where $m_2 = E^*[(Y - u(R+S))^2]$ is the second moment of the payoff of the investment net of the total financing payback.

The first term of $g$ in (29) now appears clearly as the volatility impact of asset substitution, namely the product of the "equity gamma" $\mathcal{E}''(L)$ and the second moment $m_2$ of the net marginal payoff to shareholders. When the trade is done on a shareholder break-even basis after considering financing costs, in that $\delta E^*(Y) = \delta u(R+S)$, the second moment is the risk-neutral variance of the investment payoff net of financing costs.

For the case of a dealer financing the collateral required for hedged swap positions or financing a CIP basis arbitrage, the asset substitution effect is extremely small, because the incremental asset payoff $Y$ is risk-free or nearly risk-free.

For a non-zero investment position $q$, the second-order Taylor series approximation of the incremental gain to equity value associated with the investment can now be computed explicitly as

$$qG + \frac{q^2}{2} g, \tag{30}$$

where $G$ is given by (5). This expansion captures both the first-order effects of debt overhang and the second-order effects of asset substitution.

To further interpret the "asset-substitution" effect $g$, we can consider the case in which $A$ is log-normally distributed, in that

$$A = A_0 \exp\left(\log R - \frac{\sigma^2}{2} + \sigma \mathcal{W}\right),$$

where $A_0$ is a positive constant, $\mathcal{W}$ is standard normal under $P^*$, and $\sigma$ is the volatility of the firm's existing assets. Applying the Black-Scholes formula, we have the explicit equity gamma

$$\mathcal{E}''(L) = \frac{\delta \mathcal{N}(d_2)}{L\sigma^2},$$

where $\mathcal{N}$ is the probability density of the standard normal distribution and

$$d_2 = \frac{\log A_0 - \log L + \log R - \frac{1}{2}\sigma^2}{\sigma}.$$

One can see that

$$s'(0) = \frac{R\kappa(a-b)}{(1-E^*(\phi))^2},$$

where

$$a = \frac{u(R+S)RA_0\mathcal{N}(-d_2-\sigma) - E^*(Y)L\mathcal{N}(-d_2)}{L^2}$$

and

$$b = \frac{L-\kappa R}{\kappa L} f(L)\left(E^*(Y) - u(R+S)\right).$$

In the "breakeven" case $\delta E^*(Y) = \delta u(R+S)$, we can further simplify to

$$s'(0) = \frac{R\kappa E^*(Y)(RA_0\mathcal{N}(-d_2-\sigma) - L\mathcal{N}(-d_2))}{(1-E^*(\phi))^2 L^2}.$$

## D *Proof of Proposition 2*

The proof has the following three parts.

*(i)* We have characterized the net cash flow of the package of transactions if the dealer finances the upfront payment $U(q)$ by issuing new debt. The net cash flow at time 1, from the viewpoint of the dealer, is

$$\mathcal{C}(q) = q(X-K) - q(1-\beta)(X-K)^+\mathbf{1}_B + (1-\kappa\rho(q))q(X-K)^-\mathbf{1}_{\mathcal{D}(q)}$$

where $\mathcal{D}(q) = \{A - L + qY - U(q)(R+s(q)) < 0\}$ is the event of the dealer's default with $Y = X - K - (1-\beta)(X-K)^+\mathbf{1}_B$, and $\rho(q)$ is the asset-to-debt payoff ratio

$$\rho(q) = \frac{A}{L + U(q)(R+s(q)) + q(X-K)^-}.$$

Thus, the market value of the package of transactions is

$$\mathcal{V}(q) = \delta E^*[\mathcal{C}(q)].$$

*(ii)* Suppose the dealer finances the initial investment by issuing new equity. The dealer's default event in this case is $\mathcal{D}^0(q) = \{A - L + qY < 0\}$. The cash flow $q(X-K)$ of the unsecured client-to-dealer is not paid in full at time 1 in either of the two events: (a) the event that the client defaults and $q(X-K) > 0$, in which case the dealer receives $\beta q(X-K)$ from the client, and (b) the event that the dealer defaults and $q(X-K) < 0$, in which case the client is pari passu with other creditors of the dealer, and the proportional recovery rate is

$$\kappa\rho^0(q) = \frac{\kappa A}{L + q(X-K)^-}.$$

Thus, the net cash flow at time 1, from the viewpoint of the dealer, is

$$\mathcal{C}^0(q) = q(X-K) - q(1-\beta)(X-K)^+\mathbf{1}_B + (1-\kappa\rho^0(q))q(X-K)^-\mathbf{1}_{\mathcal{D}^0(q)}.$$

The market value of the package of transactions is

$$\mathcal{V}^0(q) = \delta E^*[\mathcal{C}^0(q)].$$

*(iii)* If the dealer finances the initial investment by using cash from its balance sheet, the dealer's default event is $\mathcal{D}_0(q) = \{A + qY - L - U(q)R < 0\}$. Thus, the net cash flows at time 1, from the dealer's perspective, is

$$\mathcal{C}_0(q) = q(X-K) - q(1-\beta)(X-K)^+\mathbf{1}_B + q(1-\kappa\rho_0(q))(X-K)^-\mathbf{1}_{\mathcal{D}_0(q)},$$

where $\rho_0(q) = (A - U(q)R)/(L + q(X-K)^-)$. Thus, the market value of the package of transactions is

$$\mathcal{V}_0(q) = \delta E^*[\mathcal{C}_0(q)].$$

It is easy to see that whether the dealer finances the initial investment by issuing debt, by issuing equity, or by using existing cash, the marginal value of the package to the dealer is

$$V = \lim_{q\to 0}\frac{\mathcal{V}(q)}{q} = \lim_{q\to 0}\frac{\mathcal{V}^0(q)}{q} = \lim_{q\to 0}\frac{\mathcal{V}_0(q)}{q} = \delta(X-K) + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma(X-K)^+],$$

where the last equality is due to the fact that $A$, $L$, and $Y$ have finite expectations, allowing interchangeability of the limit and expectation.

## *E Marginal Swap Valuation to Legacy Creditors*

We first calculate the marginal value $H$ of the package of transactions to the legacy creditors by assuming the dealer finances the initial investment by issuing new debt. For an investment of $q$ units, the dealer's assets at time 1 are

$$\mathcal{A}(q) = A + q(X-K)^+ - q\mathbf{1}_B(1-\beta)(X-K)^+ + q(\tilde{K} - X) + qIR.$$

The dealer's total liabilities due at time 1 are

$$\mathcal{L}(q) = L + q(X-K)^- + qI(R + s(q)).$$

As in the proof of Proposition 1, we can show that the marginal value of the package to the existing creditors is

$$H = \delta P^*(D)(\tilde{K} - K) + \Lambda + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma\mathbf{1}_D(X-K)^+] - \delta(1-\kappa)\psi,$$

where *(i)* in the finite-state space case,

$$\psi = E^*[\mathbf{1}_D((X-K)^+ + (\tilde{K}-X) + IR)] - E^*[\gamma \mathbf{1}_D(X-K)^+],$$

and *(ii)* in the infinite-state space case,

$$\psi = E^*[\mathbf{1}_D((X-K)^+ + (\tilde{K}-X) + IR)] - E^*[\gamma \mathbf{1}_D(X-K)^+] + J$$

with $J = \lim_{q\to 0} E^*[A(\mathbf{1}_{\mathcal{D}(q)} - \mathbf{1}_D)]/q$. The existence of $J$ is guaranteed by the fact that $A$ and $L$ have a continuous joint density.

