# Black, Merton, and Scholes — Their Central Contributions to Economics

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[1]My deep admiration of Fischer Black, Bob Merton, and Myron Scholes has been for qualities that go well beyond those evident in their exceptional research contributions. I am grateful to Julie Sundqvist of the *Scandinavian Journal of Economics* for editorial guidance, and to Linda Bethel and Aileen Lee for technical assistance. Mail: Stanford, California, 94305-5015, USA; Phone: 650-723-1976; Email: duffie@baht.stanford.edu

# 1 Introduction

I will briefly summarize the central contributions to economics of Fischer Black, Robert C. Merton, and Myron S. Scholes.

Of course, the contribution that first comes to mind is the Black-Scholes option pricing formula, for which Robert Merton and Myron Scholes were awarded the Alfred Nobel Memorial Prize in Economic Sciences in 1997. I have no doubt that, because of his key role in that far-reaching formula, Fischer Black would have shared in that prize but for his recent untimely death, so I will depart from the usual convention for essays that appear in this journal on these occasions, and address the contributions of all three of these exceptional economists simultaneously, rather than giving separate treatment to Fischer Black. My goal is to give an objective and concise account of their path-breaking research and what it has offered to the theory and practice of economics.

# 2 Setting the Stage

Finance is a large, richly interwoven, widely applied, and extremely active area of economics. One of the central issues within finance is the valuation of future cash flows. While there are important alternatives, a current basic paradigm for valuation, in both academia and in practice, is that of competitive market equilibrium: The price that will apply in the market is that price which, taken as given by market participants, equates total demand to total supply.

With this year's award to Robert Merton and Myron Scholes, three fundamental contributions to the theory of financial valuation that are based on this paradigm of market equilibrium have now been closely linked to Nobel prizes. These are:

1. The portion of the Modigliani-Miller (1958) theory that deals with the irrelevance of capital structure for the market value of a corporation.

2. The Capital Asset Pricing Model (CAPM) of William Sharpe[2] (1964).

3. The Black-Scholes (1973) option pricing theory.

[2] The work of John Lintner (1965) is also often cited.

The first and third of these contributions rely on the notion of market equilibrium in only the weakest possible sense, known as "arbitrage reasoning." If, under their respective assumptions, the valuation formulas provided by these theories were not correct, then market participants would have an opportunity to create an "arbitrage," that is, to trade securities so as to make unbounded profits with no initial investment and no subsequent risk of loss. In particular, if the market price of a financial security were lower than suggested by arbitrage reasoning, arbitrageurs would ask to buy it, and in unbounded quantities. Conversely, if the market price were higher than suggested by theory, arbitrageurs would want to sell, and the more the better. In such situations, markets could not clear, and equilibrium would be impossible. Such "arbitrages" are only prevented, in theory, when the proposed valuation formulas actually apply.

While there are some close precursors in the literature, Modigliani and Miller (1958) essentially established the modern foundation in finance for arbitrage-based valuation reasoning. The Black-Scholes theory provided an extremely powerful extension of arbitrage modeling to dynamic settings.

The assumptions of any model rarely (if ever) apply literally. What might be an arbitrage in theory is sometimes difficult to carry out in practice. For example, arbitrage-based valuation models often rely on the assumptions of perfect information and the absence of transactions costs. No-arbitrage arguments are so compelling, however, that financial economists encounter almost daily reference to the Modigliani-Miller and Black-Scholes theories as central points of departure for model building or reasoned discussion of financial problems.

As shall be discussed below, even though the CAPM does not rely on arbitrage reasoning, it also played a key role in the development of the Black-Scholes formula.

# 3 Arbitrage Pricing of Options

Before getting to the focal point of our story, the Black-Scholes formula, it will be useful for readers that are newcomers to finance or unfamiliar with stochastic calculus to see the basic idea of arbitrage-based option pricing in the simplest possible setting. Ironically, this simple introductory model was only developed, by William F. Sharpe, *after* the advent of the Black-Scholes model.

Consider a financial security, say a traded stock, whose price today is 100 and whose price tomorrow will be either 102 or 98. Consider an option that grants its owner the right to purchase the stock tomorrow for 100. If the stock price tomorrow turns out to be 102, the owner of the option will (as we assume rationality and no transactions costs), exercise the right to buy for 100, and thereby benefit from exercising the option to the extent of a cash flow of $102 - 100 = 2$. If, on the other hand, the stock price turns out to be 98 tomorrow, the owner of the option will decline the opportunity to buy at 100, and the option has no cash flow in that event.

