Financial Market Innovation and Security Design: 
An Introduction*

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The article describes, in the context of theory and practice, this special issue of the Journal of Economic Theory on financial market innovation and security design. The main focus is on security design in incomplete financial markets, possibly with asymmetrically informed traders. We first survey the general equilibrium literature, which emphasizes the spanning role of securities. We then provide a unified framework encompassing the literature that employs a CARA-Gaussian setting to study the impact of financial innovation on risk-sharing and information aggregation. Journal of Economic Literature Classification Numbers G10, D52. © 1995 Academic Press, Inc.

1. INTRODUCTION

This article describes, in the context of theory and practice, this special issue of the Journal of Economic Theory on financial market innovation and security design.

* We are grateful to many, including Franklin Allen and Douglas Gale for pre-publication access to their book-length treatment of the topic; to the other authors of this issue for many helpful conversations and for their patience and willingness; to Peter DeMarzo for extensive research discussions; to Susan Schulze for editorial and administrative assistance of the first caliber; and to Karl Shell for his impetus and guidance as editor. We also thank David Cass, Phil Dybvig, David Nachman, and Kazuhiko Ohashi for useful conversations. Rahi was at Universitat Pompeu Fabra in Barcelona during the time this paper was written. He acknowledges financial support from the Spanish Ministry of Education (DGIICYT Grant PB91-0811).
Financial securities are designed to suit many motives. Entrepreneurs and firms hope to raise capital efficiently. The managers of a firm use the securities they issue on behalf of their firm to signal the firm’s potential value and opportunities, or their own abilities and efforts. Entrepreneurs may issue securities designed to maintain some of the benefits of control of their firms. Market intermediaries hope to profit from offering transactions services in previously unavailable contingent claims. Regulators consider the role of financial innovation in promoting an efficient allocation of risk and capital.

The theoretical literature covering these issues is relatively young, but growing quickly. Harris and Raviv [61, 62] have carefully surveyed the part of the theory motivated by the value of control of a corporation, or by agency costs in its management. These motives, while important, are not emphasized in this special issue. New securities are often designed in response to accounting standards, regulations, and tax codes. In our minds, these are not mundane motives. For instance, there is an exciting place still open for theories of security design focusing on the important role of accounting information (and, therefore, accounting standards) in the market for a firm’s securities or managers. The articles in this issue concentrate, however, on parts of the theory dealing with the “spanning” role of securities, and in some cases the interaction between spanning and asymmetric information. Recently, Allen and Gale [6] collected their own research on financial innovation and spanning in a book that includes an extensive survey of the topic.

At this early stage, while there are several results providing conditions for the existence of equilibrium with innovation, the available theory has relatively few concrete normative or predictive results. From a spanning point of view, we can guess that there are incentives to set up markets for securities for which there are no close substitutes, and which may be used to hedge substantive risks (Allen and Gale [3–6], Chen [32], Cuny [35], Demange and Laroque [36], Duffie and Jackson [41], Rahi [92]). Given the potential for adverse selection, we would expect issuers of securities to consider the impact of private information on the design of their securities (Amihud and Mendelson [8], Boot and Thakor [24], Brennan and Kraus [25], Constantinides and Grundy [34], Demange and Laroque [37], DeMarzo and Duffie [38], GlAESer and Kallal [53], Gorton and Pennacchi [55, 56], Nachman and Noe [82, 83], Ohashi [87], and Rahi [91]). Indeed, we might expect markets to collapse if the issuer’s information is sufficiently large relative to that of potential investors (Bhattacharya et al. [19],

1 Allen and Winton [7] summarize some additional literature in this area. Recent examples include Bagnoli and Snowden [14], Dionne and Viala [39], Fluck [47], Mello and Parsons [75], Nagarajan [84], and Sanig [96].
Rahi [91]). These are a few of the themes that emerge in this symposium issue.

Section 2 of this introductory article places in perspective the abstract problem of security design and innovation, emphasizing the articles in this issue that are cast in a general equilibrium setting (Elul [43], Chen [32], Pesendorfer [89]). Section 3 summarizes some of the key insights in a standard "linear" framework that, because of its simplicity, is frequently adopted for the analysis of security market innovation, both within this issue (Bhattacharya et al. [19], Demange and Laroque [36, 37], Hara [60], Ohashi [87], Rahi [92]) and elsewhere (Cuny [35], Duffie and Jackson [41], Hara [59], Ohashi [85, 86, 88], Rahi [91]). Section 4 attempts to catalog some of the other strands of the literature.

We next describe two basic institutional settings for financial innovation. The descriptions are not designed to maintain a particularly close parallel with paradigms already developed in the literature.

1.1. Innovation by Exchanges

Consider, for the sake of illustration, the management of a futures or options exchange, which has a responsibility to provide its members with profitable opportunities to act as brokers. The degree of profitability may be evident in the market value of a "seat," that is, a membership. The bulk of the profit is from trading commissions and from the ability to act as buyer to outside sellers and as seller to outside buyers, presumably with some average positive spread. The exchange cannot bear the costs of setting up markets in all possible contracts. We can therefore guess that exchange members hope for the introduction of those futures or options contracts that generate significant trading activity.

Suppose one fixes the scale\(^2\) of some new futures contract to be chosen, say in terms of the standard deviation of the payoff of the contract, so that volume of trade has at least some meaning as a measure of trading activity. For a given exchange member, the equilibrium\(^3\) average brokerage spread per contract and fraction of total trading volume handled may depend on the relative and absolute trading skills of the members, and on the institutional features of the exchange. If we suppose that they do not depend on the particular identity of the contract, then the members of the exchange unanimously support trading in that contract with the largest volume of trade. This homogeneity assumption is a major proviso, but

\(^2\) Integer constraints in trading units and some model of transactions costs presumably play a role in scaling. For relevant discussions, see Duffie and Jackson [41] and Allen and Gale [6].

\(^3\) For a Bertrand model of competition among futures brokers, see Saloner [94].
there is not much guidance on how the design of a particular contract would indeed influence the spreads and determine the fractions of trading volumes handled by individual brokers. In any case, volume of trade is predominant among the criteria by which success is judged by the exchanges themselves, as one can see, for example, from the regular feature stories in the newsletters of most futures exchanges. For instance, the lead story in the Summer 1994 issue (Vol. 14, Number 2) of Open outcry, published by the Chicago Mercantile Exchange, is “Seats Hit Record Prices; Volume, Open Interest Surge.” The story relates that “the surge [of seat prices] in all membership divisions this year reflects explosive volume and open interest growth.”

With multiple contracts, the exchange must be concerned with the spillover effects of innovation on trading activity in other contracts handled by the exchange, which could be positive or negative. With multiple innovating exchanges in competition, the criteria for innovation may be significantly more complicated, as exchange-wide liquidity offers a strategic advantage.

Exchange members would not favor introducing trade in contracts delivering something as obscure as, say, amethysts, if this meant giving up trade in, say, oil, German marks, or U.S. Treasury bonds. Neither would exchange members favor trade in something whose price is relatively stable, such as salt. Amethysts and salt are unlikely to present significant price risk, to consumers or businesses. One theme of the literature, going back at least to Working [106] and evident in the Milgrom–Stokey [78] no-trade theorem, is that an exchange would rarely find it attractive to introduce a security whose sole justification is the opportunity for speculation. Speculators depend for trading opportunities on the existence of hedges, or, as in the financial microstructure literature, on “liquidity traders.”

In practice, given a candidate for a new futures contract, the research department of a futures exchange typically prepares an analysis of the projected hedging demand. Potential users of the contract are interviewed, and statistical estimates are obtained of the correlation between the contract’s designed payoff and the risks associated with changes in the value of the potential users’ market commitments. Black [22] has shown an empirical link between the volume of trade of a futures contract and the ability of the contract to act as a hedge for significant, and otherwise uninsurable, economic risks. This is theoretically supported by Cuny [35], Duffie and Jackson [41], and Rahi [92], who considers as well the price-discovery role of innovation. In reality, however, there remains much guesswork. Many contracts are introduced only to fail immediately. Others have sustained success but ultimately disappear with a change in the economic circumstances that fostered their initial popularity.

See, for example, Admati and Pfleiderer [1].
<table>
<thead>
<tr>
<th>Year</th>
<th>Economic Events (') and Financial Innovations (°)</th>
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<tbody>
<tr>
<td>1971</td>
<td>° United States suspends gold convertibility</td>
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<td>1972</td>
<td>° Inflation rate at 3.3% for year&lt;br&gt;° First money market mutual funds&lt;br&gt;° Foreign currency futures</td>
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<tr>
<td>1973</td>
<td>° Floating exchange rates mark suspension of gold standard&lt;br&gt;° Oil prices quadruple to $12 (a barrel)&lt;br&gt;° Chicago Board Options Exchange established&lt;br&gt;° Black-Scholes options model published in JPE</td>
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<tr>
<td>1974</td>
<td>° Dow hits low of 570&lt;br&gt;° Commodity Futures Trading Commission created&lt;br&gt;° Inflation rate at 11% for year&lt;br&gt;° Franklin National Bank failure</td>
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<td>1975</td>
<td>° Fixed commission rates eliminated&lt;br&gt;° Japanese yen at 292 to the dollar&lt;br&gt;° Ginnie Mae futures</td>
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<td>1976</td>
<td>° Gold drops to $101 an ounce&lt;br&gt;° 90-day Treasury Bill futures</td>
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<td>1977</td>
<td>° Foreign broker-dealers permitted to obtain NYSE membership&lt;br&gt;° Long-term Treasury Bond futures&lt;br&gt;° Merrill Lynch introduces Cash Management Account</td>
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<tr>
<td>1979</td>
<td>° Inflation rate reaches 11.3% for year&lt;br&gt;° Second oil shock strikes United States during Iranian crisis&lt;br&gt;° Federal Reserve tightens money supply</td>
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<tr>
<td>1980</td>
<td>° Price of gold peaks at $875 an ounce&lt;br&gt;° Federal Reserve discount rate rises to 13%&lt;br&gt;° Inflation rate at 13.5% for year&lt;br&gt;° Home purchase revenue bonds</td>
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<tr>
<td>1981</td>
<td>° Interest rates peak at 21.5%&lt;br&gt;° Price of oil peaks at $39 a barrel&lt;br&gt;° Foreign currency swaps&lt;br&gt;° Bonds with detachable warrants offered&lt;br&gt;° First offering of an original-issue discount convertible&lt;br&gt;° First debt-for-equity swap&lt;br&gt;° Portfolio insurance invented&lt;br&gt;° Futures in Eurodollars&lt;br&gt;° Futures on bank CDs</td>
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<tr>
<td>1982</td>
<td>° Latin American debt crisis—Mexico undergoes peso devaluation&lt;br&gt;° Shelf registration starts&lt;br&gt;° Unemployment rate at 9.7% for year&lt;br&gt;° First 100-million-shares day on NYSE</td>
</tr>
</tbody>
</table>

TABLE 1—Continued

- Stock index futures tied to Value line, S & P's 500, and NYSE
- Options on Treasury Bond futures
- Options on common stock index
- Retail CDs zero coupon
- Tigers
- Second mortgage pass-through securities
- Zero coupon Eurobond issue
- Extendable notes with rates adjusted at holder's puttable option
- Federal Home Loan Mortgage Corporation offers zero coupon bond
- Treasury note futures