If the dealer instead finances the initial investment by issuing new equity, it can be shown similarly that the marginal value of the package of transactions to the dealer's legacy creditors $H^0$ is

$$H^0 = \delta P^*(D)(\tilde{K}-K) + E^*(\mathbf{1}_D)I + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma \mathbf{1}_D(X-K)^+] - \delta(1-\kappa)\psi^0,$$

where *(i)*

$$\psi^0 = E^*[\mathbf{1}_D(\tilde{K} - X + IR + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+]$$

in the finite-state space case, and *(ii)*

$$\psi^0 = E^*[\mathbf{1}_D(\tilde{K} - X + IR + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+] + \hat{J}$$

in the infinite-state space case with $\hat{J} = \lim_{q\to 0} E^*[A(\mathbf{1}_{\mathcal{D}^0(q)} - \mathbf{1}_D)]/q$.

Finally, if the dealer finances the initial investment by using cash on the balance sheet, the marginal value of the package of transactions to the dealer's legacy creditors $H_0$ is

$$H_0 = \delta P^*(D)(\tilde{K}-K) + \delta E^*[\phi(X-K)^-] - \delta E^*[\gamma \mathbf{1}_D(X-K)^+] - \delta(1-\kappa)\psi_0,$$

where *(i)*

$$\psi_0 = E^*[\mathbf{1}_D(\tilde{K} - X + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+]$$

in the finite-state space case, and *(ii)*

$$\psi_0 = E^*[\mathbf{1}_D(\tilde{K} - X + (X-K)^+)] - E^*[\gamma \mathbf{1}_D(X-K)^+] + \tilde{J}$$

in the infinite-state space case with $\tilde{J} = \lim_{q\to 0} E^*[A(\mathbf{1}_{\mathcal{D}_0(q)} - \mathbf{1}_D)]/q$.

## *F Modigliani-Miller Invariance in the Absence of Distress Costs*

If there are no deadweight frictional losses at the dealer's default (that is, if $\kappa = 1$), we have the following result.

PROPOSITION 4: MODIGLIANI-MILLER INVARIANCE. *If the fractional default recovery rate $\kappa$ is 1, then the total marginal value of the forward portfolio to the dealer is invariant to how the collateral is financed, and identical to the market value of the forward portfolio. That is,*

$$G + H = G_0 + H_0 = G^0 + H^0 = v.$$

## Appendix C Secured or Hedged Swaps

This appendix extends the results of Section III.A to treat cases involving variation margin, hedged swap positions, and margin value adjustments.

### A Variation Margin and Inter-Dealer Hedging

When a dealer trades an unsecured swap with a client, the dealer is likely to combine the position with a suitable hedge. In practice, two separate hedges would typically be used. One hedge would mitigate the risk of default of the swap counterparty, for instance using a credit default swap (CDS) referencing the counterparty. Another position would be taken as a hedge against the market risk exposure of the floating-side payment $X$.

Using the setup in Section III.A, we can incorporate the effect of hedging a swap by assuming that the hedge simply takes the form of an offsetting position paying $-Y$, where $Y$ is the net payout given by (6). As an abstract simplification, this covers both the counterparty risk and the underlying market risk $X$. The hedge is executed with another dealer, called the "hedge dealer." As is standard practice in inter-dealer transactions, the hedge requires the posting of variation margin, a running exchange of collateral that is sufficient to cover the entire present value of the transaction. In addition to providing default protection for both dealers, the variation margin mechanism provides an automatic source of cash funding of the hedge position, as we mentioned earlier.

In our one-period model, we can capture the effect of a running posting of variation margin in the following simplified way.

- At time 0, the dealer receives a cash payment from the hedge dealer equal to the fair value $\delta E^*(Y)$. The dealer immediately posts this cash amount back to the hedge-dealer as a variation margin payment, earning the risk-free rate on the associated posting of collateral. As the two initial cash payments cancel, neither the dealer nor the hedge-dealer needs any financing to instantiate the hedge transaction.
- At time 1, but before other cash flows at time 1 are paid, the collateral is refreshed. That is, the dealer receives $E^*(Y)$ back from the hedge dealer. (This is margin posted at time zero, plus the risk-free interest.) The dealer pays $Y$ to the hedge-dealer. The hedge-dealer is

assumed to be paid with priority over all other creditors.[25] As the swap itself pays $Y$, given this assumed priority, the dealer will always be able to make this payment. This abstracts from some potential loss of priority that might apply in extreme practical cases, for example in an administrative failure resolution process that could override contractual termination rights.

Netting the cash flows, the total package consisting of an unsecured asset and the hedge will pay the dealer $E^*(Y)$ at time 1, an amount that is known at time zero. As desired, the hedge removes the variability of the payment $Y$, replacing it with its fair forward value.

Assuming that the dealer finances the purchase of the client asset by issuing debt, we can now repeat the funding cost analysis shown in Section III.A. The results, found in Appendix B, are obvious. Because the hedge removes net payout variance, the covariance term in (12) disappears, and the FVA for the package consisting of the asset and its hedge is simply $g(v - d) = -\Phi$.

As we have explained, the assumption of a perfectly offsetting hedge payout of $-Y$ is an idealization. In practice, the risk associated with the client swap payoff is not completely extinguished. This allows small default covariance terms to creep back into the breakeven price $v^*$. Further, inter-dealer hedge swaps are virtually always executed at par, that is, at a fixed rate of $\tilde{K} = E^*(X)$, rather than at an arbitrary rate of $K$. We deal with this minor complication in the next section.

### *B Par Swaps and Forward Swap Rates Without Margin*

In practice, the fixed swap rate $K$ is typically negotiated so that there is no upfront payment. In this case, the swap is known as a "par-valued swap." The resulting fixed rate $K$ is often known as the "forward swap rate." In our setting, three different forward swap rates are of interest:

- The forward swap rate $\tilde{K}$ for a fully collateralized dealer-to-dealer swap. The swap has a market value of $\delta E^*(X - \tilde{K})$, so the fair forward swap rate $\tilde{K} = E^*(X)$ reflects no credit risk component. This is the benchmark forward swap rate typically shown on standardized trading screens. In practice, the risk-neutral probability measure $P^*$ used by dealers for fair valuation would typically be calibrated so as to match the risk-neutral expected payment $E^*(X)$ to the "screen rate" $\tilde{K}$, and likewise for other liquidly traded financial instruments.
- The forward swap rate $\hat{K}$ for an unsecured client swap that is executed at fair-market pricing. If we express $v$ in (7) as $v = \eta(K)$, then $\hat{K}$ is the solution in $K$ of the equation $\eta(K) = 0$.
- The forward swap rate $K'$ for an unsecured client swap that leaves shareholders indifferent to the trade. From (6) and (8), $K'$ is determined by the equation $E^*(1_{D^c} y(K')) = 0$.