Suppose, to keep the numbers simple, that the overnight interest rate is zero. At another interest rate, the following arguments would apply with slightly different numbers.

We claim that, in the absence of arbitrage, the price of the option today is 1. How can one be so precise in the absence of any additional information? Is there no role in this for the risk preferences of market participants or the probabilities that they assign to the event that the stock price goes up? Let's delay an answer to these questions for now.

Before directly addressing the arbitrage valuation, let us first find the number $a$ of shares of stock to buy and the amount $b$ to borrow so that, whether the stock price goes up or down, the net proceeds of the stock portfolio with loan repayment is equal to the cash flow from owning one option. This means that $a$ and $b$ must solve

$$102a - b = 2 \quad (1)$$

$$98a - b = 0. \quad (2)$$

The solution is $a = 0.5$ and $b = 49$. The net initial cost of this option-replicating portfolio is $100a - b = 50 - 49 = 1$. It seems unlikley that brokers would quote a price other than 1 for the option if one can make a "synthetic" version of its cash flows for 1.

In order to substantiate this claim, a simple proof by contradiction will serve. Suppose that the option were actually trading at a price of $p > 1$. If this were true, an arbitrageuer could sell the option for $p$, and replicate, at an initial cost of 1, the option's future cash flows by purchasing 0.5 shares of stock and borrowing 49. The net payoff tomorrow of the replicating portfolio meets any cash flow demanded tomorrow by the purchaser of the option, whether the stock price goes up or down. The arbitrageur has netted an initial gain of $p - 1 > 0$ with no investment and no risk. This, however, is an arbitrage! So, we must have $p \leq 1$. If, however, $p < 1$, then buying the option and selling the option-replicating portfolio (that is, short-selling 0.5 shares of stock and lending 49) constitutes an arbitrage. Thus $p = 1$ is necessary for the absence of arbitrage. One can easily check that it is sufficient for no arbitrage that $p = 1$.

Those new to this could test their understanding by solving the option valuation problem for a non-zero interest rate. (A simple daily interest rate below $-2$ percent or above 2 percent will not work. Why?)

Before we get to the actual Black-Scholes formula, let us revisit the role in this simple option-pricing example of the risk preferences of investors, and the probabilities that they may assign to positive or negative stock returns. A naive objection might be: How could the initial option price be as little as 1, for example, if there is a 99 percent chance that the stock return will be positive (in which case the option pays 2) and there are investors that are relatively close to indifferent about bearing risk? The natural, and correct, reply is that preferences as well as beliefs about the likelihoods of two states *do* indeed play a role, because they determine the initial price of the stock and the interest rate. If, for example, an investor that is close to risk-neutral believes that it is virtually certain that the stock price will be 102 tomorrow, then the initial price of the stock must be close to 102 today, not 100. As the stock price and interest rate vary with the preferences and beliefs of investors, so will the option price.

# 4 The Black-Scholes Formula

Now we are ready to see how the Black-Scholes formula works.

In the Black-Scholes model, the price of a security, say a stock, is assumed to be given at any time $t \geq 0$ by $X_t = x \exp(\alpha t + \sigma B_t)$, where $x > 0$, $\alpha$ and $\sigma > 0$ are constants, and where $B$ is a standard Brownian motion.[3] Riskless

[3]Underlying the model is a probability space. On this space, $B_0 = 0$, $B_t - B_s$ is normally distributed with zero expectation and variance $|t - s|$, and the increments of $B$ are independently distributed. Except for technicalities that can be found, for example,

borrowing and lending is possible at the constant continuously compounding interest rate $r$. For future reference, $x$ is the initial stock price, $\sigma$ is referred to as the *volatility* of the stock, and, because $E(X_t) = e^{(\alpha+\sigma^2/2)t}$, we call $\mu \equiv \alpha + \sigma^2/2$ the expected rate of return on the stock.

Consider an option that grants its owner the right, but not the obligation, to buy the stock at a given exercise date $T$, and at a given exercise price $K$. Trading is permitted at arbitrary frequency and there are no transactions costs. The information available to investors at any time $t$ is the history of the stock price up to that time. Certain minor technical assumptions apply.