<table>
<thead>
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<th>1983</th>
<th>1984</th>
<th>1985</th>
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<td>Wings</td>
<td>Run on Continental Illinois bank, nation's eighth largest</td>
<td>Colts</td>
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<td>Dates</td>
<td>Dutch auction rate preferred stock</td>
<td>Stripped floating rate notes with a cap</td>
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<tr>
<td>Cats</td>
<td>Fannie Mae zero coupon</td>
<td>CARS</td>
</tr>
<tr>
<td>CMOs</td>
<td>Fannie Mae 35 zero coupon subordinated cap debenture</td>
<td>Zero coupon sterling issue</td>
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<tr>
<td>Libor-based floating rate notes</td>
<td>Synthetic bonds</td>
<td>New hybrid bond-dual series discount bonds</td>
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<td>Swap equity of American company for foreign debt</td>
<td>Eurobond discount mortgage-backed bonds</td>
<td>Flexible Credit Account</td>
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<tr>
<td>Stilts</td>
<td>Zero coupons by mortgages</td>
<td>Floating rate securities—capped, Mini/Max, mismatched, partly paid</td>
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<td>S &amp; P's 100 index futures</td>
<td>STAR</td>
<td>Non-dollar FRNS</td>
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<td>Shoguns</td>
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<td>Yen-denominated Yankees</td>
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<td>ECU-denominated securities</td>
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<td>Dual currency yen bonds</td>
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<td>Down Under bonds</td>
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<td>Variable duration notes</td>
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<td>Collateralized securities—multifamily pass-through, leaseback</td>
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<td>Commercial mortgage pass-throughs</td>
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<td>Cross-collateralized pooled financing</td>
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<td>Pooled nonrecourse commercial mortgage</td>
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<td></td>
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<td>Daily adjustable tax-exempt securities</td>
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<td></td>
<td></td>
<td>Municipal put option securities</td>
</tr>
</tbody>
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TABLE I—Continued

- Periodically adjustable rate trust securities
- UPDATES
- Options on Eurodollar futures
- Options on Treasury note futures
- Japanese government yen bond futures
- ECU warrants
- European-style options
- Range forward contract
- US dollar index
- Options on cash five-year Treasury notes

1986
- Dow hits all-time high of 1910
- Federal funds rate at 6.8%
- Budget deficit surges to $230.2 billion
- Unemployment rate for year at 7%
- Inflation for year at 1.9%
- West German deutsche mark drops to 2.07 to the dollar
- Japanese yen drops to 154 to the dollar
- Price of oil dips to $10 a barrel
- US broker-dealers join London and Tokyo exchanges
- SYDS
- Marketed preferreds
- Euro MTNs
- Real-estate master limited partnership
- Extendable bonds—step up or put coupon bonds
- Universal Commercial Paper
- Oil-indexed bonds
- Municipal Receipts

Table I, reproduced here from Matthews [73], shows that many types of securities, both exchange-traded and otherwise, emerge with changes in the institutional features of the economy. Finnerty [46] provides a more extensive table, giving his own interpretation of the incentives for innovation for each new security. These innovation-inducing changes are not always characterized by some new or significantly greater economic risk to be hedged. Instead, in a large fraction of cases, the motivating event is new regulation, a change in fiscal or monetary policies of governments, or adjustments in accounting standards or tax codes.

There is an effective first-mover advantage for innovators, both because of the pre-emptive value of liquidity⁵ and also because regulatory approval

⁵ Once liquidity (measured in terms of trading activity) is established in a given contract, it may be difficult to obtain a foothold for a competing contract with potentially superior hedging characteristics. See Working [106], Cuny [35] and, for a related first-mover model, Anderson and Harris [9].
may be withheld for contracts that have close substitutes already in existence. (Perhaps regulators view liquidity as a public good.) Indeed, it is not unusual in the United States for a futures exchange to obtain regulatory approval for a contract without actually introducing active trade, perhaps viewing the potential to immediately begin trade as a barrier to entry by other exchanges.

Sandor [95] describes the case of innovation of plywood futures. Johnston and McConnell [67], Manaster and Tashjian [72], and Tashjian et al. [99] review the significance of delivery options in futures contract design, a feature of the Ginnie Mae (mortgage pass-through) futures contract that was ultimately responsible for its demise. Townsend [102] provides a general conceptual framework for the organizational design and regulation of futures markets, including the impetus for contract design.

1.2. Investment Banking and New Financial Products

By a recent count, there are now over 1200 different types of derivative securities in use, most of which are traded in the over-the-counter (OTC) market, rather than on an exchange. Most OTC derivatives, and many other new forms of financial securities, some of which are shown in Table I, are introduced by securities firms.

The innovator, often an investment bank, usually acts as an intermediary. In many cases, however, what is sold by the intermediary to one customer is bought by the intermediary only in a synthetic form. The advent of derivative hedging methods based on Merton's [76, 77] replication approach to the Black-Scholes [23] option pricing model has permitted intermediaries to make markets by carrying large positions that net to a small market exposure. In so doing, the intermediary may remain exposed to significant credit risk.

In addition to innovation via the intermediation of new derivative securities, a securities firm also innovates through its underwriting business, acting as a design and pricing consultant, as well as marketing agent, to firms that will issue a new financial product, usually as a vehicle for raising capital. In marketing the new product for the issuer, the securities firm will frequently buy the product itself, or equivalently guarantee the price to the issuer, and profit to the extent that it can sell the product for a higher price to investors. Taxes and accounting standards are often a motivating factor in the design.

A major example of innovation by securities firms is the creation of asset-backed securities such as collateralized mortgage obligations

4 Elizabeth Tashjian's name was Elizabeth Johnston.
(CMOs). An asset-backed security is one of a family of securities whose cash flows are collectively backed by some asset, not necessarily securitized itself. For a CMO, the backing asset is a pool of mortgages. (For a review of the basic features of CMOs, see Bartlett [17] and Carron [26].) Other examples of backing assets are portfolios of credit-card receivables or auto-loan payments. The issuer of an asset-backed security is often motivated to securitize assets because of regulatory restrictions on the size of the issuer's balance sheet relative to its capital. For example, risk-based capital requirements make it advantageous for a bank to securitize some of its credit-card portfolio in order to liberate capital for purposes of additional intermediation. For further discussion of the institutional background of the securitization of assets, see, for example, Schwarz [97]. Allen and Gale [3, 5, 6], Chen [32], DeMarzo and Duffie [38], and Pesendorfer [89] may be thought of as models of the asset securitization process.

An empirical study by Tufano [103] distinguishes among possible hypotheses regarding the advantages of financial innovation for underwriting by a securities firm:

1. The innovator hopes to establish an effective monopoly in the underwriting of a class of financial products, allowing it to sell these products at a higher premium over cost than otherwise possible.

2. By initiating the product design, the innovator hopes to capture more of the market for underwriting new issues of that type of financial product than it otherwise would. Its underwriting spreads are not wider than those of copy-cat investment banks.

Tufano finds that the data favor the second hypothesis over the first. Patent or copyright protection is difficult to obtain for financial products. The first-mover advantage for an investment bank is the expertise, and reputation for expertise among potential issuers, that can be obtained through innovation. This expertise includes the ability to exploit the properties of the financial product to the benefit of the issuer (for example, obtaining the most efficient tax shield if the product is designed for tax avoidance), the ability to price the product in the market accurately, and knowledge of the market of potential investors in the product. Not only will the innovator gain reputation, it may also have lower costs as a result of its expertise, according to Tufano. First-mover advantages in financial product innovation abate with time, but apparently survive long enough to create an incentive for the larger investment banks to develop new products.

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8 See Allen and Gale [6, Chap. 3] for some of the differences in incentives for financial innovation and for general product innovation, as the latter is treated in mainstream industrial-organization models such as Farrell and Saloner [45].
McConnell and Schwartz [74] give a case history of the innovation by Merrill Lynch, in 1985, of the LYON, or "Liquid Yield Option Note." The LYON was described by The Wall Street Journal as "one of Wall Street's hottest and most lucrative corporate finance products." The story of the LYON, a zero-coupon corporate bond that is callable, puttable, and convertible, provides an example of how subtle and complex the process of innovation may become. McConnell and Schwartz describe how Lee Cole, an options manager at Merrill Lynch, noted a pattern of retail investment in call options on common stocks that, on its own, appeared unusually risky.

"In reviewing customers' consolidated accounts, however, Cole observed that many options customers also maintained large balances in the CMA accounts [Cash Management Accounts] while making few direct equity investments. From these observations, Cole deduced a portfolio strategy: Individuals... were willing to risk a fraction of their funds in highly volatile options as long as the bulk of their funds were largely safe from risk in their CMA accounts. ... He leaped to the further inference that funds used to buy options came largely from the interest earned on CMA accounts. ... Cole drafted a memorandum describing in general terms a corporate security that would appeal to this segment of the retail customer market. ... Cole's intent was to design a security that would allow corporations to tap a sector of the retail market whose funds were currently invested in government securities and options. ... The LYON... approximates the features of the trading strategy as perceived by Cole. [74, p. 42].... By offering what amounts to a continuous option position, such a convertible [the LYON] would have the added appeal to investors of potentially large transactions cost savings"[74, p. 44].

The story here is not entirely at variance with the motivation of the models of Allen and Gale [3, 5, 6], Chen [32], Madan and Souabra [69], and Pesendorfer [89], who rely on market frictions (in their case, short sales constraints) as a source of profit to corporate issuers. The success of the LYON also depended in part on the use of valuation methods of the sort implicit in Merton's proof [77] of the Black-Scholes [23] option pricing formula. Without the ability to convince an issuer (or investor) that it would receive (or pay) a "fair" value for such a complex security, the LYON would have had difficulty reaching the market. Gale [50] offers a model of security standardization that is relevant to this issue of aversion to complexity in security design, because, for example, of the costs associated with the analysis of complex securities.

1.3. Modeling Financial Innovation

Most security innovation models have two stages:

(a) Given the innovated securities and those already present, the determination of equilibrium prices and allocations.

(b) Given the correspondence mapping the securities to be chosen for innovation to the resulting set of security market equilibria, optimization by one or more innovators.

Among the theoretical difficulties is finding a reasonable, determinate, and tractable model for security market equilibrium. Except for cases in which innovation leads automatically to complete markets, or in special parametric examples, models of financial innovation must contend with indeterminacy and non-existence of equilibrium. For surveys of the relevant literature on incomplete security markets, see Cass [29], Duffie [40], Geanakoplos [51], Magill and Quinzii [70], and Magill and Shafer [71]. Most of the literature on financial innovation has sidestepped these difficulties by restricting attention to securities with payoffs in terms of a single numeraire commodity.