[25] This effective priority over standard debt claims follows from exemptions for swaps from automatic stays in bankruptcy or other insolvency proceedings. Even under proposed methods for resolving the failure of a systemically important dealer that would apply the effect of an automatic stay on swap terminations, the dealer's swaps would likely retain priority over ordinary creditors, who would be "bailed in." This would fully prioritize swap counterparties except in the most extreme scenarios, in which even the cancellation of all debt subject to bail-in is insufficient to re-capitalize the dealer.

Neither $\hat{K}$ nor $K'$ depend on the financing strategy used by the dealer. Without an upfront, no financing is required, putting aside for now the issue of initial margin, which we will get to later in this section. Here, $\hat{K}$ and $K'$ differ only because the DVA benefit on the swap is excluded from $K'$.

LEMMA 1: *ORDERING OF FORWARD SWAP RATES. Suppose that either (a) the dealer's default indicator $1_D$ is uncorrelated (under $P^*$) with the swap payment $Y$, or (b) the swap position is fully hedged by an inter-dealer swap. Then $K' \leq \hat{K}$ and $K' \leq \tilde{K}$.*

In a model with several time periods, even a position with no upfront cash payment may involve a funding value adjustment. For example, consider a position entered a time zero with no upfront payment, requiring a significant positive expected cash payment by the dealer at some intermediate date or dates, before compensating payments are later received by the dealer. A common example of this is a long-dated swap issued in an environment with a steeply sloped yield curve. As we will explain in more detail in Section IV, such a position can be associated with a substantial funding value adjustment.

### *C Par Swaps with Initial Margin, and Margin Value Adjustment (MVA)*

Par-valued swaps require no upfront funding and therefore have no FVA in our one-period setting. This situation changes with the introduction of initial margin, whether on the client swap itself or on the hedge swaps. In fact, it is becoming increasingly common to encounter swap agreements that require one or both counterparties to post risk-based initial margin, providing an additional layer of credit risk protection beyond variation margin. For instance, such agreements are routinely required by CCPs and are supposed to be mandatory under the Dodd-Frank act for all inter-dealer trades executed after September 2016 (see BCBS (2013)). European regulators have delayed the application of this rule to 2017. Because initial margin always implies a positive initial cash outlay, even for par-valued swaps, funding valuation adjustments for margin will inevitably result in costs to dealer shareholders.

To be concrete, we consider the funding cost impact on the shareholders of a swap dealer that hedges an unsecured par-valued swap with a par-valued hedge transaction that requires the dealer to post initial margin. In summary, the swaps dealer in question is contemplating a pair of transactions consisting of:

*(i)* An uncollateralized swap with a client, by which the dealer pays a fixed rate $K$ in exchange for a floating payment $X$, for a net contractual receivable at time 1 of $X - K$. We take $K$ as given for now, and assume that the client swap terms involve no initial exchange of cash. The terms of trade for the swap are thereby captured entirely by the fixed-side payment $K$.

*(ii)* A hedge-motivated fully collateralized swap with another dealer or a central counterparty, by which the dealer has a net receivable at time 1 of $\tilde{K} - X$, at the fair forward swap rate $\tilde{K} = E^*(X)$. As before, we suppose that the hedge swap involves variation margin and no

net initial payment. In this case, however, the swap additionally requires the dealer to post a specified cash initial margin of $I > 0$. The recipient of the margin, typically either a CCP or a third-party custodian, invests the margin in risk-free assets, paying the dealer $RI$ back at time 1 (unless the dealer defaults). As a simplification, we assume that the margin agreement is sufficient to ensure that both of the counterparties to the hedge swap are fully secured against loss.

The hedge swap payout $\tilde{K} - X$ is not an exact match for the client swap, except in the unlikely case that $K = \tilde{K}$. We do not consider a CDS hedge against default, but our results can be trivially extended to this case.[26] Our results are unaffected if the initial margin $I_q$ for a position of size $q$ is not necessarily proportional to $q$, provided that the per-unit margin has some limit $I \equiv \lim_{q\downarrow 0} I_q/q$. Likewise, our results remain as stated if the swap fixed-side terms $K$ and $\tilde{K}$ depend on $q$, provided only that they converge with $q$ to limits denoted $K$ and $\tilde{K}$, respectively. These generalizations are avoided merely for notational simplicity.

We carry over all notation from Section III.A. Once again, the effect of any pre-existing positions between the swap counterparties is considered only in the appendix. We model variation margin in the same manner as in Section C.A, so that the net payment at time 1 on the hedge swap is $E^*(X - \tilde{K}) - (X - \tilde{K}) = \tilde{K} - X$. Before considering the impact of dealer default, the package of swap transactions therefore has a per-unit cash flow to the dealer at time 1, including the return of the margin with interest, of

$$Y = RI + \tilde{K} - K - \gamma(X - K)^+.$$

The initial required per-unit cash investment $u$ is merely the initial margin $I$, because the swaps themselves are all executed without upfront payments.

Assuming that the initial margin is funded by debt issuance, Proposition 1 implies that the marginal value of the transaction to the dealer's shareholders is

$$G = \delta P^*(D^c)(\tilde{K} - K) - \delta E^*[1_{D^c}\gamma(X - K)^+] - \Lambda, \tag{31}$$

where $\Lambda = \delta P^*(D^c)SI$ is the funding cost adjustment for the payment of initial margin, known in industry practice as the margin value adjustment (MVA). In this simplest of settings, the value adjustment $\Lambda$ for initial margin is the initial market value of the component of net margin-funding interest expense $SI$ that is borne by shareholders at time 1. The shareholders bear the entire expense $SI$ if the dealer does not default, and bear none of the expense if the dealer defaults.

We also calculate the total market value of the package of swap transactions. For a position of $q$ units, the initial margin payment generates cash flow of $-qI$ to the dealer at time 0. At time 1, the payment of the hedging swap, including the return of margin with interest, is $q(\tilde{K} - X) + qIR$.

[26] As we have already seen in Section C.A, adding a CDS hedge essentially removes the covariance effects in the CVA term. For instance, the term $\delta E^*[1_{D^c}\gamma(X - K)^+]$ in (31) would become $\delta P^*(D^c)E^*[\gamma(X - K)^+]$.

The payment of the client-to-dealer swap to the dealer is $q(X-K)$ before considering default. The cash flow $q(X-K)$ is not paid in full at time 1 in either of two events: *(i)* the client defaults and $q(X-K)>0$, in which case the dealer receives $\beta q(X-K)^+$ from the client; and *(ii)* the dealer defaults and $q(X-K)<0$, in which case the client is *pari passu* with the other creditors of the dealer, and the swap client receives $\mathcal{R}(q)q(X-K)^-$, where, based on (2),

$$\mathcal{R}(q)=\frac{\kappa(A+q(\tilde{K}-X)+qIR)}{L+q(X-K)^-+qI(R+s(q))}$$

is the fractional recovery of the dealer's assets in default on the event that $X-K<0$. The numerator of $\mathcal{R}(q)$ is the amount of the dealer's assets that are recovered if the dealer defaults and $X-K<0$. The denominator is the aggregate liabilities of the dealer, which include the legacy liabilities $L$, the liabilities due to financing the initial margin, which is $qI(R+s(q))$, and the liabilities to the swap client, which is $q(X-K)^-$. By assumption, $A+qIR+q\,(X-K)^+$ is always sufficient to pay the amount $q(\tilde{K}-X)^-$ due on the secured hedge.