Now, at what price will the option be sold, assuming that there are no arbitrages? For purposes of future reference, let

$$C(x, r, \mu, T, \sigma, K) = e^{-rT} E[\max(X_T - K), 0)] \tag{3}$$

denote the expected discounted payoff of the option, for given parameters $(x, r, \mu, T, \sigma, K)$. This is *not*, in general, the price of the option. Sprenkle (1961), in effect, showed that

$$C(x, r, \mu, T, \sigma, K) = xe^{(\mu-r)T} N(d(x, \mu, T)) - e^{-rT} K\, N(d(x, \mu, T) - \sigma\sqrt{T}), \tag{4}$$

where $N(\,\cdot\,)$ is the cumulative standard-normal distribution function and, for any $(x, y, T)$,

$$d(x, y, T) = \frac{\log(x/K) + (y + \sigma^2/2)T}{\sigma\sqrt{T}}.$$

Stochastic calculus[4] can be used to show the following fact: One can invest a total of $C(x, r, r, T, \sigma, K)$ at time 0 and, at each time $t$ between 0 and $T$, hold $N(d(X_t, r, T - t))$ shares of the stock, always borrowing or lending cash flows as necessary to finance the position between 0 and $T$, and be left at time $T$ with a position in cash and stock whose market value is exactly $\max(X_T - K, 0)$, the payoff of the option. (This is analogous to the replication strategy shown in Section 3.)

From the definition (3) of $C(\,\cdot\,)$, this initial cost $C(x, r, r, T, \sigma, K)$ of replicating the option, called the Black-Scholes option pricing formula, would be

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in Karatzas and Shreve (1988), these properties define a standard Brownian motion. It is noteworthy that Brownian motion was first given an effective definition by Bachelier (1900) in a study of security price behavior that included an option pricing formula not entirely unlike that of Black and Scholes.

[4] Karatzas and Shreve (1988) offer a good textbook treatment of stochastic calculus.

the expected discounted payoff of the option, *if the mean rate of the return of the stock were the riskless rate $r$.*

Using the same logic as in Section 3, assuming that there are no arbitrages, the option must trade in the market for its initial replication cost, $C(x, r, r, T, \sigma, K)$. If the option were selling for some amount $p$ strictly larger than $C(x, r, r, T, \sigma, K)$, then one could sell the option, invest in the replicating strategy, and take away an initial riskless profit of $p - C(x, r, r, T, \sigma, K)$. The net cash flow at expiration is zero since the payoff of the replicating strategy precisely covers the claim against the option. This would be an arbitrage. Conversely, if $p < C(x, r, r, T, \sigma, K)$, the opposite strategy of buying the option and selling the replicating strategy is an arbitrage. Indeed then, the arbitrage-free price of the option is $C(x, r, r, T, \sigma, K)$, which is familiar from (4) as the Black-Scholes option pricing formula.

As far as the sufficiency of the Black-Scholes formula for the absence of arbitrage, one must place only some reasonable limits on the class of allowable trading strategies. For example, as shown by Dybvig and Huang (1988), it is enough to insist that an investor should not be given unlimited credit.

# 5 History of the Black-Scholes Formula

The best two available written sources on the history of the development of the formula are Black (1989) and Bernstein (1992), the latter being based on extensive interviews of those involved. The accounts given in these two sources are consistent with each other, with other published sources including the published form of the original paper by Black and Scholes (1973) presenting the formula, and with what has been told to me anecdotally. The story goes roughly as follows.

Fischer Black, one of many who had looked at this problem,[5] began with the idea of applying the Capital Asset Pricing Model at each instant of time, for investments over an infinitesimally small period of time. This allowed him to derive a partial differential equation (PDE) for the option price $c(x, t)$ that

[5]Those who had attacked some version of the problem prior to Black and Scholes included Bachelier (1900), Sprenkle (1961), Samuelson (1965), and Samuelson and Merton (1969), who, in effect, derived the reservation price of an investor with a particular utility function.