The main points of departure in the literature are the innovator’s objectives and price conjectures, and the potential for private information. In Allen and Gale [3, 5, 6], Chen [32], Demange and Laroque [37], DeMarzo and Duffie [38], Gale [50], Madan and Soubra [69], Pesendorfer [89], and Rahi [91], the innovator maximizes the utility of the proceeds of the sale of the new issue, considering as well the utility of retained cash flows and (in some cases) the cost of obtaining the assets collateralizing the issue. In other cases (Bisin [20, 21], Che and Rajan [31], Cuy [35], Duffie and Jackson [41], Hara [59, 60], Heller [65], Ohashi [85, 87, 88], and Rahi [92]), the innovator is an intermediary who plans to profit purely from the provision of transactions services, as described in Subsection 1.1. The case of a ”social planner” has also been considered (Cass and Citanna [30], Demange and Laroque [36], Duffie and Jackson [41], Elul [43, 44], Ohashi [85, 88], Rahi [92]).

2. General Equilibrium Models of Financial Innovation

This section gives a brief sketch of selected general equilibrium formulations and results, emphasizing those appearing in this symposium issue. We begin with a standard model of general equilibrium in a given set of security markets, then review some of the ideas in the articles by Elul [43], Chen [32], and Pesendorfer [89] that are played out in a like setting. The basic modeling approach of Allen and Gale [3, 5, 6] is outlined as a point of departure for the literature on security design in a general equilibrium setting.
2.1. Typical Setup

In studying financial innovation, one often begins with a standard setup for general equilibrium in incomplete markets, in which there is a finite set \( \{1, \ldots, \Omega \} \) of states of nature, with trading at time 0 of \( L \) commodities and a finite number of securities, and trading at time 1, in each given state, of \( L \) commodities. The consumption set \( C \subset \mathbb{R}^L \times (\mathbb{R}_+^L)^{\Omega} \) has a typical element \( c \) representing an initial commodity bundle \( c_0 \in \mathbb{R}^L \) and a state-contingent commodity bundle \( c_\omega \in \mathbb{R}_+^L \) at time 1 in state \( \omega \). Each agent \( h \) in some set \( \mathcal{H} \) of households has preferences over \( C \) represented by a utility function \( u^h: C \to \mathbb{R} \) satisfying typical regularity conditions. The initial distribution of consumption to agents is given by a function \( e: \mathcal{H} \to C \).

Most models in the literature take as given a set \( \mathcal{R} \) of potential financial structures. Each financial structure \( R \) in \( \mathcal{R} \) is a finite subset of \( \mathbb{R}_+^\Omega \), a collection of \# \( R \) securities, each characterized by a vector in \( \mathbb{R}_+^\Omega \) of state-contingent payoffs of a fixed numeraire commodity,\(^{10}\) say commodity number 1. For each financial structure \( R \), there is an associated admissible set \( \Theta(R) \subset \mathbb{R}^{\#R} \) of portfolios. For example, one can enforce short sales constraints on certain securities by suitable restrictions on \( \Theta(R) \).

Given a price vector \( p \in \mathbb{R}^L \times (\mathbb{R}_+^L)^{\Omega} \) for spot consumption and a price vector \( q \in \mathbb{R}^{\#R} \) for securities, a budget-feasible choice for agent \( h \) is a pair \((c, \theta) \in C \times \Theta(R)\) such that

\[
\begin{align*}
    p_0 \cdot (c_0 - e(h)_0) + q \cdot \theta &\leq 0 \\
    p_\omega \cdot (c_\omega - e(h)_\omega) &\leq \theta \cdot R_\omega, \quad \omega \in \{1, \ldots, \Omega\},
\end{align*}
\]

where \( R_\omega \) denotes the vector of payoffs of the securities in state \( \omega \).

For agent \( h \), given prices \((p, q)\), a budget-feasible choice \((c, \theta)\) is optimal if there is no budget-feasible choice \((c', \theta')\) such that \( u^h(c') > u^h(c) \). A market

\(^{10}\) As pointed out by Hart [63], if securities pay off bundles of different commodities, then equilibrium may fail to exist without restrictions such as short sales limits (as imposed by Radner [90]), because of the potential for discontinuous changes in the span of the securities’ income payoffs with changing spot commodity prices. While generic existence of equilibrium has been shown (see Duffie and Shafer [42] and other results cited in the surveys by Geanakoplos [51], Magill and Quinzii [70], and Magill and Shafer [71]), it is nevertheless convenient and customary in the financial innovation literature to assume that securities pay off in a numeraire commodity, or indeed, as in the majority of papers in this issue, that \( L = 1 \).

The alternative is to study purely financial securities, paying in units of account. This allows for existence without short sales constraints, as shown by Cass [27] and Werner [105], but also allows a broad scope for indeterminacy of equilibrium as demonstrated by Balasko and Cass [15], Cass [28], Geanakoplos and Mas-Colell [52], and others cited in the survey by Cass [29]. Since we have fixed commodity number 1 as a numeraire, neither existence nor local indeterminacy is an issue, whether we treat securities as purely financial or not. Pesendorfer [89] takes purely financial securities but does not fix a numeraire commodity, allowing him to study the role of financial innovation in reducing indeterminacy.
equilibrium, given parameters \((u, c, \Theta, \Theta(R))\), consists of prices \((p, q)\) and allocations \(c: \mathcal{H} \to C\) and \(\Theta: \mathcal{H} \to \Theta(R)\) of consumption and securities such that, for all \(h\) the choice \((c(h), \Theta(h))\) is optimal for agent \(h\) given these prices, and such that markets clear:

\[
\sum_{h \in \mathcal{H}} c(h) - e(h) = 0; \quad \sum_{h \in \mathcal{H}} \Theta(h) = 0, \tag{1}
\]

where \(\sum_{h \in \mathcal{H}}\) denotes summation when \(\mathcal{H}\) is finite, and otherwise denotes integration, with suitable measurability and integrability conditions on \(c, e,\) and \(\Theta,\) taking \(\mathcal{H}\) to be a particular measure space. (In some cases, the supply of securities is not 0, as assumed above, and adjustments to the market clearing condition apply.)

The parameter of particular interest in a study of innovation is the financial structure \(R.\) Somehow, \(R\) is influenced by some agents or institutions, or merely by the modeler, so as to achieve some aim or study the comparative effects of different security structures.

2.2. Elul

As an illustration of how delicate the welfare effects of innovation might be, Elul [43] shows that the addition of a new security may have almost arbitrary effects on agents’ utilities. Specifically, Elul takes

- \# \(\mathcal{R}\) to be \(J\) or \(J + 1\) for some fixed integer \(J \geq 1.\)
- \(\Theta(R) = \mathbb{R}^{*,R}\) for all \(R\) (no portfolio restrictions).
- \(\mathcal{H} = \{1, \ldots, H\}\), for some finite number \(H\) of agents.

For some fixed subset \(\bar{R} \subset \mathbb{R}^J\) of \(J\) original securities, Elul considers innovation of an arbitrary \((J + 1)\)-st security. In effect, Elul takes

\[
\mathcal{R} = \{R \in \mathbb{R}^{O \times (J + 1)} : R_{\omega j} = \bar{R}_{\omega j}, 1 \leq \omega \leq \Omega, 1 \leq j \leq J\},
\]

where \(R_{\omega j}\) denotes the payoff of security \(j\) in state \(\omega,\) and likewise for \(\bar{R}_{\omega j}.\)

The parameters defining the underlying set \(\mathcal{E}\) of economies are:

- the original security payoffs \(R \in \mathbb{R}^{O \times J}.
-\)
- the utility functions \((u^1, \ldots, u^H)\) (the set of which is given a finite-dimensional parameterization via quadratic perturbations).
- the endowments \((e(1), \ldots, e(H))\).

Giving the set \(\mathcal{E}\) of economies the structure of a finite-dimensional smooth manifold, Elul shows, under typical technical conditions, that for a generic (that is, full measure, open) subset of \(\mathcal{E}\) one can move any equilibrium vector \((u^1(c(1)), \ldots, u^H(c(H)))\) of utilities associated with the set \(\bar{R}\) of
original securities in an arbitrary direction in $\mathbb{R}^H$, merely by varying the choice of the $(J+1)$-st "new" security. For example, innovation can generically make all agents strictly worse off, or all agents strictly better off, or favor any group of agents over any other. Elul [44] and, independently, Cass and Citanna [30] have results along similar lines.

One of the conditions imposed by Elul is that markets are "sufficiently incomplete," in that the number $J$ of originally marketed securities is sufficiently small\(^\text{11}\) relative to the number of states of the world and the number of agents. We know that some minimum degree of market incompleteness is required because, when an additional security completes the markets for contingent claims, that is, $\text{span}(R) = \mathbb{R}^2$, the equilibrium allocation becomes\(^\text{12}\) Pareto efficient, a point made obvious by Arrow [12]. In Elul's model, one needs a degree of market incompleteness large enough to allow for an arbitrary perturbation of the vector of equilibrium utilities through innovation alone. In order to allow for a smooth perturbation of equilibrium utilities with innovation, Elul restricts attention to new securities that are only marginally desirable to trade. The (generically) minimum degree of market incompleteness for Elul's results can then be ascertained by counting the number of equations defining equilibrium, the specified direction of movement of equilibrium utilities, and the marginal indifference to trading the new security. This oversimplifies; for a proper account of his approach and results, one should read Elul's article.

2.3. The Issuer's Problem

Of course, Elul's result is not based on any particular motive for innovation (beyond moving utilities in a specified direction), whereas market innovators typically have in mind some economically reasonable objective of their own, as modeled in the remainder of this section.

Continuing abstractly for the moment, an issuer is restricted to financial structures in some given collection $\mathcal{R}$. The issuer takes $(u, e, \Theta, \mathcal{R})$ as given and maximizes some objective function $\pi : \mathcal{R} \to \mathbb{R}$. Typically, $\pi$ is endogenous and involves conjectures regarding the equilibrium associated with each possible financial structure $R$ in $\mathcal{R}$. The various models differ in the nature of these conjectures, particularly with regard to equilibrium

\(^{11}\) Specifically, Elul assumes that $\Omega \geq H(J + 2) + H(J + 1)(J + 2)/2$. In a slightly different model, Cass and Citanna [30] use a different incompleteness condition, namely that $\Omega - J \geq H + 1$, under which the addition of two securities will produce the desired welfare perturbation; or that $\Omega - J \geq 2H - 1$, under which adding a single security will suffice.

\(^{12}\) Zame [107] shows, however, that with an infinite number of states, the equilibrium allocation need not not approach efficiency as securities are added to the point of complete markets.
security prices, and also differ regarding the nature of competition among innovators.

2.4. Allen and Gale

Allen and Gale [3, 5, 6] focus on short sales restrictions as a source of value to securitization of an issuer's assets. Because of such restrictions, two portfolios of securities paying the same total amount may sell for different prices. In a simplification of their 1988 article that appears in Chap. 4 of their recent monograph, Allen and Gale take:

- \( L = 1 \) commodity and the consumption set \( C = \mathbb{R} \times \mathbb{R}^{Q} \).
- \( \Theta(R) = \mathbb{R}^{+N} \) (no short sales).
- A set of \( B \) of basic financial structures. Each basic financial structure \( B \) in \( \mathcal{B} \) is a finite subset of \( \mathbb{R}^{Q} \), representing a portfolio of securities satisfying the collateralization constraint

\[
\sum_{z \in N} z = Z,
\]

where \( Z \in \mathbb{R}^{Q} \) denotes the collateral of state-contingent numeraire commodity available to an issuer. For example, the issuer could be a bank issuing collateralized mortgage obligations against a given pool of mortgages whose total cash flows are represented in the model by \( Z \). While each \( B \) in \( \mathcal{B} \) is finite, Allen and Gale allow the set \( \mathcal{B} \) of basic financial structures to be infinite in some versions of their model.