Following the definitions of Section II.B, the net actual cash flow at time 1 of the package of swap transactions is

$$\hat{\mathcal{C}}(q)=q(\tilde{K}-X)+qIR+q(X-K)-q\gamma(X-K)^++q1_{\hat{\mathcal{D}}(q)}(1-\mathcal{R}(q))(X-K)^-,$$

where

$$\hat{\mathcal{D}}(q)=\{A+q(\tilde{K}-K)-q\gamma(X-K)^+-L-qIs(q)<0\}$$

is the event of the dealer's default.

The total market value of the package of transactions is

$$\mathcal{V}(q)=-qI+\delta E^*(\hat{\mathcal{C}}(q)).$$

One can see that the initial payment $I$ of margin at time 0 and the return payment of $RI$ at time 1 have offsetting impacts on the total market value of the swap. When considering the marginal value of the transaction to shareholders, however, the computation (eq:G3) shows the crucial impact on shareholder value of of financing the initial margin.

Similar to the case of Proposition 2, the marginal value of the swap,

$$v=\left.\frac{\partial\mathcal{V}(q)}{\partial q}\right|_{q=0}=\delta(\tilde{K}-K)-\delta E^*[\gamma(X-K)^+]+\delta E^*[\phi(X-K)^-], \tag{32}$$

is decomposed into the present value of the gross swap spread $\tilde{K}-K$, less the CVA, plus the DVA. As anticipated, the per-unit fair market value $v$ of the combined swap position does not depend on the amount $I$ of required initial margin, nor does $v$ depend on how the margin was financed. As we have noted, however, this invariance of valuation to the financing of initial margin is contrary to current dealer valuation practice.

Appendix B calculates the impact of the value $H$ of the package on the legacy creditors. If there are no default distress costs, we have usual value-conservation identity $H + G = v$.

The fair-market level of the spread $\tilde{K} - K$ between the two swap rates, obtained from (32) by setting $v$ equal to zero, is

$$\mathcal{S} = E^*[\gamma(X - K)^+] - E^*[\phi(X - K)^-], \tag{33}$$

which is merely the net risk-neutral expected default loss on the client swap (loss from client default net of loss from dealer default). The swap spread $\mathcal{S}' = \tilde{K} - K$ that makes the dealer's shareholders indifferent to the trade is instead obtained from (31) by setting $G = 0$, leaving

$$\mathcal{S}' = SI + \frac{E^*[1_{D^c}\gamma(X - K)^+]}{P^*(D^c)}.$$

In order to generate positive shareholder returns in this setting, the dealer must be able to identify hedged swap positions at fixed swap rates that improve on fair-market rates by $\mathcal{S}' - \mathcal{S}$. In gauging how difficult this may be for the dealer's swap desk, we suppose that the dealer's default event is uncorrelated under $P^*$ with the client default loss $\gamma(X - K)^+$. The dealer must then be able to improve on fair-market swap rates by at least

$$\mathcal{S}' - \mathcal{S} = SI + E^*(\phi)E[(X - K)^-].$$

For the typical (small) credit spreads of major dealers, and for small risk-free interest rates (that is, $R$ near 1), we have the Taylor approximation $S \simeq E^*(\phi)$, and thus

$$\mathcal{S}' - \mathcal{S} \simeq S\left(I + E^*[(X - K)^-]\right), \tag{34}$$

where the first term originated from the margin funding costs and the second from the DVA. This is the adjustment to the swap quote necessary to overcome effect of value impact on shareholders shown by Equation (13).

Because initial margins set by CCPs or in the inter-dealer swap market are standardized, the right hand side of (34) is the dealer's credit spread $S$ multiplied by some positive swap-specific amount that does not depend on the identity of the dealer.

# Appendix D Multi-Period Model

We generalize the basic model of Section III to 2 periods with 3 dates $t = 0, 1, 2$. New information is revealed at the interim date 1 through observation of a collect $Z$ of random variables. All uncertainty is resolved at date 2. We let $E_1^*$ denote expectation under $P^*$ conditional on $Z$. We assume that the one-period gross risk-free returns are $R_0$ and $R_1$ at time 0 and 1, respectively. We don't require $R_1$ to be constant. Thus, the fair market value of cash flows of $\{C_t\}_{t=1}^2$ is defined as $\sum_{t=1}^2 E^*(\delta_t C_t)$, where $\delta_1 = 1/R_0$ and $\delta_2 = 1/(R_0 R_1)$.

We consider a dealer whose pre-existing assets have payoffs at time 2 are given by some random variable $A$. The firm has short-term liabilities $L_1$ that expire at time 1 and long-term liabilities $L_2$ that expire at time 2. We assume that the dealer liquidates a portion of its legacy assets to pay back the maturing liabilities $L_1$ at time 1 and pay out dividend $\pi_1$, which is also a random variable. If the liquidation value of asset is not enough to cover $L_1$, the dealer defaults, which we denote the event as $D_1$. We let $W$ denote the payoff at time 2 of the liquidated assets. As a result, the firm defaults at time 2 in the event $D_2 = \{A - W < L_2\}$. In the dealer's default events $D_1$ and $D_2$, we assume all liabilities are *pari passu* with each other, and the recovery rates of assets are some constant $\kappa_1$ and $\kappa_2$, respectively. We let $\tau_D$ denote the dealer's default time. If the dealer survives at time 2, that is, $\tau_D = \infty$, the firm is liquidiated and the remaining cash flows are attributed to shareholders after paying back creditors. Thus, the total value of the firm's equity is $E^*[\delta_1 \mathbf{1}_{\{\tau_D>1\}}\pi_1] + E^*[\delta_2 \mathbf{1}_{\{\tau_D>2\}}(A - W - L_2)]$. The total value of the dealer's liabilities is

$$E^*[\delta_1 \mathbf{1}_{\{\tau_D>1\}} L_1 + \delta_1 \mathbf{1}_{\{\tau_D=1\}} \kappa_1 E_1^*(A)/R_1] + E^*[\delta_2 \mathbf{1}_{\{\tau_D>2\}} L_2 + \delta_2 \mathbf{1}_{\{\tau_D=2\}} \kappa_2 (A - W)].$$

We assume either *(i)* finite states of the world, or *(ii)* infinitely many states of the world with standard continuity conditions of $(A, W, L_1, L_2)$ as in Section II. As in Section II, the dealer's marginal credit spread at time 0 for short-term (one-period) debt is

$$S_0 = \frac{E^*(\phi_1) R_0}{1 - E^*(\phi_1)},$$

where $\phi_1 = \mathbf{1}_{D_1}(L_1 + E_1^*(L_2)/R_1 - \kappa_1 E_1^*(A))/(L_1 + E_1^*(L_2)/R_1)$. If the dealer survives at 1, the dealer's marginal credit spread at time 1 for one-period debt is

$$S_1 = \frac{E_1^*(\phi_2) R_1}{1 - E_1^*(\phi_2)},$$

where $\phi_2 = \mathbf{1}_{D_2}(L_2 - \kappa_2(A - W))/L_2$.