would apply at any time $t < T$ and at any stock price $x$ for that time. This, now famous, PDE is

$$c_t(x,t) + c_x(x,t)rx + \frac{1}{2}c_{xx}(x,t)\sigma^2 x^2 - rc(x,t) = 0, \tag{5}$$

where subscripts indicate partial derivatives in the customary way, with the obvious boundary condition

$$c(x,T) = \max(x - K, 0). \tag{6}$$

Black had found this PDE by 1969 or earlier but could not initially solve it. He did note, however, that the solution could not allow any role for the coefficient $\mu$, the expected rate of return expected rate of return on the stock! With this in mind, Black and Scholes teamed up at MIT. They noted that since the PDE did not involve $\mu$, any expected return for the stock would generate the same option price, including the riskless rate of return $r$. Then they noted that if the stock could be treated as having a riskless rate of return, then, by applying the CAPM instant by instant, so could the option because (under the assumption that the option price is a smooth function of the stock price) changes in the option and stock prices over infinitesimal periods of time are perfectly correlated. Using Sprenkle's calculation (4), this would imply the explicit option valuation $C(x,r,r,T,\sigma,K)$, the discounted expected payoff of the option that would apply if the stock had the riskless expected rate of return $r$ and if the option payoff could be discounted at a risk-free rate. Sure enough, this solution satisfied the PDE (5).

As Black (1989) put it, referring to himself and Scholes, "We had our option formula." He continued, "As we worked on the paper, we had long discussions with Robert Merton, who was also working on option valuation. Merton made a number of suggestions that improved our paper. In particular, he pointed out that if you assume continuous trading in the option or stock, you can maintain a hedged position between them that is literally riskless. In the final version of the paper, we derived the formula that way, because it seemed to be the most general derivation." This generous acknowledgment of Merton's contribution to the more general derivation, indeed the derivation that truly revolutionized modern financial theory, is consistent with the acknowledgment of Merton's contribution given in Black and Scholes (1973) (in their footnote numbered 3). To be precise, there is no need to rely, as Black and Scholes had originally, on market equilibrium under the strong assumptions of the CAPM. Instead, the simple assumption of no arbitrage would suffice. Merton's no-arbitrage argument appears, along with the CAPM-based argument, in the finally[6] published form of Black and Scholes (1973).

In yet another source,[7] Merton's contribution is acknowledged, with Black's statement that "A key part of the option paper that I wrote with Myron Scholes was the arbitrage argument for deriving the formula. Bob gave us that argument. It should probably be called the Black-Merton-Scholes paper."

Merton went on to write his 1973 paper, "Rational Option Pricing," another landmark contribution that elaborated on the Black-Scholes approach to option valuation in many ways. Merton generously attempted to delay the publication of his own paper until the earlier paper of Black and Scholes, after surprising resistance from journal editors, could finally be published (apparently with help of Merton Miller and Gene Fama) in *The Journal of Political Economy* in 1973. Later, Merton (1977) derived a more theoretically sound and concrete version of his replication argument, based on actual trading strategies rather than the more ephemeral notion of returns over "infinitesimal periods." The argument sketched out in Section 2 is essentially that of Merton (1977), and is now the standard derivation.

There is an additional important contribution in Black and Scholes (1973). They observed that because of limited liability, the equity of a corporation may itself be treated as an option on the total asset value of the firm, and thereby priced by the same methodology. This observation is at the core of modern corporate finance, and was apparently made independently by Merton (1973b). Because Merton, having overslept, missed a presentation of this idea by Black and Scholes, neither team was aware of the other's progress of this problem.

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[6]Because of the difficulty that Black and Scholes had in getting their original paper [Black and Scholes (1973)] published their second paper on this topic, Black and Scholes (1972), actually appeared in print before the first.

[7]See *MIT Management*, 1988, Fall, page 28. This quote came to my attention in Bernstein (1992), page 223.

# 6 The Significance of Black-Scholes Today

The option pricing methods of Black, Merton, and Scholes are now being taught to almost every MBA student and to most graduate, and many undergraduate, students in economics. Many investors and major corporations use these methods for planning, purchasing, pricing, or accounting purposes. In addition, to valuing straight put and call options, corporations use Black-Scholes modeling to value executive stock compensation plans, real production options, warrants, convertible, securities, debt, and so on. (In fact, for many of these applications, the methods are sometimes applied inappropriately.)

Before the advent of Black-Scholes, option markets were sparse and thinly traded. Now they are among the largest and most active security markets. The change is attributed by many to the Black-Scholes model, since it provides a benchmark for valuation and (via the arbitrage argument) a method for replicating or hedging options positions. One now can buy options on most of the major exchange-traded commodities, foreign currencies, stock indices, and government bonds. None of these markets existed in any active form before 1973. Over-the-counter options can be obtained from major investment banks on almost any important index, even if there is no commodity or security underlying the index.