- All possible securities appearing in basic financial structures are present for trade by agents. That is, \( \mathcal{B} \) is a singleton containing only the union \( R \) of all \( B \) in \( \mathcal{B} \), and firms have no influence over the set \( R \) of securities that are actually available for trade.

An issuer conjectures a price \( Q(z) \) for each security \( z \) in \( R \) and solves the problem

\[
\max_{B \in \mathcal{B}} \sum_{z \in R} Q(z) - T(B),
\]

where \( T(B) \) is a fixed cost in initial commodity for setting up the financial structure \( B \). The objective is thus to maximize the market value of the firm, the total value of its issued securities net of the costs of issuing. (Issuing costs are taken out of the issuer's supply of commodity, which is "private," in the sense that it is not considered in the market clearing conditions).
In order to obtain a model of innovation that is consistent with the existence of equilibrium, Allen and Gale adopt a measure\(^{13}\) space \((\mathcal{I}, \mathcal{F}, \mu)\) of issuers of total "mass" \(\mu(\mathcal{I}) = 1\).

Initially, Allen and Gale take \(\mathcal{F}\), and therefore, \(\mathcal{R}\), to be finite. Given the security price conjecture \(Q: \mathcal{R} \rightarrow \mathbb{R}\), a financial selection is a measurable function \(\Gamma: \mathcal{I} \rightarrow \mathcal{F}\) indicating the choice of basic financial structure by each issuer such that, for \(\mu\)-almost every issuer \(i\), the basic financial structure \(\Gamma(i)\) solves (2). Since issuers are identically defined, it must then turn out that all basic financial structures actually chosen by issuers are equally profitable.

An equilibrium is a collection \((p, q, c, \theta, \Gamma)\), where \(\Gamma\) is a financial selection given the security price conjecture \(Q\) determined by the announced security price vector \(\mathbf{q} \in \mathbb{R}^\#\mathcal{R}\), and where \((p, q, c, \theta)\) is a market\(^{14}\) equilibrium, defined as in Subsection 2.1 with a modification of the market clearing condition (1) to account for the supply of securities associated with \(\Gamma\):

\[
\sum_{h \in \mathcal{R}} c(h) - e(h) = \hat{Z}, \quad \sum_{h \in \mathcal{R}} \theta(h) = \gamma,
\]

where \(\hat{Z} := (0, Z)\) and, for any \(j \in \{1, \ldots, \#\mathcal{R}\}\), we let \(\gamma_j\) denote the total number of units of security \(j\) issued. That is, if security \(j\) is in basic financial structure \(B\), then \(\gamma_j := \mu(\{i \in \mathcal{I}: \Gamma(i) = B\})\). (Two securities in different basic financial structures with the same payoffs are treated as distinct securities.)

Under technical conditions, Allen and Gale provide a proof of existence of equilibrium.

Consider an equilibrium \((p, q, c, \theta, \Gamma)\). Assuming that utilities are smooth, the marginal rates of substitution of any consumer \(h\) of state-contingent consumption, relative to initial consumption, are uniquely defined by the vector \(\mathbf{m}^h \in \mathbb{R}^\Omega\) given by

\[
\mathbf{m}^h = \frac{\partial u^h(c(h))}{\partial c_{\omega}} \cdot \frac{\partial c_{\omega}}{\partial c_0}, \quad \omega \in \{1, \ldots, \Omega\}.
\]

If consumer \(h\) is not bound by the short sales constraint on a given security with payoff \(z \in \mathcal{R}\), then the first-order conditions for optimality of the portfolio choice for consumer \(h\) imply that the price of this security is \(\mathbf{m}^h \cdot z\). If consumer \(h\) is not bound by a short sales constraint on any security in a basic financial structure \(B\), it follows from adding up that the

\(^{13}\) Allen and Gale assume the space of issuers to be countable, but nevertheless take the fraction of issuers that choose a particular structure as chosen from potentially any point in the continuum \([0, 1]\).

\(^{14}\) Since there is but a single commodity in this model, \(p_\ell = 1\) for all \(\ell\).
market value of that structure must be $m^b \cdot Z$, which is the same for all such basic financial structures. If no agent is bound by a short sales constraint, it follows that the only financial structures that appear in this equilibrium must be those $B$ with minimal transactions cost $T(B)$. A version of the model without short sales constraints would therefore be relatively uninteresting.

Even with unrestricted short sales, of course, financial innovation would have important welfare implications. Indeed, merely the incompleteness of markets implies that marginal rates of substitution are not typically equated among consumers, and therefore that there is an important spanning role for the particular financial structures chosen by firms. In the Allen–Gale model, however, neither consumers nor firms consider the set of traded securities as variable. For example, issuers do not attempt to identify unspanned risks and cater directly to the implied needs for insurance. They merely issue the package of securities fetching the highest announced market value (net of issuing costs). Equilibrium prices give consumers an incentive to hold in non-zero amounts only those securities actually issued by firms.

With binding short sales constraints, those agents with the highest marginal valuation for a security will determine its market value. That is, given an equilibrium $(p, q, c, \theta, \Gamma)$, the conjectured security pricing functional $Q: \mathbb{R} \to \mathbb{R}$ can be defined by

$$Q(z) := \sup_{h \in \mathcal{X}} m^h \cdot z.$$  

As Allen and Gale state\textsuperscript{15} the point, “one is breaking the firm into pieces and selling the pieces to the clientele that values it most. It is this ability to increase the value of the firm that provides the incentive to innovate and allows the cost of innovation to be covered.” Indeed, rather than taking the price of each security as given while solving (2), an issuer could as well solve (2) by taking as given the marginal valuation functional $Q: \mathbb{R}^D \to \mathbb{R}$, as extended to $\mathbb{R}^{D^2}$ by (3). The resulting definition of equilibrium would be equivalent to the original one.

One of the key characteristics of the equilibrium security design in Allen and Gale [3, 6] is the “extremal” nature of the design. One may think of some securities as convex combinations of others. Extremal securities are those that cannot be formed as convex combinations of others. An extremal set of securities has, in some sense, a maximal market span, in that the set of feasible portfolio payoffs that can be achieved is maximal. The extremality of optimal designs is an issue revisited by Madan and

\textsuperscript{15} See Allen and Gale [6, p. 73].
Soubra [69], who show that, with a richer formulation of marketing costs for new securities, the optimal securities need not be extremal, and one can in fact recover the optimality of standard non-extremal structures such as debt-equity, in certain situations.

Allen and Gale also provide a notion of constrained optimality of the choice of securities issued by firms. Their equilibrium satisfies this constrained optimality property, essentially because prices are a guide to firms of the securities beneficial to consumers, somewhat along the lines of standard proofs of the first welfare theorem. The fact that prices and available markets are not conjectured by the firm to vary with the firm’s innovation plans plays an important role in this result, in contrast to the failures of constrained optimality found by Che and Rajan [31], Heller [65], and Pesendorfer [89].

Che and Rajan [31] and Heller [65] take the perspective of Hahn [58] and Foley [48]: There are profit-maximizing market-making firms providing transactions services. A trade can only be conducted through the intermediation of the market maker. Hahn [58] and Foley [48] did not consider security markets specifically. For Che and Rajan [31] and Heller [65], market makers choose the subset of securities in which they will make markets, based on the prices that they conjecture will be obtained with each choice of markets. As there are setup costs for transactions services, not all markets will be set up, and inefficiencies may arise. Examples include cases in which it would be constrained efficient to conduct trade in two complementary securities, but a market maker will not introduce one of these if the other is not already available.

2.5. Chen

Chen [32] models financial intermediaries that create new securities collateralized by old securities. As with Allen and Gale, short sales constraints (weakened to allow simply a lower bound on the position held in a given security) imply that, even when the linear span of existing securities is complete, innovation may be profitable because it can reduce the cost of market frictions, in this case, short sales constraints. Some of this reduction in costs is captured by profit-maximizing innovators. Grossman [57] had given an informal discussion of this sort of motive for innovation.

Chen takes as a starting point a given finite set $R$ of securities and a market equilibrium $(p, q, c, \theta)$, with $\Theta(R)$ defined by lower bound constraints on each security position. The equilibrium security pricing functional $Q$, as defined by (3), is assumed by innovators to apply as well to the securities that they will innovate. In effect, an innovator takes agents’ marginal rates of substitution as fixed with the introduction of a security, and extends $Q$ from $R$ to $\mathbb{R}^\theta$ by (3).
The innovator chooses a portfolio \( \varphi \in \mathbb{R}_+^k \) of existing securities as collateral for a finite collection \( B \subset \mathbb{R}^2 \) of new securities, meaning that
\[
\sum_{z \in B} z_\omega = \varphi \cdot R_\omega, \quad \omega \in \{1, \ldots, \Omega\}. \tag{4}
\]
If (4) is satisfied, we say that \((B, \varphi)\) is a feasible innovation package. The innovator solves
\[
\max_{(B, \varphi) \in P} \sum_{z \in B} Q(z) - \varphi \cdot q - T(B),
\]
where \(P\) denotes the set of feasible innovation packages, and as before \(T(B)\) is a setup cost for the innovation choice \(B\).

Chen's main goal is to characterize the pricing functional \(Q\), which he shows is positive (with strictly monotonic preferences) and sublinear (a consequence of the lower bound constraints on security portfolios). He relates \(Q\) to a lower bound for market valuation provided by the mere assumption of no arbitrage.

2.6. Pesendorfer

Pesendorfer [89] models intermediaries as introducers of new securities collateralized by a fixed set of "standard" securities and by portfolios of securities innovated by other intermediaries. Like Allen and Gale [3, 5, 6], Pesendorfer studies a notion of equilibrium in which all consumers optimize, all intermediaries optimize, and all markets clear. Unlike Allen and Gale, however, issuers explicitly consider whether the securities they may issue are already traded, and if not make price conjectures based on the reservation prices of consumers. As a consistency condition, there is no incentive in equilibrium to introduce securities that are not already introduced.

Pesendorfer's objective is to prove the existence of, and characterize, an equilibrium. In our description of Pesendorfer's model, we take significant liberties with notation and emphasis mainly in order to simplify the connection with the models described earlier. One should see Pesendorfer's article for a proper exposition of his own model.

For each issuer \(i \in \mathcal{I} = \{1, \ldots, I\}\), there are effectively three sets of securities to consider: the standard set \(A \subset \mathbb{R}_+^2\), the intermediary's own innovated set \(B_i \subset \mathbb{R}_+^2\), and the set \(B_{-i} \subset \mathbb{R}_+^2\) of securities innovated by other intermediaries. The set of all innovated securities is \(B := \bigcup_{i=1}^I B_i\). In Pesendorfer's model, the number of securities actually innovated is finite without loss of generality, although the menu of possible securities is
infinite. Given $A$ and $B$, a feasible innovation by intermediary $i$ is a collection $(D, \varphi, \zeta, \psi, \nu)$ satisfying the collateralization restriction

$$D \left[ \psi + \int y \, d\nu(y) \right] \leq B - \varphi + A \zeta,$$

where

$D$ is the matrix in $\mathbb{R}^{D \times D}$ of payoffs of the candidate set of securities for innovation by intermediary $i$, not necessarily the same as the announced set $B$, that is taken as given by other agents. (In equilibrium, $D = B$.) Here and below, we use the same symbol for the set of securities and for the matrix of state-contingent payoffs of the securities, taking a fixed ordering of the securities.

