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A LIQUIDITY-BASED MODEL OF SECURITY DESIGN

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We consider the problem of the design and sale of a security backed by specified assets. Given access to higher-return investments, the issuer has an incentive to raise capital by securitizing part of these assets. At the time the security is issued, the issuer's or underwriter's private information regarding the payoff of the security may cause illiquidity, in the form of a downward-sloping demand curve for the security. The severity of this illiquidity depends upon the sensitivity of the value of the issued security to the issuer's private information. Thus, the security-design problem involves a tradeoff between the retention cost of holding cash flows not included in the security design, and the liquidity cost of including the cash flows and making the security design more sensitive to the issuer's private information. We characterize the optimal security design in several cases. We also demonstrate circumstances under which standard debt is optimal and show that the riskiness of the debt is increasing in the issuer's retention costs for assets.

KEYWORDS: Security design, financial innovation, capital structure, asymmetric information.

1. INTRODUCTION

THIS PAPER INVESTIGATES THE PROBLEM faced by a corporation that raises capital by issuing a security backed by a fixed set of assets. The choice of a security design has important consequences. First, to the extent that the security does not pay out all asset cash flows, the issuer raises less capital. Second, security designs may differ in their market liquidity. By “liquidity” we are referring to the possibility of inelastic demand for the security, so that the quan-

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tity sold may affect the price. The issuer's design problem is thus to choose a security that balances the issuer's desire to raise capital against illiquidity costs.

In order to formulate the security-design problem, we develop a model of liquidity that is based on asymmetric information. The security will be sold by the issuer, or in many cases by an underwriter, that may have private information regarding the distribution of the cash flows of the underlying assets. An attempt to sell a security backed by these assets therefore exposes the issuer to a standard "lemons" problem, as described by Akerlof (1970). That is, investors rationally anticipate that the amount sold is greater when the private information implies a low value for the security. This, in turn, implies that investors offer a lower price if the issuer puts a larger quantity of the security up for sale. Thus, private information held by the seller leads naturally to a downward-sloping demand curve for the security, and hence a liquidity problem for the issuer.

The magnitude of this liquidity problem depends in a natural way on the security design. For instance, if the asset cash flows always exceed some positive level, the issuer could offer riskless debt. Because the value of riskless debt is independent of any private information, there would be no lemons problem. Riskless debt may not be optimal for the issuer, however, since issuing risk-free debt generally forces the issuer to retain a large portion of the asset cash flows, reducing the expected amount of capital that could be raised. On the other hand, if the issuer creates a security that pays out all of the cash flows generated by the assets (that is, a full equity claim on the assets), none of the cash flows remain unsecured, but the issuer suffers a lemons cost when liquidating the security.

Our goal is to characterize the class of security designs that is optimal given this tradeoff. We do not limit the potential securities to debt and equity, but include general securities whose payoffs may be contingent on arbitrary public information. An obvious question is whether reasonable assumptions lead to the optimality of standard debt or some other simple security designs.

The overall structure of our model is roughly as follows. The issuer holds some assets that generate cash flow X . In order to raise capital, the issuer designs a security backed by these assets. The security is a claim to a nonnegative payment, F , possibly contingent on the public information, S . In order to focus on the role of private information, we take the probability distributions of X and S to be exogenously given, which ignores for simplicity such effects as moral hazard on the part of the issuer.

The issuer may be a bank facing minimum regulatory capital requirements that lead to a preference for receiving cash over holding risky assets.² The issuer

²See, for example, Dewatripont and Tirole (1995), Hill (1996), and Picker (1996). Picker states that "issuing asset-backed securities is an efficient way to fund asset growth. It provides immediate liquidity and, by getting loans off the balance sheet, reduces the amount of capital needed to meet minimum capital adequacy requirements." Dewatripont and Tirole (1995, p. 173) point out that "prudential regulation and in particular minimal net worth requirements provide the banks with an incentive to sell risky assets when they do not meet these capital adequacy requirements. This explains a fair share of sales by financial intermediaries."

could be a broker-dealer whose core business and comparative advantage is intermediation rather than long-term investment. Alternatively, the issuer may be a firm with alternative profitable investment opportunities and limited access to credit. (Under this last interpretation, we are restricting the security to be a claim on a fixed collection of assets of the issuer, such as a pool of mortgages, and not on assets that might be purchased with the proceeds of the sale of the security in question.)