The Black-Scholes approach has been extended to a wide variety of instruments with embedded options such as caps, floors, collars, collateralized mortage obligations, knockout options, swaptions, lookback options, barrier options, compound options and the list goes on and on. Indeed, there is nothing that restricts the approach to options, as opposed to other contingent claims. For example, the same arguments used in the previous section apply for a contingent claim paying $g(X_T)$ at time $T$, for any function $g : [0, \infty) \rightarrow \Re$ satisfying technical conditions. One merely substitutes the PDE boundary condition (6) with $c(x, T) = g(x)$. In many cases, of course, the contingent claim's price cannot be computed explicitly, and it is now standard operating procedure to use such numerical techniques as finite-difference solution of the associated PDE, or Monte Carlo integration of the associated "risk-neutral" expectation. Almost every major bank and trading firm has a team of specialists that use advanced Black-Scholes methods. All of these methods have their genesis in the work of Black, Merton and Scholes.

Aside from their use in pricing, the methods developed by Black, Merton, and Scholes are widely applied to financial risk management. The idea that the option can be priced by finding a trading strategy that replicates its payoff is frequently used to hedge a given security, or even to hedge a given cash flow that is not traded as a security. If one is to receive an untraded option payoff, for example, the risk inherent in that payoff can be eliminated by selling the replicating strategy previously described. This converts the risky cash flow at expiration into an initial cash flow. Indeed, the ability to hedge the value of an option on the entire S&P 500 portfolio in this manner, under the rubrick of "portfolio insurance," was accused by some of having significantly contributed to the stock market crash of 1987. Investment banks routinely sell securities with embedded options of essentially any variety requested by their customers, and then cover the combined risk associated with their net position by adopting dynamic hedging strategies.

The approach of Black, Merton, and Scholes also allows one to use market option price quotations as a guage of market volatility. For example, the Black-Scholes formula $C(x, r, r, T, \sigma, K)$ can be inverted to recover the volatility parameter $\sigma$ implied by the option price. The fact that volatility is not actually constant, but rather varies over time with uncertainty, has instigated a new generation of option-pricing formulas allowing "stochastic volatility," but based on the same Black-Scholes approach.

In addition to these important practical applications (pricing, synthesis of untraded cash flows, hedging, and information discovery) the option pricing work of Black, Scholes, and Merton has led to important theoretical work on optimal portfolio choice and multi-period equilibrium in financial markets. The key to this work is the observation that, in Black-Scholes setting, one can replicate not only the option payoff, but any stream of cash flows that depends on the path taken by the stock price. The required initial investment is the expected discounted cash flow, after replacing the expected return of the stock with the riskless rate of return. Harrison and Kreps (1979) later obtained an essentially definitive extension of the Black-Scholes model and the general notion of "risk-neutral valuation," following in part on ideas appearing in Cox and Ross (1976). The Harrison-Kreps generalization of the Black-Scholes modeling approach allowed Cox and Huang (1989) to give important extensions of Merton's (1971) model of optimal portfolio choice in a multi-period setting.[8] The idea is that the dynamic program that Merton

[8]See, also, Karatzas, Lehoczky, and Shreve (1987).

solved can be replaced with a static calculus of variations problem. One merely replaces the complicated dynamic budget constraint with a static constraint that the expected discounted consumption payoff of the investor (after replacing the expected rate of return on all securities with the riskless rate) must be equal to the initial wealth of the investor. This approach applies not only in Merton's setting, but in significantly more general settings.

The replication arguments used to derive the Black-Scholes formula have also been applied to general equilibrium modeling in multi-period financial markets. Arrow (1953) showed that security markets are an efficient method for allocating risk because they allow one to replace a complete set of contingent claims markets with a sparser set of financial security markets. The payoff of any contingent claim can be replicated by a portfolio of basis securities. His one-period model has been extended in a series of papers by various authors to multi-period settings by using the dynamic replication arguments used to prove the Black-Scholes formula. Literally, an infinite-dimensional space of possible consumption streams that investors might wish to obtain can be synthesized by trading a small number of securities. This allows one to convert a complicated multi-period general equilibrium problem into a single period problem.