$\varphi$ is a portfolio in $\mathbb{R}_{+}^{E_{-i}}$ of purchases by intermediary $i$ of securities innovated by other intermediaries.

$\zeta$ is the portfolio in $\mathbb{R}^{A}$ of standard securities purchased by $i$.

$\psi$ is the portfolio in $\mathbb{R}_{+}^{D}$ of intermediary $i$'s own securities sold to other intermediaries.

$\nu$ is a measure on $\mathbb{R}_{+}^{D}$ describing the “retail marketing plan” of the intermediary, the cross-sectional distribution of sales of portfolios of the innovated securities to consumers.

The set of securities actually tradable is $R = A \cup B$, the union of all standard and innovated securities. Standard securities are available to all at a given price vector $q_{A} \in \mathbb{R}^{A}$. Marketing an innovated security to a consumer is assumed to require a fixed setup cost to any intermediary of $b$ (in the numeraire commodity, at time 0). This cost is passed along to consumers as a brokerage fee. Thus, given a security price vector $(q_{A}, q_{B}) \in \mathbb{R}^{A} \times \mathbb{R}^{B}$, the total charge to a consumer for a portfolio $\theta = (\theta_{A}, \theta_{B}) \in \mathbb{R}^{A} \times \mathbb{R}_{+}^{B}$ of standard and innovated securities is

$$q_{A} \cdot \theta_{A} + q_{B} \cdot \theta_{B} + b \eta(\theta_{B}),$$

where, for any Euclidean vector $y$, $\eta(y)$ denotes the number of non-zero elements in $y$.

Pesendorfer assumes the following conjectures by intermediaries for the prices of innovated securities. For innovated securities already in existence, those in $B$, retail portfolio prices are as given by the vector $q_{B}$. Institutional prices, those charged for innovated securities sold by one intermediary to another, are given by some equilibrium “wholesale” pricing functional $\nu: \mathbb{R}_{+}^{D} \to \mathbb{R}$. For a portfolio of securities not currently in existence, the intermediary's conjectured price (including brokerage fees) is the maximum reservation price that would be paid by any consumer for the portfolio,
taking all other securities and all prices as given. Specifically, given the
announced set $B$ of innovated securities and a candidate innovation plan
$(D, \varphi, \zeta, \psi, v)$ for intermediary $i$, we can treat $D$ as the disjoint union of
$D \setminus B$, those securities in $D$ that are not in $B$, and $D \cap B$. Likewise,
the measure $v$ on $R^D$ describing the retail marketing plan can be decomposed
by projection into measures $v_{D \setminus B}$ on $K(D \setminus B) := R^{\sigma(D \setminus B)}$ and $v_{D \cap B}$ on $K(D \cap B) := R^{\sigma(D \cap B)}$. A portfolio $y$ of securities in $K(D \cap B)$ is assumed to
have a retail market value of $Q_y$ determined by the announced price vector
$q_y$ (gross of brokerage commissions). For any portfolio $y$ in $K(D \setminus B)$, corresponding to the set of securities not currently available for
trade, we let $\rho(y | D, B, q, p)$ denote the supremum, over the set of con-
sumers, of the amount of initial numeraire commodity that would be
offered by a consumer in order to obtain this portfolio, given the possibil-
ities for trade on existing markets represented by $(B, q, p)$. (For
details, see Pesendorfer's article.)

Taking as given a spot commodity price vector $\rho$, an announced set $B$
of innovated securities, a retail security price vector $q$, and a wholesale
security pricing functional $v$ for innovated securities, a feasible innovation
plan $(D, \varphi, \zeta, \psi, v)$ for intermediary $i$ generates the net profit

$$\pi_i(D, \varphi, \zeta, \psi, v \mid B, q, v, p) := \int \left[ b\eta(y) + Q_y(y) \right] \, dv_{D \cap B}(y)$$

$$+ \int \rho(y | D, B, q, p) \, dv_{D \setminus B}(y)$$

$$- b \int \eta(y) \, dv(y) - q_A \cdot \zeta - \varphi \cdot V(B \_ \_ i)$$

$$+ \psi \cdot V(D) - T_i(p, q, v, B, D, \varphi, \psi, v), \quad (5)$$

whose terms correspond, respectively, to

1. Revenue from retail sales of existing innovated securities.
2. Revenue from retail sales of proposed additional innovated
   securities.
3. Setup costs for making retail sales.
4. Cost of standard securities used for collateral.
5. Cost of innovated securities purchased from other intermediaries
   for collateral, where, for any set $F \subset R^D$ of innovated securities,
   $V(F)$ denotes the vector in $R^F$ of prices on the wholesale market, according to
   the announced pricing functional $v$. 

6. The market value received for innovated securities sold to other intermediaries.

7. An innovation technology cost that depends on all prices and on retail as well as wholesale transactions of innovated securities. Pesendorfer places structure on the technology cost functional $T$, that we do not review here.

Intermediaries’ equilibrium conjectures regarding the reservation value $\rho(\cdot)$ to be received for non-traded securities are to be taken as marginal\textsuperscript{16} valuation, as in Chen’s model.

Given $(B, q, v, p)$, a feasible innovation plan for intermediary $i$ is optimal for intermediary $i$ if there is no other with higher profit. An equilibrium for a given economy defined by the parameters $(u, c, A, T, b)$ is a collection

$$(p, q, v, c, \theta, (B_i, \varphi_i, \xi_i, \psi_i, v_i))$$

such that:

- For each agent $h$, given the spot price vector $q$, the set $B$ of innovated securities, and the vector $q = (q_A, q_B)$ of retail security prices, the plan $(c(h), \theta(h))$ is optimal. The definition of optimality is the same as that in our formulation of equilibrium in Subsection 2.1, with the exception that a consumer must pay the fixed brokerage commission $b$ for each type of innovated security purchased.

- For each intermediary $i$, given $(B, q, v, p)$, the innovation plan $(B_i, \varphi_i, \xi_i, \psi_i, v_i)$ is optimal.

- Wholesale innovated securities markets clear, $\sum_{i \in I} \beta_i = 0$, where $\beta_i$ is the portfolio in $\mathbb{R}^{\#B}$ made up of the portfolio $\varphi_i$ of securities in $B_i$, and of the portfolio $-\psi_i$ of securities in $B_i$, in the obvious way.

\textsuperscript{16} This perspective is only consistent with the nature of the reservation valuation functional $\rho(\cdot)$ because of Pesendorfer’s assumption that the set $\mathcal{H}$ of agents is a non-atomic measure space made up of a finite number of types of agents. That is, it may happen that the agent whose reservation value attains the supremum across agents for a given portfolio $y$ of non-traded securities in the support of $\nu_{p, a}$ is also of the same type as the agent attaining the supremum reservation value for some other such portfolio $y'$. While a single consumer of this type would in general not be willing to pay $\rho(y) + \rho(y')$ for the combined portfolio $y + y'$, one can allocate $y$ to some consumer of this type and $y'$ to some other consumer of this type and still obtain the marginal valuation $\rho(y) + \rho(y')$ for the total portfolio $y + y'$, as implicit in (5). Given concavity of utilities, if the total mass of $\nu_{p, a}$ is positive, marginal valuation by intermediaries is overly optimistic. The equilibrium consistency condition that $\mathcal{B}$ is the set of securities chosen for innovation implies that the mass of $\nu_{p, a}$ is in fact zero in equilibrium. In general, intermediaries have more incentive to innovate in this model, and similar marginal valuation models such as Chen [32], than they would if they computed the revenue that they would receive from private placement of non-marketed securities after other markets have cleared.
• Standard securities markets clear: $\sum_{h \in \mathcal{H}} \theta(h) a + \sum_{i \in \mathcal{C}} \xi_i = 0$.
• Retail innovated securities markets clear: For all $i$ in $\mathcal{F}$,

$$\int y \, dv_i(y) = \sum_{h \in \mathcal{H}} \theta(h; B_i),$$

where $\theta(h; B_i)$ denotes the portfolio of securities in $B_i$ purchased by consumer $h$, as determined by $\theta(h)$.

Consumption markets clear automatically in equilibrium since security markets clear and there is but a single commodity ($L = 1$). Under technical conditions, Pesendorfer demonstrates the existence of equilibrium. He goes on to characterize equilibria in several ways. As do Allen and Gale [3, 5, 6], Pesendorfer studies conditions under which, fixing spot commodity prices, innovation is constrained efficient, in the sense that there is no alternative innovation plan and redistribution of security portfolios that results in an allocation of consumption to consumers that strictly Pareto dominates that given by the equilibrium allocation. Pesendorfer shows that constrained efficiency should not be expected unless there is but one intermediary ($I = 1$), or unless there is no setup cost for marketing innovated securities to consumers ($b = 0$). As with Che and Rajan [31] and Heller [65], a source of inefficiency is complementarities among the innovations available to different intermediaries. Pesendorfer goes on to show that the degree of indeterminacy of equilibrium allocations usually associated with purely financial securities (discussed in Footnote 10) can be reduced significantly by the process of innovation. For example, if innovation costs are small, then the implications of indeterminacy for equilibrium allocations are small.

3. The Exponential-Normal Setting

In this section we review models with a variable financial structure that exploit parametric assumptions of normality and constant absolute risk aversion. We present below a simple framework of security trading in an asymmetric information environment that encompasses most of the literature in this category, including the symposium articles by Bhattacharya et al. [19], Demange and Larroque [36, 37], Hara [60], Ohashi [87], and Rahi [92]. The framework is no doubt quite restrictive, but has the advantage of admitting closed-form solutions. This allows a relatively transparent analysis of the impact of alternative financial structures on risk-sharing opportunities and information revelation in asset markets. In particular, we are able to develop explicit characterizations of optimal security design,
both from the social planning perspective, and from the point of view of the innovators of securities such as exchanges and entrepreneurs. These results are an important step in deepening our understanding of financial innovation, but much remains to be done before we can expect the theory to yield practical prescriptions for regulators or innovators.

3.1. The Basic Framework

All random variables are defined on a fixed probability space\(^{17}\) \((\Omega, \mathcal{F}, P)\). All normally distributed random variables belong to a linear space \(\mathcal{N}\) of joint normally distributed random variables on \(\Omega\), endowed with an inner product in the usual way: For \(g, h \in \mathcal{N}\), \((g \mid h) := \text{cov}(g, h)\). Throughout this section, matrices, vectors, and vector-valued random variables are distinguished by boldface type. The following notational convention is used for covariance matrices: \((V_g)_{ij} := \text{cov}(g, h_j)\), \((V_{gh})_{ij} := \text{cov}(g_i, h_j)\), and, for univariate \(g\), the \(j\)th component of the column vector \(V_{gh}\) is \(\text{cov}(g, h_j)\). The symbol \(\dagger\) denotes transpose.