In any case, we suppose that, by the time of sale of the security F , the seller may have received private information regarding the distribution of F . Thus, investors are naturally concerned that they are being sold a “lemon.” We show that a natural signalling equilibrium exists in which the seller faces a downward-sloping demand curve for the security. Intuitively, the seller receives a high price for the security only by demonstrating a willingness to retain some fraction of the issue. Because the seller prefers cash over long-term assets, retention is a credible, but costly, signal. This “liquidity cost” varies with the severity of the lemons problem, which in turn depends on the sensitivity of the security’s value to the seller’s information.

We are able to characterize an optimal security design in several settings. One setting we consider assumes that the information learned by the seller has a “uniform worst case” (a weaker statistical assumption than the monotone likelihood ratio property), and restricts attention to securities whose payoffs are monotone in the cash flow X . In this case, we show that an optimal contract is of the form $F = \min(X, d)$, for some constant d . It is natural in certain situations, elaborated below, to interpret this as a standard debt contract. Intuitively, a debt contract has the lowest sensitivity to private information held by the issuer. The optimal security, however, is not riskless debt. Instead, the issuer increases the face value of the debt to trade off the lemon’s premium of risky debt against the retention cost associated with the unsecuritized portion of the cash flows. In fact, if the issuer’s retention cost is sufficiently high (for example, if the firm has sufficiently profitable alternative investment opportunities), then the “pass-through” security $F = X$ is optimal (or equivalently $d = \infty$), which we can interpret as an equity contract.

The model fits well several important applications. One example is the design of asset-backed securities. Here we interpret X as the cash flows of the underlying collateral, or asset pool, and F as a security, or “tranche,” that is backed by the pool. In this context, the private information of the seller may be knowledge of aspects of the asset pool,³ or special expertise in pricing the securities.⁴ Our conclusions imply that an optimal tranche will consist of a senior claim against the pool. The issuer retains the residual portion plus any

³For example, a collateralized loan obligation (CLO) package structured by SBC/Warburg/Dillon-Read in 1997 is backed by a package of roughly one billion dollars (notional) of private loans. The borrowers, under the original loan provisions, are to remain “anonymous” to investors. The senior tranche of the CLO structure protects its investors to a significant extent from default by borrowers.

⁴See, for example, Bernardo and Cornell (1997) for evidence of this in the CMO market.

unsold fraction of the senior tranche. In fact, this is generally consistent with observed behavior in these markets.

Another context in which we believe our model fits rather closely is the design of corporate securities that are then sold by an informed underwriter. Due to its superior information, the underwriter retains some fraction of the issue in order to “support” the price. Tying up capital in this fashion is costly for the underwriter, and these costs are passed along to the issuing firm. Thus, the issuing firm designs a security to minimize these costs. We note that, in the corporate context, the presence of the underwriter is important for our model: the issued security does not have a claim against the profits earned nor the fraction of the issue retained by the underwriter.⁵ Of course, in this setting we are also abstracting from agency and incentive issues regarding the firm’s production decision.

Finally, we generalize the basic model in several directions. For instance, we allow for cases in which the issuer possesses private information before the security design is chosen. We also show that the issuer may have an incentive to “split” the security and issue both a risky and a riskless security.

Related Literature

Several papers are closely related to ours. Perhaps the closest relationship is with Myers and Majluf (1984), who point out the impact of the lemons problem for a firm that issues equity to finance a fixed investment opportunity. They show that if management has private information regarding the firm’s value, then only those firms with values in the lower tail of the distribution choose to issue equity and invest. Better firms retain their equity and pass up the investment opportunity, rather than issue underpriced equity. Myers and Majluf argue that debt, whose value may be less sensitive to management information, is less subject to this lemons problem, and may therefore be a preferred financing vehicle.

Our model differs from that of Myers and Majluf in several important ways. First, in their model, the firm must raise a fixed amount of capital. This leads to a pooling equilibrium in which firms either issue and invest, or retain their equity and forgo the project. Our model allows for a variable scale of investment. This leads to a continuous retention decision and a separating equilibrium in which better firms retain more and invest less than do firms with lower valuations for the security. Our signalling model of security issuance is therefore more like the model of Leland and Pyle (1977), although Leland and Pyle model the cost of retention as risk-bearing by the owner-entrepreneur, rather than reduced investment.

A second major difference is that Myers and Majluf focus on the issuance of equity by the firm. They briefly (and less formally) consider the use of debt. In this paper we consider the full range of possible security designs for the firm.

⁵In the absence of an underwriter, the available cash flows X should depend on the profits from selling the security as well as the retained portion of the issue.