In this two-period setting, a swap is a contract promising *(i)* floating payment $X_1$ in exchange for fixed payment $K_1$ at time 1, and *(ii)* floating payment $X_2$ in exchange for fixed payment $K_2$ at time 2, before considering the effect of counterparty default. We let $C_1 \equiv X_1 - K_1$ and let $C_2 \equiv X_2 - K_2$. We focus on the payer swap, that is, the positive cash flow of this contract is an asset to the dealer, whereas the negative cash flow is a contingent liability. A swap position of size $q$ requires the dealer to make an upfront payment of $U(q)$. We assume $u = \lim_{q \downarrow 0} U(q)/q$ exists. Results for the reverse case are obvious by analogy.

The supporting calculations for the following results are similar to Appendix B and are omitted for brevity.[27]

[27] The calculations will be provided to readers upon request.

## *A Valuing Unsecured Swaps with Upfront*

In this section, we extend the results in Section III.A. That is, the client swap is assumed to be fully unsecured. For simplicity, we assume that at the interim period, swap counterparties default after the coupon payment.[28] We let $\tau_C$ denote the swap client's default time. At the client's default, the dealer recovers a fraction $\beta_1$ and $\beta_2$ of any remaining contractual amount due to the dealer at time 1 and time 2, respectively. We also suppose that there are no pre-existing positions between the swap client and the dealer. The effect of netting the new swap flows against those of the legacy positions with the same client is analyzed in Appendix F.

We have the following natural extension of the basic one-period swap valuation model in Section III.A.

PROPOSITION 5: *Whether the dealer finances any net payments by issuing debt, issuing equity, or using existing cash on its balance sheet, the marginal market value of the swap is well-defined by*

$$v = E^*\left(\sum_{t=1}^{2}\delta_t C_t - u\right) + E^*\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_D=t,\tau_C>t-1\}}\phi_t V_t^-\right) - E^*\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t,\tau_D>t-1\}}(1-\beta_t)V_t^+\right), \tag{35}$$

*where* $V_1 = E_1^*(C_2)/R_1$ *and* $V_2 = C_2$.

As in the single-period model, the swap value (35) includes two credit-related adjustments for the default free value, $V_0 = E^*(\delta_1 C_1) + E^*(\delta_2 C_2)$, for default. The CVA is

$$E^*\left[\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t,\tau_D>t-1\}}(1-\beta_t)V_t^+\right]$$

and the DVA is $E^*\left[\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_D=t,\tau_C>t-1\}}\phi_t V_t^-\right]$. The market value of the same swap from the viewpoint of the swap client is of course $-v$.

Now, we analyze the marginal value of the new swap to shareholders of the dealer, we assume that the positive financing requirement is financed by issuing short-term (one-period) debt. Likewise, any net positive cash flow to the dealer is used to retire short-term debt.

PROPOSITION 6: *If the dealer issues debt to finance net payments and uses received cash to retire outstanding debt, then the marginal value of the swap to the dealer's shareholders is well defined by*

$$G = E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2}\delta_t C_t - u\right)\right] - E^*\left[\mathbf{1}_{\{\tau_D>2\}}\left(\sum_{t=1}^{2}\delta_t \mathbf{1}_{\{\tau_C=t\}}(1-\beta_t)V_t^+\right)\right] - \Phi(u), \tag{36}$$

*where*

$$\Phi(u) = E^*\left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} u S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} u R_0 S_1\right] - E^*\left[\delta_2 \mathbf{1}_{\{\tau_D>2,\tau_C>1\}} C_1 S_1)\right],$$

[28] This assumption is valid for the purpose of marginal analysis.

*is the debt funding valuation adjustment.*

As in Section III.B, if the swap is executed at the "conventional" upfront,

$$u^* = V_0 - c^* = V_0 - E^* \left( \sum_{t=1}^{2} \mathbf{1}_{\{\tau_C = t\}} \delta_t (1 - \beta_t) V_t^+ \right),$$

then the marginal value of the swap portfolio to the dealer's shareholders is

$$G = \operatorname{cov}\left( \mathbf{1}_{\{\tau_D > 2\}}, \sum_{t=1}^{2} \delta_t C_t - \sum_{t=1}^{2} \mathbf{1}_{\{\tau_C = t\}} \delta_t (1 - \beta_t) V_t^+ \right) - \Phi(u^*). \tag{37}$$

In practice, $c^*$ is often known as *Unilateral Credit Valuation Adjustments* (UCVA),[29] and it is different from the CVA in (35) as it does not take into account the dealer's default. In the case that the dealer's default is independent of the swap cash flows, the shareholder value is

$$G = -\Phi(u^*).$$

In analogy with (13), for a small spread $S$, we see that the dealer's indifference quote is approximately $u^* - \Phi(u^*)$.

### *B Inter-dealer Hedge, Initial Margin, and MVA*

In this subsection, we consider a swap dealer hedges the unsecured swap with a fully collateralized inter-dealer swap that requires the dealer to post both initial margin and variation margin. We assume that the hedge-motivated collateralized swap with another dealer or a central counterparty has a net receivable of $-C_1 = K_1 - X_1$ at time 1 and a net receivable of $-C_2 = K_2 - X_2$ at time 2. The hedging swap requires the dealer to post both cash initial margin of $I_0$ and $I_1$, and variation margin $M_0$ and $M_1$ at time 0 and time 1, respectively. We follow the same variation margin mechanism as in Section C.A,and we assume that $M_0 = V_0 = E^* \left( \sum_{t=1}^{2} \delta_t (X_t - K_t) \right)$ and $M_1 = V_1 = E_1^*(X_2 - K_2)/R_1$, the standardized margin payment that equal to the market value of the hedging swap. We assume this hedging swap is transacted at the fullly collateralized value $V_0$.

We have the following natural extension of the basic one-period swap valuation model with inter-dealer hedge.

PROPOSITION 7: *If the dealer issues debt to finance margin payments and uses received margin to retire outstanding short-term debt obligations, then the marginal value of the swap portfolio to the dealer's shareholders is well defined by*

$$G = E^* \left[ \mathbf{1}_{\{\tau_D > 2\}} (V_0 - u) \right] - E^* \left[ \mathbf{1}_{\{\tau_D > 2\}} \left( \sum_{t=1}^{2} \delta_t \mathbf{1}_{\{\tau_C = t\}} (1 - \beta_t) V_t^+ \right) \right] - \Phi(u) - \Psi,$$

[29] See Albanese and Andersen (2014) for details on UCVA.