The setting is a single-good economy with uncertainty and asymmetrically informed agents. These agents have access to an incomplete set of asset markets that allows some risk-sharing and aggregation of information. The sequence of events is as follows: At the \textit{ex ante} stage the collection of tradable securities is determined. At the \textit{interim} stage agents observe their private signals and trade the available securities in a rational expectations equilibrium. Finally, at the \textit{ex post} stage, all uncertainty is resolved, the assets pay off, and consumption takes place.

We will be more specific later regarding the design of securities at the \textit{ex ante} stage. For now let us focus on the \textit{interim} stage, taking as given a financial structure consisting of \(m\) assets with payoffs in \(\mathcal{N}\). The asset payoff and price vectors are denoted by \(f\) and \(q\) respectively. In addition to these \(m\) assets, there is a riskless bond whose price and payoff are normalized to one. This assumption may be dropped if the assets under consideration are futures contracts. In either case, a portfolio \(\Phi \in \mathbb{R}^m\) yields a net payoff of \(\Phi^\dagger (f - q)\).

There are \(n\) agents, with von Neumann–Morgenstern utility functions displaying constant absolute risk aversion (coefficient \(r_i\) for agent \(i\)). Agent \(i\) has an initial risky position \(e_i := x_i p_i\), where \(p_i\) is a random variable\(^{18}\) in \(\mathcal{N}\). The nature and interpretation of \(x_i\) will vary with the model under

\(^{17}\) Note that, unlike the previous section, the state space is not finite.

\(^{18}\) The notation \(p_i\) is suggestive of the possibility that the risk that agents face arises from randomness in futures spot prices, as is often the case with users of futures markets. This interpretation should not be taken literally, however, since the \(p_i\)'s are exogenous in the model presented here.
consideration. For the moment, it suffices to think of \( x_i \) either as a scalar or as a random variable in \( \mathcal{X} \) whose value is known at the time of trading. An asset position \( \Phi_i \), leaves agent \( i \) with net wealth

\[
w_i := x_i p_i + \Phi_i^T (f - q).
\]

(6)

The information set of agent \( i \) is \( Y_i \), a collection of random variables that includes \( x_i \) and the asset price vector \( q \). She faces the optimization problem

\[
\max_{\Phi_i \in \mathcal{K}} E[-\exp(-r w_i)],
\]

(7)

where \( w_i \) is given by (6), and \( \mathcal{K} \) is the space of \( Y_i \)-measurable random variables valued in \( \mathbb{R}^m \). Conditional on \( Y_i \), any choice of \( \Phi_i \), leaves net wealth \( w_i \) normally distributed.\(^{19}\) Therefore, agent \( i \)'s expected utility is

\[
E[-\exp(-r w_i)] = -E[E[-\exp(-r w_i) | Y_i]]
\]

\[
= -E\left[ \exp\left( -r_i \left[ E(w_i | Y_i) - \frac{r_i}{2}\text{Var}(w_i | Y_i) \right] \right) \right].
\]

(8)

Let

\[
\chi_i := E(w_i | Y_i) - \frac{r_i}{2}\text{Var}(w_i | Y_i).
\]

(9)

The problem (7) reduces to choosing a portfolio in \( \mathbb{R}^m \) to maximize \( \chi_i \) pointwise for each realization of the information \( Y_i \). From (6) and (9),

\[
\chi_i = x_i E(p_i | Y_i) + \Phi_i^T \left[ E(f | Y_i) - q \right] - \frac{r_i}{2} \left[ x_i^2 \text{Var}(p_i | Y_i) + \Phi_i^T \text{Var}(f | Y_i) \Phi_i + 2x_i \Phi_i^T \text{cov}(f, p_i | Y_i) \right].
\]

(10)

The solution to (7) is now easily obtained. If the agent is a price-taker,

\[
\Phi_i = \text{Var}(f | Y_i)^{-1} \left[ E(f | Y_i) - q - r_i \text{cov}(f, p_i | Y_i) \right],
\]

(11)

provided \( \text{Var}(f | Y_i) \) is nonsingular. From (10) and (11), we can deduce that in equilibrium

\[
\chi_i = x_i E(p_i | Y_i) - \frac{r_i}{2} x_i^2 \text{Var}(p_i | Y_i) + \frac{r_i}{2} \Phi_i^T \text{Var}(f | Y_i) \Phi_i.
\]

(12)

This expression will be useful later in making welfare comparisons.

\(^{19}\) Assuming that \( (f, p) \) is joint normal conditional on \( Y_i \), which will indeed be the case in equilibrium.
3.2. Equilibrium

In equilibrium the optimal asset positions of agents sum to 0 identically. That is, markets clear for each realization of private information:

$$\sum_{i=1}^{n} \Phi_i = 0.$$  \hspace{2cm} (13)

This gives us a rational expectations equilibrium with learning from prices since agents know the asset price function $q: \mathcal{A} \rightarrow \mathbb{R}^m$ and condition their trades on the realization of asset prices. Of particular interest is the amount of information revealed by prices.

3.2.1. Fully Revealing Equilibria

The following is a useful benchmark case. Let $x_i = 1$ for all $i$. Agent $i$’s private information is represented by a vector of signals $s_i$, with each signal a random variable in $\mathcal{A}$. Then $Y_i = (s_i, q_i).$ We look for linear equilibria of the form

$$q = \bar{q} + Qs,$$

where $s := (s_1, s_2, ..., s_n)$, and $Q$ is a matrix of the appropriate dimension. In our setting an equilibrium is fully revealing if, for every agent $i$, moments conditional on her information $Y_i$ are equal to the corresponding moments conditional on $s$. We will use “hats” and “tildes” to denote moments conditional on $s$ and $q$ respectively. (For example, $\hat{E}_t := E(f | s)$ and $\tilde{V}_t := \text{Var}(f | q_t).$) Let $r := (\sum_{i=1}^{n} r_i^{-1})^{-1}$ be the harmonic mean of the risk aversion coefficients of agents, and $\xi := \sum_{i=1}^{n} \rho_i$ be the aggregate endowment. Using (11) and (13), we can show the following:

**Proposition 1.** There exists a fully revealing equilibrium with the price function given by

$$q = \hat{E}_t - r \tilde{V}_{t\xi}. \hspace{2cm} (14)$$

Equilibrium prices have the standard CAPM form. Given our normality assumptions, the risk premia are nonstochastic. The case of no trading on information, discussed in the security design context in an earlier survey by Duffie [40], can be seen as a special case wherein conditional moments are replaced by the corresponding unconditional moments.

3.2.2. Partially Revealing Equilibria

Here we explore a particular variation of the basic setup that yields partially revealing equilibria. This kind of model is studied by Bhattacharya et al. [19] and Rahi [91].
There are two agents \((n = 2)\). Agent 1 is the informed agent (the "insider"). She observes an \(l\)-vector of signals \(s \sim N(0, V_s)\). Furthermore, \(x_i = \Gamma s\), for some non-zero vector \(\Gamma\) in \(\mathbb{R}^l\), and \(p_1\), also of mean zero, is independent of \(s\). Thus the insider's endowment \(e_1\) is the product of two independent random variables \(x_1\) and \(p_1\) in \(\mathcal{V}\); where \(p_1\) can be interpreted as the normalized value of the endowment, about which no information is available at the time of trading; and \(x_1\) can be viewed as a scale parameter whose value can be perfectly inferred given the insider's private information. Agent 2 (the "outsider") is an uninformed agent and has no hedging motive \((x_2 = 0)\). Then \(Y_2 = q\). In the rest of this subsection, we will drop the subscripts of \(x_1\) and \(p_1\) to lighten the notation. No confusion should arise, since we will be speaking only of the insider's endowment.

Let \(y := (y_1, ..., y_{l-1})\) be an orthonormal basis for the orthogonal complement of \(x\) in the linear subspace of \(\mathcal{V}\) spanned by \(s_1, ..., s_l\). Then, any asset payoff vector \(f \in \mathcal{V}^m\) can be written in the form

\[
    f = \bar{f} + a p + b y + Cy + De,
\]

where \(\bar{f}, a, b, C, D\) are coefficient vectors or matrices of the appropriate dimension; and \(e \sim N(0, I_m)\) is independent of \((p, x, y)\) (or, equivalently, of \((p, s)\)). The distributional assumptions can be summarized as

\[
    (p, x, y, e) \sim N[0, \text{diag}(V_p, V_x, I_{(l-1)+m})].
\]

It is convenient to refer to \(y\) as the extraneous private information of the insider (that is, information unrelated to her own endowment), and to \(e\) as the extraneous noise in the asset payoffs.

We are interested in linear equilibria of the form

\[
    q = \bar{q} + \Delta \Phi f.
\]

The price function is thus restricted to be measurable with respect to the informed agent's asset demand function. It must also be consistent with market clearing ((13)). Since the outsider's asset demand \(\Phi_2\) depends only on the price vector \(q\), this condition implies that the coefficient matrix \(\Delta\) is nonsingular.

The insider is assumed to behave monopolistically, taking into account the impact of her asset demands on equilibrium prices. Given (16) and the fact that \(x\) is \(s\)-measurable, \(Y_i = s\). Using (10) and (16), we can derive the insider's demand function (as before, we use "hats" to indicate moments conditional on \(s\)):

\[
    \Phi_1 = M^{-1}[\hat{E}_t - \hat{q} - r_1 x \hat{V}_{pt}],
\]

\[
    (17)
\]
where
\[ \mathbf{M} := r_1 \mathbf{\hat{V}}_f + \Lambda + \Lambda^T, \]
provided \( \mathbf{M} \) is positive definite. Note that a necessary condition for the insider’s maximization problem to have a solution is that \( \mathbf{M} \) be positive semidefinite. Generically,\(^{20}\) however, \( \mathbf{M} \) is nonsingular, so that, for a generic subset of economies, positive definiteness of \( \mathbf{M} \) is both necessary and sufficient for the demand function to be well defined.

The outsider is a price-taker (he can be thought of as a representative agent for a large number of uninformed investors). His demand function is easily inferred from (11) (recall that “tildes” denote moments conditional on \( \mathbf{q} \)):
\[ \Phi_2 = \frac{1}{r_2} \mathbf{\hat{V}}_f^{-1}(\mathbf{\hat{E}}_f - \mathbf{q}), \tag{18} \]
provided that \( \mathbf{\hat{V}}_f \) is nonsingular. This will generically be satisfied. Consequently, one may conjecture that, generically, an equilibrium exists if and only if \( \mathbf{M} \) is positive definite. This is indeed the case, as we will see shortly.

Since \( \Lambda \) is nonsingular, observing prices \( \mathbf{q} \) is equivalent to observing the insider’s demand \( \Phi_1 \). Define
\[ \tau := \mathbf{\hat{E}}_f - \mathbf{\hat{f}} - r_1 x \mathbf{\hat{V}}_f. \tag{19} \]
Then, from (17),
\[ \tau = \mathbf{\hat{q}} - \mathbf{\hat{f}} + \mathbf{M} \Phi_1, \tag{20} \]
so that, if \( \mathbf{M} \) is positive definite, observing \( \mathbf{q} \) is in fact equivalent to observing \( \tau \). The signal \( \tau \) that the outsider receives does not fully reveal the insider’s information in general, because of the latter’s unobserved hedging demand which depends on \( x \) (compare (19) with the signal (14) in the fully revealing case).