Each possible design induces an associated lemons problem along the lines of the Myers-Majluf intuition, and also may be more or less effective at securitizing the available cash flows. We look for a security design that optimally trades off these effects. Under special restrictions we identify this security to be risky debt. If, however, the firm's investment opportunities are sufficiently profitable, equity may be optimal in our setting. In a sense, our work generalizes the Myers-Majluf intuition, and offers some support for their "pecking-order hypothesis."

Nachman and Noe (1991, 1994) model a setting similar to that of Myers and Majluf, in which the firm must raise a fixed amount of capital. Again, this leads to a pooling equilibrium. They allow for a general set of securities, and show, under distributional assumptions, that all types of firms may pool and issue the same risky debt security.

An important difference between our work and that of Nachman and Noe is that we model the security-design decision as an ex-ante problem faced by the issuer.⁶ Thus, the firm's choice of security design is not a signal of the firm's private information. Rather, the optimal security design is that which is most efficient, ex ante, at resolving the lemons problem that the seller will face ex post. (This difference also distinguishes our approach from much of the other signalling literature on security design, such as Brennan and Kraus (1987), Constantinides and Grundy (1990), Rebello (1995), and others.) Our approach may be appropriate in several settings. First, there is an advantage in issuing "standardized securities," that work well for many types of issuers.⁷ Second, there are often significant lags between the design of the security and its actual issuance, as is the case, for instance, with the use of "shelf registrations" of securities. For example, there may be significant design costs that are optimally spread out over several different issues. (See Hill (1996) and Tufano (1989).) In these cases, significant information is likely to be learned between the initial design of the security and its issuance. Finally, our model applies to situations in which the designer of the security may differ from the actual seller. For example, it may be that underwriters are charged with selling the securities while holding private information. Indeed, we expect sophisticated underwriters to have an ability to price complex securities, such as certain collateralized mortgage obligations (CMOs), in a manner that reflects superior informational technology, but has little to do with any private information that may have been available to the issuer when the security is designed.⁸ Likewise, future holders of the security may liquidate their positions in the secondary market. Alternatively,

⁶We do, however, generalize our model in Section 6 to allow for cases in which the issuer learns some information prior to the design of the security. We show that the main conclusions of our model are not affected provided there is significant new information that is only available to the issuer after the security design has been chosen.

⁷See Gale (1992b) for an alternative model of standardized securities based on the informational costs of learning about new securities.

⁸For empirical evidence of significant variation in the pricing of sensitive CMOs exposed to risks about which most information is public, see Bernardo and Cornell (1997), who suggest differences in pricing technology.

the security design may be an innovation by an investment bank, which then markets the structure to several different client-issuers.

In this sense, the timing of information for our model is in the spirit of the costly-state-verification literature, initiated by Townsend (1979). In that model, a security design is chosen *ex ante* in anticipation of the *ex-post* costs of asymmetric information. The asymmetric-information problem arises from the fact that the cash flow X can be verified only at a cost. (For an application of this approach to corporate capital structure, see Cantillo (1994).) In our model, enforcement of the security payoff is not a problem; the asymmetric information is regarding the distribution of X . Thus, while Townsend shows that debt may be optimal because it minimizes the need to verify, in our model debt may be optimal because its value is least sensitive to the issuer's private information.

This timing is also shared by the security-design models of Demange and Laroque (1995) and Rahi (1996). In those models, an entrepreneur designs a security whose payoff is linear in the fundamental uncertain factors in the economy. The security design is chosen *ex ante* to trade off the agent's insurance needs with the desire to earn speculative profits, in an extension of the Grossman (1976) rational-expectations model. Our informational timing is also like that of Zender (1991), who examines the implications of security design for corporate control.

Boot and Thakor (1992) show that there is value added by splitting an asset into two separate securities, one informationally sensitive, the other less so. In their model, this follows from the fact that, given a cost to becoming informed, an issuer must concentrate a sufficiently large amount of private information in one component of the asset structure in order to induce the potentially informed investors into the market. The issuer does not retain assets, and there is no lemons premium since the issuer is assumed to delegate the sale to an uninformed investment bank, who merely chooses a price that clears the market. There are two possible outcomes for the value of the firm, which reduces the optimal decomposition to two securities.