*where*

$$\Phi(u) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} u S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} V_1 S_1\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} (u - V_0)\right],$$

*is the funding value adjustment, and*

$$\Psi = E^* \left(\delta_1 \mathbf{1}_{\{\tau_D>1\}} I_0 S_0\right) + E^* \left(\delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} I_1 S_1\right)$$

*is the margin value adjustment.*

In the special case that the unsecured swap is executed at the default-free market value, that is, $u = V_0$, the FVA is

$$\Phi(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} V_0 S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} V_1 S_1\right].$$

*C Imperfect Variation Margin and FVA*

So far, we have assumed that the client swap is fully unsecured. It is also of interest to consider the case that the client swap requires both counterparties to post some variation margin. To be concrete, we assume the client swap requires some "imperfect" variation margin, so that $m_0$ and $m_1$ are the amount of variation margin in the dealer's possenssion at time 0 and time 1, respectively. We assume this client swap is hedged with the same fully collateralized inter-dealer swap in Section D.B.

By direct algebra, the FVA in this case is

$$\begin{aligned}\Phi(u) =& E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} (V_0 - m_0) S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} (V_1 - m_1) S_1\right] \\ &+ E^* \left[\delta_1 \mathbf{1}_{\tau_D>1} (u - V_0)\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} (u - V_0)\right].\end{aligned}$$

In the case that the client swap is execuated at the default-free market value $V_0$, then the FVA is

$$\Phi(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D>1\}} (V_0 - m_0) S_0 + \delta_2 \mathbf{1}_{\{\tau_D>2, \tau_C>1\}} (V_1 - m_1) S_1\right].$$

If the "imperfect" margin becomes "perfect", that is, if $m_0 = V_0$ and $m_1 = V_1$, then the FVA $\Phi(V_0) = 0$.

*D Cash Management Strategy and Asymmetric FVA*

Our denition of FVA is symmetric, in the sense that cash inflows and outflows are assumed to be financed or to reduce financings, respectively, at a spread of S. For the case of cash inflow, this implicitly assumes that there is always some short-term unsecured debt to roll over whose total amount can be reduced by swap cash inflows.

Now, we consider the case that the cash outflows are financed with unsecured debt and cash inflows are invested at the risk-free rate. All else are equal as in Section D.B. Correspondingly, we can calcuate the "asymmetric funding value adjustment" (AFVA) as

$$\tilde{\Phi}(u) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} u^+ S_0\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} (V_1 + u - V_0)^+ S_1\right].$$

If the unsecured swap is executed at $u = V_0$, then the AFVA is

$$\tilde{\Phi}(V_0) = E^* \left[\delta_1 \mathbf{1}_{\{\tau_D > 1\}} V_0^+ S_0\right] + E^* \left[\delta_2 \mathbf{1}_{\{\tau_D > 2, \tau_C > 1\}} V_1^+ S_1\right].$$

# Appendix E The Continuous-Time Reduced-Form Model

This appendix provides additional details underlying the continuous-time reduced-form model of Section IV.

## A Technical Assumptions

We fix our probability space, $(\Omega, \mathcal{F}, P^*)$ and a filtration $\{\mathcal{F}_t : t \geq 0\}$ of sub-$\sigma$-algebras of $\mathcal{F}$ satisfying the usual conditions, as defined by Protter (2005). We take the short-rate process $r = \{r_t : t \geq 0\}$ to be progressively measurable and adapted, and such that $\int_0^t |r_s|\, ds$ is finite almost surely for all $t$. As usual, we let $E_t^*$ denote conditional expectation with respect to $\mathcal{F}_t$.

All probabilistic statements to follow are with respect to our valuation probability measure $P^*$. This means, by definition, that the market value at time $t$ of a fully collateralized claim to some payment $C$ at some bounded stopping time $T \geq t$ is by definition $E_t^*(\delta_{t,T}\, C)$, where $\delta_{t,u} = e^{-\int_t^u r(s)\, ds}$ for any times $t$ and $u \geq t$. Here, $C$ is measurable with respect to $\mathcal{F}_T$ and such that $e^{-\int_0^T r_s\, ds} C$ has a finite expectation with respect to $P^*$.

Before considering the effect of incremental cash flows associated with a new position, the derivatives dealer defaults at a stopping time $\tau_D$ with intensity process[30] $\lambda_D$. An unsecured claim of size $C$ on the dealer's estate at default is paid $(1 - \ell_D(\tau_D))C$, for some proportional loss process[31] $\ell_D$ taking outcomes in $[0, 1]$. This implies that the dealer's short-term credit spread at time $t$ is $S_t = \lambda_D(t)\ell_D(t)$. That is,[32] each unit of the dealer's short-term unsecured debt can be continually renewed, or "rolled over," until any fixed time $U$, or until default, whichever comes earlier, by making continual floating-rate interest payments at the adjusting rate $r_t + S_t$, and by making a final payment of 1 at time $U$ in the event that default occurs after time $U$. In the event that the default time $\tau_D$ is before $U$, each unit of this debt recovers $1 - \ell_D(\tau_D)$ at default.

[30] The default time $\tau_D$ of the dealer is doubly stochastic driven by a sub-filtration $\{\mathcal{G}_t : t \geq 0\}$ of $\{\mathcal{F}_t : t \geq 0\}$ to which the short-rate process and all payment processes that we consider are adapted. See Duffie (2001), Chapter 11, for details.

[31] We assume that $\ell_D$ is a predictable process. One can generalize so as to get essentially the same result, under mild regularity, by replacing $\ell_D$ with the dual predictable projection of a loss-given-default random variable.

[32] This follows from the fact that a martingale $\mathcal{M}$ is defined by

$$\mathcal{M}_t = E_t^* \left( \int_0^U \delta_{t,u}(r_u + S_u) 1_{\{\tau_D > u\}}\, du + 1_{\tau_D > U} \delta_{0,U} \; + \; 1_{\tau_D \leq U}\, \delta_{t,\tau_D} \ell_D(\tau_D) \right).$$

The same result applies if $U$ is any given bounded stopping time relative to the driving sub-filtration $\{\mathcal{G}_t : t \geq 0\}$.

Similarly, a given client swap counterparty has default risk characterized by a default time $\tau_C$ with intensity process[33] $\lambda_C$, and by a proportional loss-given-default process $\ell_C$.

The CVA and DVA definitions and calculations shown in Section IV.A, from Duffie and Huang (1996), differ from the so-called "unilateral" CVA and DVA, which are given, respectively, by

$$\Pi_c' = E^* \left( 1_{\{T > \tau_C\}} \delta_{0,\tau_C} \ell_C V_t^+ \right)$$

and

$$\Pi_d' = E^* \left( 1_{\{T > \tau_D\}} \delta_{0,\tau_D} \ell_D V_t^- \right).$$

See Albanese and Andersen (2014) for details. The unilateral definitions abstract from the fact that the dealer's default is irrelevant if the customer has already defaulted, and vice versa.

## *B Computational Analysis*

We provide the computational analysis underlying the numerical examples in Section IV.B of XVAs for an unsecured semi-annual plain-vanilla interest rate swap. We assume the swap has a maturity of 10 years and that the coupon payment dates are $\{t_i\}_{i=0}^N$, where $t_i = i\Delta$ with $\Delta = 0.5$. At time $t_i$, a payer swap to the dealer has a contractual payment of $C_i = \Delta(X_{i-1} - K)$, where $X_{i-1}$ is the LIBOR rate fixed at time $t_{i-1}$ and $K$ is the fixed coupon rate. The first LIBOR fixing is assumed to take place at $t_0 = 0$, and the last coupon time is $t_N = 10$.