Since moments with respect to \( \mathbf{q} \) are the same as moments with respect to \( \tau \), we can use (18) and (20) to calculate the outsider’s demand function in terms of \( \Phi_1 \) and \( \mathbf{q} \). Then, invoking the market clearing condition, we can solve for \( \mathbf{q} \). Finally comparing coefficients with (16), we can show that
\[ r_1 \mathbf{\hat{V}}_f + \Lambda + \Lambda^T = \mathbf{U}^{-1}(\Lambda - r_2 \mathbf{\hat{V}}_f), \tag{21} \]
\(^{20}\)We parametrize the economy by the risk aversion coefficients of agents and the coefficients of the asset payoff vector (15). “Generically” means “except for a closed subset of Lebesgue measure zero” in the appropriate Euclidean space.
where \( U := V, V_{-1} \) is the outsider's "update matrix" or "matrix of regression coefficients" (see, for example, Anderson [11, Chap. 1]). Generically, a linear equilibrium exists if and only if (21) has a nonsingular solution \( \Delta \) such that both sides of the equation are positive definite. This problem is studied by Bhattacharya et al. [19]. They derive conditions on \( U \) under which a linear equilibrium exists. In fact, they show that these are precisely the conditions under which there exists any equilibrium, linear or not.

**Proposition 2.** Generically, an equilibrium exists if and only if every eigenvalue of \( U \) is less than 1/2. Furthermore, this is precisely when a linear equilibrium exists as well.

The eigenvalue condition can be interpreted as saying that an equilibrium fails to exist when the insider's motive for trade is primarily informational. This can be seen most clearly in the single-security case, to be discussed shortly. Proposition 2 provides a necessary and sufficient condition for the viability of an asset structure. Perforce, this condition must be satisfied by any endogenously determined collection of assets. Bhattacharya et al. [19] do not study the security design problem, however. To do that, in fact, Proposition 2 needs to be sharpened, since it only holds for a (generic) subset of assets. This can be done when there is only one asset, as was shown by Bhattacharya and Spiegel [18] in an earlier paper. For this case (15) may be written as

\[
f = \tilde{f} + ap + bx + c k^tx + de,
\]

where \((\tilde{f}, a, b, c, d) \in \mathbb{R}^5 \) and \( k \in \mathbb{R}^{t-1} \). An equilibrium is nontrivial if it entails a non-zero amount of trade. The following result appears in Rahi [91]:

**Proposition 3.** A nontrivial equilibrium exists if and only if

\[
r_1^2 a^2 V_s V_x^2 > b^2 V_s + c^2 k^T k.
\]

Given (22), there is a unique linear equilibrium.

We will refer to a situation in which no trade takes place, either because an equilibrium does not exist or because the equilibrium is trivial, as a market breakdown, following Bhattacharya and Spiegel [18]. The above proposition shows that there is a market breakdown if and only if condition (22) is violated. This happens when the informational motive of the insider, as measured by the right-hand side of (22), is too strong relative to her hedging motive, measured by the left-hand side. The implications for security design will be discussed in Subsection 3.4.2.
3.3. Efficiency Characterizations

We now revert to the perspective of a variable financial structure, albeit keeping the number of assets fixed and restricting their payoffs to be joint normal with endowments and signals. This subsection deals with welfare issues. We use a notion of efficiency proposed by Duffie and Jackson [41]. A financial structure is constrained efficient if there is no other collection of assets that leads to a Pareto dominating allocation in equilibrium. Stronger efficiency concepts have been employed by Demange and Laroque [36] and Hara [59]. An allocation is efficient in Hara's sense if it cannot be Pareto dominated by an allocation obtainable in a competitive equilibrium for an alternative financial structure, and supplemented by riskless wealth transfers. Demange and Laroque allow redistribution of all assets, so that their notion of constrained efficiency is the strongest, and the one commonly found in the general equilibrium literature (see Section 2).

Explicit characterizations of constrained efficiency are available for the case in which prices are fully revealing with respect to asset payoffs. For convenience of exposition, we will assume that welfare weights are non-zero for every agent, and that there are no redundant assets \( m \leq n \). To avoid double subscripts, we will use only \( i \) for \( p_i \), when the latter appears as a subscript; for example, \( V_{r_i} := \text{cov}(p_i, g) \).

3.3.1. Pure Exchange

Let us first consider the benchmark case discussed in Subsection 3.2.1. The vector of risky endowments of the agents is denoted by \( p \sim \mathcal{N}(\bar{p}, V_p) \). Using Proposition 1, (11) gives us agent \( i \)'s asset demand,

\[
\Phi_i = \tilde{V}_f^{-1} \left( \frac{r_i}{\tilde{r}_i} \tilde{V}_t - \tilde{V}_{rt} \right),
\]

which is nonstochastic in our Gaussian setting. Also, from (8), (9), and (12), agent \( i \)'s ex ante expected utility in equilibrium (which we will henceforth refer to as agent \( i \)'s equilibrium utility) is monotonically increasing in \( \Phi_i^\top \tilde{V}_f \Phi_i \). A constrained efficient financial structure maximizes a weighted average of equilibrium utilities of agents. (It can be shown that the set of equilibrium utilities is convex.) This observation leads to the following characterization\(^\dagger\):

**Proposition 4.** A financial structure \( f \) is constrained efficient if and only if it is of the form

\[
f = f + A(p - \bar{p}) + B_\xi,
\]

\(^\dagger\) For a multiperiod analogue of this result, see Ohashi [88]
where the rows of $\Lambda$ span the subspace generated by eigenvectors corresponding to the $m$ largest eigenvalues$^{22}$ of

$$
\mathbf{V}_p^{-1} \left[ \sum_{i=1}^n \lambda_i \left( \frac{\mathbf{r}_i}{\mathbf{V}_p} - \mathbf{y}_p \right) \left( \frac{\mathbf{r}_i}{\mathbf{V}_p} - \mathbf{y}_p \right)^T \right],
$$

for some positive weights $\lambda_1, \ldots, \lambda_n$.

The proposition can be proved using techniques in Rahi [92]. Since the financial structure is assumed to be in the Gaussian class, the form (23) simply means that a constrained efficient financial structure does not contain any extraneous noise. A complementary result appears in Demange and Laroque [36]. For general concave financial utility functions, they show that eliminating noise expands the utility possibility set (allowing arbitrary redistributions of assets). In fact, the eigenvector result above is very similar to Propositions 2.2 and 2.3 in the Demange–Laroque paper,$^{23}$ even though their concept of efficiency is stronger, and they have no trading on information. Clearly, in the fully revealing case there is no role for information in affecting the optimal financial structure. This is not true, however, if private information is relevant to real investment decisions, which leads us to consider an economy with production.

3.3.2. Production

Rahi [92] provides a characterization of constrained efficiency for a production economy. It fits into the basic setup described in Subsection 3.1 as follows. The $n$ agents are producers. Producer $i$ chooses a level of investment $x_i$. The stochastic return per unit investment is $p_i$. The vector of risky returns is $\mathbf{p} \sim N(\bar{\mathbf{p}}, \mathbf{V}_p)$. The assets are futures contracts which are priced in a risk neutral fashion:

$$
\mathbf{q} = E(\mathbf{f} \mid \mathbf{s}).
$$

(24)

This may be interpreted as the result of trading by risk neutral speculators who have inside information about production returns, represented by $s \in \mathcal{A}^t$. Alternatively, we may think of prices being set by competitive, risk neutral, informed market makers. Producers have no private information but learn from prices: $\mathbf{y}_p = \mathbf{q}$. The price function (24) implies that in equilibrium there is no asymmetric information regarding the futures contracts being traded. However, if $l > m$, futures prices do not fully reveal all the information relevant to investment decisions.

---

$^{22}$ If an eigenvalue $\lambda$ has multiplicity $\ell$, we simply think of these as distinct eigenvalues, taking $\ell$ linearly independent eigenvectors corresponding to $\lambda$.

$^{23}$ Their results are dressed up differently, being expressed in terms of coordinates with respect to an orthonormal basis for the span of agents' endowments.
Using (24) and (11), and the fact that the equilibrium is fully revealing with respect to asset payoffs, the optimal futures position of producer $i$ is

$$
\Phi_i = -x_i \hat{V}_f^{-1} \hat{v}_{if}.
$$

(25)

Substituting this into the agent's utility expression (12), we can solve for the optimal investment level

$$
x_i = \frac{\hat{E}_i}{r_i V_i (1 - R_{iq}^2 - R_{i-f}^2)}
$$

(26)

where

$$
R_{iq}^2 := \frac{1}{V_i} V_{iq} V_{iq}^{-1} V_{iq}
$$

is the square of the multiple correlation coefficient between $p_i$ and the random vector $g$. It turns out that $R_{iq}^2$ and $R_{i-f}^2$ have the natural interpretation of being measures respectively of the quality of information and hedging of futures contracts (from the point of view of producer $i$). Rahi [92] shows that a producer's equilibrium utility is monotonically increasing in hedging and informational quality. A financial structure is called hedging-efficient (respectively informationally efficient) if there is no other financial structure that provides higher hedging (respectively informational) quality for at least one producer without lowering it for others. The following result extends Proposition 4 to the case of a production economy:

**Proposition 5.** A constrained efficient financial structure is both hedging-efficient and informationally efficient. A financial structure $f$ is hedging-efficient if and only if it is of the form (23), and the rows of $A$ span the subspace generated by eigenvectors corresponding to the $m$ largest eigenvalues of

$$
\hat{V}_f^{-1} \left[ \sum_{j=1}^{n} \gamma_j \hat{V}_{ip} \hat{V}_{ip}^\top \right],
$$

for some positive weights $\gamma_1, ..., \gamma_n$. A financial structure $f$ is informationally efficient if and only if the vectors $\text{cov}(f_j, s)$, $j = 1, ..., m$, span the subspace generated by eigenvectors corresponding to the $m$ largest eigenvalues of

$$
\left[ \sum_{i=1}^{n} \delta_i \hat{V}_{ia} \hat{V}_{ia}^\top \right] \hat{V}_a^{-1},
$$

for some positive weights $\delta_1, ..., \delta_n$. 
What is noteworthy about this proposition is the separation between the risk-sharing and information-transmission roles of the assets. This is essentially a consequence of the fact that equilibrium prices are fully revealing with respect to asset payoffs. Referring to (23), hedging quality is independent of \( \mathbf{B} \). The matrix can therefore be chosen to control informational quality independently of hedging quality.