With some additional concepts, many of these ideas apply even if there are not enough securities to replicate every possible consumption stream, a situation known as "incomplete markets."

# 7 Other Major Contributions

As indicated in part by the attached lists of publications, Black, Merton, and Scholes are responsible for a tremendously large and important body of ideas and papers going well beyond the Black-Scholes formula. I have listed below only those that I think of as extremely important to the development of financial markets or theory. Even without any of these additional contributions, the discovery of the Black-Scholes formula and the method by which it was derived constitute an exceptionally strong justification for the award of the Nobel Memorial Prize in Economic Sciences.

(1) Black (1972) developed an extension of the Capital Asset Pricing Model that applies without the existence of a riskless security. The new "zero-

beta" model replaces the riskless rate of return in the famous "beta formula" with the expected rate of return on a portfolio uncorrelated with the market portfolio.

(2) Black (1976) examined the pricing of commodity contracts, and in particular extended the Black-Scholes model to the case of options on futures or forwards.

(3) Black, Derman, and Toy (1991) developed a model of the valuation of term-structure securities (those whose payoffs depend on the history of the term structure of interest rates) that is now an industry standard. The model is in broad spirit much like the Black-Scholes model (in its binomial form, developed by Cox, Ross and Rubinstein (1979)), and has important computational advantages in everyday work on "Wall Street." An important aspect of the model is the fact that it is constructed so that, in principle, its parameters can be computed from the current term structure and from the current prices of options on treasury bonds, much in the way that the volatility parameter of the Black-Scholes model can be computed from the option price.

(4) Merton (1969, 1971) found a path-breaking method of solving the problem of optimal consumption and portfolio choice in a continuous-time setting. His method involved reduction of the problem to a partial differential (Hamilton-Jacobi-Bellman) equation for the investor's indirect utility function for wealth. This formulation, a breakthrough in its own right, may well have influenced the way that Black and Scholes approached the option pricing problem, in choosing a continuous setting with the same stock price model assumed by Merton, and reducing the valuation equation to a PDE. To this day there is a virtual industry of researchers extending Merton's model in many different ways. Merton's model is also widely referred to among specialists working in mathematics as the best and most elegant textbook example of a stochastic control problem.

(5) Merton (1973a) offered the first major extension of equilibrium capital asset pricing theory to a multi-period setting. By taking the approach used in his 1971 paper on optimal investment and consumption behavior, and allowing for a multi-variate Markov state process for

the market environment, Merton was able to show how the equilibrium expected rate of return of a given security depends not only on the covariance of the return with that of the market portfolio (as in the one-period CAPM), but also on the covariance of the return with changes in the state variables of the economy. This is the essence of the dynamic equilibrium problem: Investors are concerned not only with their in the next period, but also with how their opportunities to generate wealth in much later periods will depend on state variables in the next period. Breeden (1979), based in part on work by Rubinstein (1976), was later able to reduce Merton's solution to an elegant formula showing that the multi-period CAPM is in fact the same as the Sharpe-Lintner CAPM, once one substitutes covariance between returns and aggregate consumption for covariance between returns and aggregate wealth (the payoff of the market portfolio). (Of course, aggregate consumption and aggregate wealth are the same in the one-period setting of Sharpe-Lintner.)

(6) Among Merton's most important extensions of the Black-Scholes formula are: (i) his work on American options and on options on stock paying dividends, among many other applications, in Merton (1973b); (ii) his extension to the case of discontinuous stock price processes in Merton (1976), which is important also for showing that the model would not in the future be confined to the setting of Brownian motion; and (iii) the conversion in Merton (1977) of the original Black- Scholes-Merton no-arbitrage pricing argument from one based on instantaneous returns to one based on dynamic replicating strategies.

(7) Scholes did important work on dividends and their impact on the valuation of common stock in Black and Scholes (1974), Miller and Scholes (1978), and Miller and Scholes (1982).

(8) Scholes and Williams (1977), in a study of how to estimate betas (in the sense of the CAPM) from non-synchronous data, provided an important and widely cited (and taught) contribution to empirical methods in finance.

(9) Scholes is one of the leading experts on employee stock compensation plans. His textbook, *Taxes and Business Strategy,* co-authored with

Mark Wolfson, is the first of its kind in a critical and under-studied area of finance.

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