We use the overnight index swap (OIS) rate as a benchmark for the instantaneous risk-free rate $r_t$, corresponding to a risk-free discount of

$$p(t,u) = E_t^*(\delta_{t,u}) = E_t^* \left( e^{-\int_t^u r_s \, ds} \right),$$

where $E_t^*$ denotes conditional expectation at time $t$ under $P^*$. As a result, the default-free market value of the payer swap is

$$V_t = 1_{t < t_{\eta(t)}} \Delta \left( X_{\eta(t)-1} \right) - K \right) p \left( t, t_{\eta(t)} \right) + E_t^* \left( \sum_{i=\eta(t)}^{N-1} e^{-\int_t^{t_{i+1}} r_u \, du} \Delta(X_i - K) \right).$$

## *C Term Structure Model*

We use a one-factor Hull-White term structure model for the short rate $r_t$, as given in Section IV.B, implying that $r_t$ is normally distributed with conditional distribution given $r_s$ of $\mathcal{N}\left(m(s,t), v(s,t)\right)$, where, with $f_t = -d\log(p(0,t))/dt$,

$$m(s,t) = f_t + e^{-\alpha(t-s)}(r_s - f_s) + e^{-\alpha t} \frac{\sigma^2}{2\alpha^2} \left( e^{\alpha t} - e^{\alpha s} + e^{-\alpha t} - e^{-\alpha s} \right),$$

[33]The counterparty default time $\tau_C$ is jointly doubly stochastic with $\tau_D$, and driven by the same sub-filtration $\{\mathcal{G}_t : t \geq 0\}$.

and $v(s,t) = \sigma^2/(2\alpha)\left(1 - e^{-2\alpha(t-s)}\right)$. The associated discount factor at time $t$ for cash flows at $T > t$ is

$$p(t,T) = \frac{p(0,T)}{p(0,t)} e^{-\frac{1}{2}G(t,T)^2\theta_t - (r_t - f_t)G(t,T)}, \tag{38}$$

where $\theta_t$ was defined in (23) and $G(t,T) = (1 - e^{-\alpha(T-t)})/\alpha$.

For simplicity, we assume that the spread $\epsilon$ between the LIBOR rate and the OIS rate is constant over time. Thus, the LIBOR rate is

$$X_i = \Delta^{-1}\left(p(t_i, t_{i+1})^{-1}(1 + \epsilon\Delta) - 1\right).$$

For notational simplicity, we define an annuity factor by

$$a(t;j) = \sum_{i=j}^{N-1} p(t, t_{i+1})\Delta,$$

an OIS forward yield by

$$y(t;j) = \frac{p(t,t_j) - p(t,T_N)}{a(t;j)},$$

as well as a LIBOR forward yield by $y_L(t;j) = \epsilon + (1 + \epsilon\Delta)y(t;j)$. By direct algebra, the default-risk-free version of the swap has market value

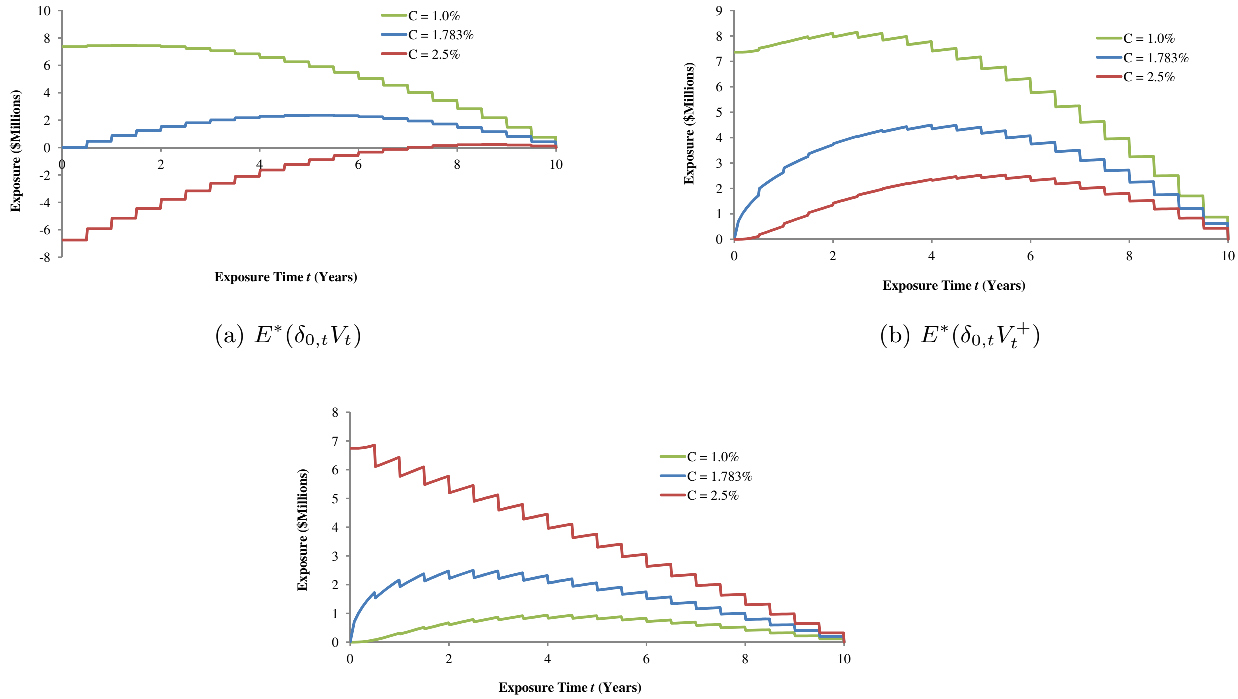
$$V_t = 1_{t < t_{\eta(t)}}\left[\Delta\left(X_{\eta(t)-1}\right) - K\right) p(t, t_{\eta(t)})\right] + a(t,\eta(t))\left(y_L(t,\eta(t)) - K\right). \tag{39}$$

## *D CVA, DVA, FVA, and MVA Calculations*

For the numerical examples in Section IV.B, we assume that the swap client has a constant default intensity of $\lambda_C = 4\%$. We also assume that the dealer has a constant default intensity of $\lambda_D = 2\%$. We assume that the proportional loss process $\ell_C$ and $\ell_D$ are also constant, and $\ell_C = \ell_D = 50\%$. This implies a credit spread $S_D = 1\%$ for the dealer. We further assume that dealer default and client default are independent of each other and of the state of interest rates. Thus, the CVA, DVA and FVA are, respectively,

$$\Pi_c = \ell_C \lambda_C \int_0^{T=10} E^*(\delta_{0,t} V_t^+) e^{-(\lambda_C + \lambda_D)t}\, dt,$$
$$\Pi_d = \ell_D \lambda_D \int_0^{T=10} E^*(\delta_{0,t} V_t^-) e^{-(\lambda_C + \lambda_D)t}\, dt,$$
$$\Phi = S_D \int_0^{T=10} E^*(\delta_t V_t) e^{-(\lambda_C + \lambda_D)t}\, dt.$$

As $V_t$ is driven by a single-factor Gaussian model, the expected values in these integrals are easy to compute from equations (38) and (39); they are shown in Figure 1 below for the payer swaps in our numerical example, using three different fixed coupon levels.