3.4. Security Design

In this subsection we review the positive theory of security design in the exponential-normal framework. There are two strands of the literature depending on who designs the securities. We will first discuss the case of futures contracts introduced by exchanges. Then we will analyze optimal security design from the point of view of a risk averse entrepreneur. In both cases we will assume that the designer knows the mapping from financial structures to equilibrium allocations.\(^{24}\)

3.4.1. Futures Innovation

Our point of departure is Duffie and Jackson [41] (henceforth \( \text{D–J} \)), in which futures exchanges choose contracts to maximize trading volume. In our setting, the D–J model essentially corresponds to the pure-exchange, fully revealing case. D–J show that a volume-maximizing contract for an exchange is such that (roughly speaking) the contract payoff is perfectly correlated with the unhedged endowment risk. A synopsis can be found in the Duffie survey [40]. In practice, liquidity appears to be an important factor in determining the popularity of a futures contract. Cuny [35] has a model to this effect. Hedgers are concerned not only with the hedging quality of contracts but also with the price impact of altering their positions. Exchanges maximize revenue from the sale of seats (right to trade) to speculators. Cuny obtains a maximal eigenvector characterization of futures innovation (cf. Subsection 3.3.3) which shows how exchanges account for both hedging and liquidity in choosing their contracts. For a summary of these findings, see Duffie [40].

In the pure-exchange, fully revealing case, if we use the normalization \( \bar{\mathbf{V}}_{\text{f}} = 1 \) for the asset payoffs, the equilibrium utility of agent \( i \) is increasing in the sum of her squared asset positions, \( \mathbf{\Phi}^T \mathbf{\Phi} \), (see Subsection 3.3.1.). Therefore, we have the following:

**Proposition 6.** A set of futures contracts that maximizes the sum of squared trading volume, \( \sum_{i=1}^{n} \mathbf{\Phi}_i^T \mathbf{\Phi}_i \), is constrained efficient.

\(^{24}\) In either case, a necessary condition for a set of securities to be innovated is that some of these securities are traded with positive probability. Ohashi [86] provides a characterization and several examples.
A straightforward corollary is D-J's efficiency result for the case of a single contract \((m = 1)\) chosen by a volume-maximizing exchange.\(^{25}\)\(^{26}\) In an oligopolistic situation with several competing exchanges, however, inefficiencies may arise due to coordination failure, as is shown by D-J in an example. One way to implement a constrained efficient financial structure with many assets has been suggested by Ohashi [85]. If a single monopolistic exchange creates all the contracts and charges a quadratic transactions fee, then fee revenue maximization approximates the maximization of the sum of squared trading volume as the fee tends to 0.

In a model of commission revenue maximization based on the D-J framework, Hara [60] shows that an exchange simultaneously choosing several different futures contracts may have an incentive to increase its total commission revenue by designing securities that are arbitrarily close to having a redundancy. Specifically, provided the number of securities to be innovated is sufficiently large, the supremum of the equilibrium commission revenue, over all possible proportional commission fees and choices of a fixed number of securities, need not be achieved. Instead, the exchange can come arbitrarily close to the supremum by choosing a sequence of sets of securities, and a decreasing sequence of monopolistic commission fees, such that consumers are induced to buy portfolios of the securities made up of unboundedly large short and long positions, as the security payoffs approach linear dependency. Hara provides a concrete example of this phenomenon and gives a proof that the design problem does have a solution in this setting if the number of securities to be innovated is no larger than 2.

Tashjian and Weissman [100] extend the D-J framework to consider endogenously determined transactions fees with multiple contract design, and show that this may lead to a strict preference by an exchange for correlated contract payoffs. As an example, they provide an empirical analysis of the complex of correlated futures contracts related to soybeans.

Rahi [92] (see Subsection 3.3.2) extends the D-J model to the case of asymmetric information and production. We can see from (25) and (26) that producers in Rahi's model trade futures (and invest) more aggressively the better are the hedging and informational qualities of these contracts. The most transparent case is that in which a monopolistic exchange maximizes a weighted average of the expected trading volumes in its contracts. Since the exchange can control the informational quality of futures markets independently of their hedging effectiveness, we have:

\(^{25}\)Hara [59] shows that, relative to his stronger definition of efficiency, a volume-maximizing contract is inefficient for an open and dense subset of economies parametrized by agents' risk aversion coefficients.

\(^{26}\)This efficiency result does not carry over to a multiperiod setting. D-J give an example. See also Ohashi [88].
PROPOSITION 7. A volume-maximizing monopolistic futures exchange chooses an informationally efficient financial structure.

Rahi [92] also provides sufficient conditions for informational efficiency in the oligopolistic setting. Corresponding conditions for hedging-efficiency are more difficult to derive. In light of our discussion of the pure-exchange case, however, hedging-efficiency is unlikely to hold except in special circumstances. Moreover, hedging- and informational efficiency are not sufficient for constrained efficiency. We may conclude that decentralized contract choice by volume-maximizing futures exchanges is not constrained efficient, in general.

Thus far we have only treated the case in which equilibrium is fully revealing with respect to asset payoffs. A characterization of volume-maximizing contracts when there is partial revelation is not yet available. A first step in this direction is taken by Ohashi [87]. Ohashi's model does not fit in our framework since he employs noise traders to obtain partially revealing equilibria. However, a look at an investor's optimal asset position (11) reveals that his trading is motivated by hedging considerations (captured by the covariance term) and by speculation (arising from differences in his own expectation of the asset payoffs, \( E(f \mid Y_t) \), and the price vector, \( q \)). This suggests that an exchange may have an incentive to design an asset structure which reveals less information, in order to encourage speculative trade. Ohashi analyzes the choice between two futures contracts and a single index contract (which is a linear combination of the two) and finds that, if investors have accurate private information on different sources of uncertainty, and their hedging needs are small, the exchange prefers the index contract. The increase in hedging-related trade from introducing two contracts is more than offset by the loss in volume due to the symmetrization of investors information in equilibrium. An important insight that emerges from this is that asymmetric information among investors may lead to an incomplete set of futures markets, even if futures contracts can be created and enforced costlessly.

3.4.2. Security Design by an Entrepreneur

We now turn to the case in which the security designer is a risk averse entrepreneur who anticipates that she will have inside information at the time of trading on the asset market. She chooses a financial structure that maximizes her \textit{ex ante} expected utility in equilibrium (equilibrium utility, for short). This problem is studied by Rahi [91] and Demange and Laroque [37]. We are interested here only in risk-sharing issues and the role of prices in revealing information. Other aspects of entrepreneurial and corporate security design, involving signaling through choice of capital structure, or motivated by agency costs and corporate
control considerations, have been extensively explored in the literature. These developments are surveyed in Harris and Raviv [61, 62].

Rahi [91] employs a single-security version of the adverse selection model of Bhattacharya et al. [19] (see Subsection 3.2.2). The entrepreneur issues an asset which she subsequently trades with a rational outside investor. She behaves monopolistically in the asset market. For a given asset, the equilibrium is described in Proposition 3. Clearly, it does not pay the insider to design a security that is too sensitive to her private information, since this results in a market breakdown. Rahi obtains a closed-form solution for the equilibrium utility of the entrepreneur in terms of the asset payoff parameters and finds that the adverse selection problem she faces is in fact more severe than Proposition 3 suggests:

**Proposition 8.** The entrepreneur's equilibrium utility $\mathcal{U}$ is monotonically decreasing in the weights that the asset payoff assigns to extraneous private information and extraneous noise. Specifically,

$$\frac{\partial \mathcal{U}}{\partial (c^2)} \leq 0 \quad \text{and} \quad \frac{\partial \mathcal{U}}{\partial (d^2)} \leq 0.$$ 

The derivatives are 0 if and only if there is a market breakdown (condition (22) is violated).

Thus it is optimal to issue an asset for which the equilibrium is fully revealing. The following is a complete characterization:

**Proposition 9.** The class of assets that are optimal for the entrepreneur is given by the set

$$\{ f = \tilde{f} + a p \mid (\tilde{f}, a) \in \mathbb{R}^2, a \neq 0 \}.$$ 

Designing an asset in this class is equivalent to issuing equity in the entrepreneur's payoff $e$. In equilibrium, a constant proportion $2r_2/(r_1 + 2r_2)$ of $e$ is retained by the entrepreneur, the rest being held by the outsider.

The entrepreneur does no better than just issuing equity, which is what she would have done in the absence of any private information. Rahi goes on to study the case in which the insider is a price-taker in the asset market and finds that the optimal security is still equity.

Demange and Laroque [37], on the other hand, find that the tradeoff between exploiting superior information and paying a higher risk premium to outside investors (the "lemons" cost) has no clear resolution. The crucial difference in their model is the presence of noise traders who serve to camouflage the insider's information from rational outsiders. Since noise
traders are not rational, no lemons cost is incurred in trading with them. Consequently, the entrepreneur typically prefers an asset that affords her some informational advantage.

4. Other Literature

Certain strands of the literature are directed at specific types of financial innovation. For example, Aziz et al. [13] and Kanemasu et al. [68] study the incentive to strip bonds into securities formed by the component coupons and principal. Gorton and Pennacchi [56], on the other hand, examine the incentives to issue composite securities made up of separately traded components, such as index-based security baskets and closed-end mutual funds. In their analysis of the pricing efficiency of Primes and Scores, Jarrow and O'Hara [66] provide a discussion of the innovation of these securities.

A reasonably extensive literature studies the empirical impact of the innovation of derivative securities on the volatility of the price of the underlying asset. In many cases, increased volatility is taken as a symptom of reduced social welfare, although this link is rarely explored. Zapatero [108] provides a theoretical example and cites some of the other relevant literature.

Townsend [101] has one of the first demonstrations of the optimality of standard debt contracts. In his model, the reason for issuing standard debt is costly state verification. Beginning with Myers [80] and Myers and Majluf [81], a large body of literature has focused on the use of standard debt contracts to mitigate the impact of a lemons premium stemming from the adverse selection that arises when the seller of the issue has better information about the prospects of the firm than the buyer. Recent theoretical justifications include those of DeMarzo and Duffie [38] and Nachman and Noe [82, 83]. A broader issue is the optimality of a “pecking order” (see Myers [80]) for corporate debt, under which a firm is financed as equity, senior (first priority) debt, and subsequent tiers of hierarchically prioritized junior debt.

Anderson and Sundaresan [10] study the implications of bankruptcy and taxes for the design of corporate debt covenants. Freeman and Tabellini [49] provide conditions under which, for risk-sharing purposes, it is optimal to have nominal as opposed to inflation-indexed securities.

Chichilinisky and Wu [33] describe a setting in which several layers of hierarchical securities may be required to insure against default. The first layer consists of contingent claims, as usual. A second-layer security is a contract providing insurance for the event of default on the payment due on a first-layer security. Third-layer securities provide insurance against
default of second-layer securities, and so on. The resulting limit set of securities need not, in general, generate complete markets.

There is a closely related literature on the design of bilateral financial contracts, not necessarily in the form of traded securities. Examples include Aghion and Bolton [2], Barnea et al. [16], Gollier [54], and Hart and Holmstrom [64]. The distinction between traded financial securities and private bilateral contracts has grown rather weak. For example, many OTC derivatives are thought of as traded securities, as they are widely quoted, relatively liquid, and homogeneously defined. In fact, since such derivatives are normally contracts between two specific parties with tightly circumscribed opportunities for retrading, they are more in the nature of private contracts and are legally treated as such.

General discussions of the role and history of financial innovation can be found in Finnerty [46], Miller [79], Ross [93], Silber [98], Van Horne [104], and Mason et al. [109].

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