(a) $E^*(\delta_{0,t}V_t)$

(b) $E^*(\delta_{0,t}V_t^+)$

(c) $E^*(\delta_{0,t}V_t^-)$

Figure 1: Exposure profiles for 10-year payer swap.

We write

$$F_t \equiv E_t^* \left( \sum_{i=\eta(t)}^{N-1} e^{-\int_t^{t_{i+1}} r_u \, du} \Delta (X_i - K) \right),$$

and $\mathcal{D}_{t,t+l} \equiv F_{t+l} - E_t^*(F_{t+l})$, where $l$ is assumed to be two weeks. When calculating the MVA, we assume that the margin $I_t$ is set as the 99th percentile of $\mathcal{D}_{t,t+l}$. As $\mathcal{D}_{t,t+l}$ is here very closely approximated by Gaussian random variable, the computation of $I_t$ is straightforward. The resulting MVA is

$$\Psi = S_D \int_0^{T=10} E^*(\delta_t I_t) e^{-(\lambda_C + \lambda_D)t} \, dt.$$

In Figure 2 we show $E^*(\delta_t I_t)$ for our numerical example. Notice how the initial margin decreases over time as the duration of the swap shrinks as it approaches the final maturity.

# Appendix F The Effect of Netting with Legacy Positions

In this section, we extend the results in Section III.A to the case in which the dealer has a pre-existing swap position with the swap client.

The dealer purchases a new unsecured swap from a client, which is identical to that in Section III.A. This same client already has a legacy swap position with the dealer, whose contractually

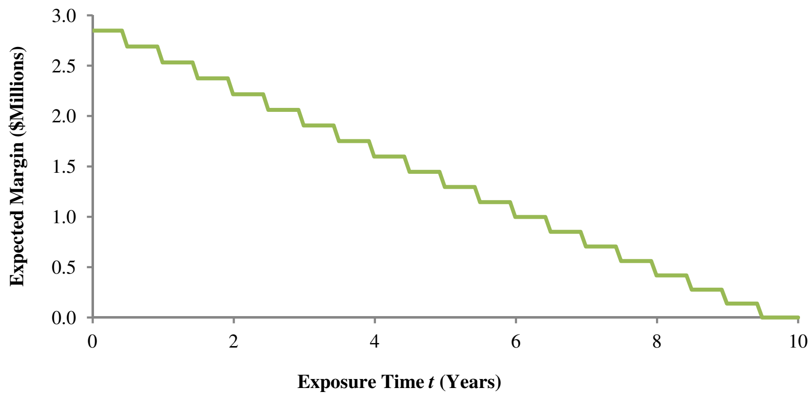



Figure 2: Margin exposure profile $E^*(\delta_{0,t} I_t)$ for 10-year payer swap.

promised payment is $c_0$ and requires the dealer to make an upfront payment of $u_0$. As has been our convention, the positive cash flow of this contract is an asset to the dealer, whereas the negative cash flow is a contingent liability.

As in the main context, we characterize the marginal value of the new swap investment for the dealer's legacy shareholders and legacy creditors (excluding the swap counterparty). We also characterize the marginal market value of the new swap investment. As we have noted, this first-order valuation approach is sufficiently accurate to analyze the investment, except for the cases in which the size of the the investment is large relative to the dealer's entire balance sheet. To this end, we compute the first-order valuation effects of the aggregate positions and the legacy swap with the client. The difference between the two is the first-order valuation of the new swap investment.

## *A Market Value*

As explained by (Mengle (2010)), in the event of counterparty default, the ISDA agreement requires one nets every position held with each counterparty before establishing the default claim. We let $B$ denote the client's default event, which is assumed to be independent of the swap trades for simplicity. By direct analogy with calculations in Appendix B, the marginal market value of the new swap is well defined by

$$V = -u + \delta\left(E^*(X-K) + E^*\left[\phi\left((X-K+c_0)^- - c_0^-\right)\right] - E^*\left[\gamma\left((X-K+c_0)^+ - c_0^+\right)\right]\right), \quad (40)$$

and $V$ is invariant to whether the dealer finances the swap by issuing debt, issuing equity, or using existing cash on its balance sheet. That is, $\delta E^*[\gamma((X-K+c_0)^+ - c_0^+)]$ and $\delta E^*[\phi((X-K+c_0)^- - c_0^-)]$ are the incremental CVA and DVA due to the new swap position.

## *B Shareholder Value*

From now on, we focus on the case that the dealer finances swap positions by issuing new debt. From Proposition 1, the first-order valuation effect to shareholders of the swap portfolio is

$$G_a = \delta E^*[\mathbf{1}_{D^c}(X - K + c_0)] - \delta E^*[\mathbf{1}_{D^c}(u_0 + u)(R + S)] - \delta E^*[\mathbf{1}_{D^c}\gamma(X - K + c_0)^+].$$

Similarly, the first-order valuation effect of the legacy swap to shareholders is

$$G_0 = \delta E^*(\mathbf{1}_{D^c}c_0) - \delta E^*[\mathbf{1}_{D^c}u_0(R + S)] - \delta E^*(\mathbf{1}_{D^c}\gamma c_0^+).$$

Thus, the marginal value of the new swap to the shareholders is

$$G = G_a - G_0 = \delta E^*[\mathbf{1}_{D^c}(X - K)] - \delta E^*[\mathbf{1}_{D^c}u(R + S)] - \delta E^*[\mathbf{1}_{D^c}\gamma((X - K + c_0)^+ - c_0^+)].$$

## *C Legacy Creditor Value*

We also consider the marginal value of the new swap to the dealer's existing creditors (excluding the swap client). To this end, we characterize the first-order effect of the legacy swap, and we characterize the first-order effect of the swap portfolio. Thus, the marginal value of the new swap to the dealer's legacy creditors is

$$\begin{aligned} H = {} & \delta E^*[\mathbf{1}_D(X - K)] - \delta E^*(\mathbf{1}_D uR) + \delta E^*(\mathbf{1}_{D^c}uS) + \delta E^*[\phi((X - K + c_0)^- - c_0^-)] \\ & - \delta E^*[\gamma \mathbf{1}_D((X - K + c_0)^+ - c_0^+)] - \delta(1 - \kappa)J, \end{aligned}$$

where

$$J = \lim_{q \to 0} E^* \left( \frac{\mathcal{A}(q)\mathbf{1}_{\mathcal{D}(q)} - \mathcal{A}_0(q)\mathbf{1}_{\mathcal{D}_0(q)}}{q} \right),$$

and $J$ is well defined by the same argument in Appendix B.

In the special case of no distress costs ($\kappa = 1$), we have $V = G + H$.