Gorton and Pennachi (1990) present a model in which informed insiders may collude at the expense of outsiders. Outsiders can respond by pooling their capital in the form of an intermediary that issues equity and debt. Under conditions, outsiders will buy the debt, and insiders will buy the equity rather than collude. In this sense, the Gorton-Pennachi model also supports the notion that splitting assets into appropriate securitized components can mitigate the lemons premium involved with asymmetrically informed traders. Gorton and Pennachi are particularly interested in the example of bank deposits.

Glaeser and Kallal (1993) show that an issuer of securities improves the value to be received from intermediaries for an asset by designing the asset in a manner that increases the costs of becoming informed about its payoff. This induces the intermediary to remain uninformed, and thereby avoid a lemons premium at the time of sale. The cost of obtaining asset information is taken to be an exogenous index, and is treated in the context of comparative statics. (We consider the issue of endogenous information acquisition by the issuer in

DeMarzo and Duffie (1997).) This model again supports the notion that the design of an asset can reduce the associated lemons premium.

We have generalized the basic model of this paper to consider the design of an entire family of securities backed by the cash flows of a given firm (see DeMarzo and Duffie (1993)). Examples include the various tranches of a structure of collateralized mortgage obligations backed by a given pool of mortgages, or the subordination structure for the bonds of a corporation. Because of the potential incentive for the issuer or underwriter to signal the quality of one issue with the amount sold of another, the problem of simultaneous design of multiple securities warrants a more detailed treatment than possible here. In this paper, we concentrate solely on the case of designing a single issue backed by a given set of cash flows. This raises a sufficiently rich set of questions and answers to merit its own treatment.

There is a large and growing body of literature on financial innovation and security design that we have not discussed here. Some of this is motivated by the value of control of a corporation, or agency costs in the management of the firm, and is carefully surveyed by Harris and Raviv (1992). Another large part of the literature is motivated by spanning risks. For surveys, see Allen and Gale (1994) and Duffie and Rahi (1995). In this paper we ignore spanning issues and assume that both issuers and investors are risk-neutral. For a broad survey of the security-design literature based on costly state verification, adverse selection, allocation of control rights, allocation of risk, and information acquisition, see Allen and Winton (1996).

The remainder of the paper is organized as follows. Section 2 presents a formulation of the abstract security-design problem. Section 3 contains an analysis of the equilibrium market demand for a given security design. In Section 4, we characterize properties of the optimal design in a general setting. Section 5 presents a special case in which we show the optimality of standard debt. Section 6 considers cases in which the issuer has payoff-relevant information prior to the design of the security. Section 7 contains concluding remarks, including some mention of extensions treated in DeMarzo and Duffie (1997).

2. THE BASIC SECURITY-DESIGN PROBLEM

The model's participants consist of an issuer and a set of outside investors. The issuer owns assets that generate future cash flows given by a nonnegative bounded random variable⁹ X . We suppose that all agents are risk-neutral, and (for notational convenience) we normalize the market interest rate to zero. Therefore, in the absence of capital market imperfections, the market value of the issuer's assets would be $E(X)$.

Risk-neutrality by the issuer is a restrictive assumption, which we make in order to focus on liquidity, as opposed to risk-sharing, motives for security design. In fact, many motivating examples involve the sale of securities by

⁹All random variables in the model are on a given probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

previously incorporated and publicly traded firms. For such cases, the firm does not come directly equipped with an attitude toward risk. With liquid capital markets, for example, Modigliani and Miller (1958) and Stiglitz (1974) showed that a publicly traded firm is indifferent to certain kinds of financial risk. Of course, with market imperfections, financial risk may matter to a firm. Indeed, we find in this paper that it matters in part because risky assets are less liquid than riskless assets, if the firm is better informed than investors about those risks. In particular, our model allows one to derive a value function for risky assets that displays a specific kind of risk attitude.

We assume that the issuer has an incentive to raise cash by selling some portion of these assets. In particular, we suppose that the issuer discounts future cash flows at a rate that is higher than the market rate. That is, there exists a discount factor $\delta \in (0, 1)$ that represents the fractional value to the issuer of unissued assets. For example, if $\delta = 0.98$, the issuer is indifferent between keeping one dollar worth of assets and selling these assets for 98 cents in cash. This assumption is natural. For example, the issuer may face credit constraints or, as for many banks and others in the financial services industry, binding minimum-capital requirements. (See footnote 2.) Under certain market imperfections, a firm may strictly prefer to raise capital for new investment opportunities from a sale of assets. (In this case δ is the discount factor corresponding to the rate of return on these new investments, relative to the return on current assets.) Thus, if the issuer retains the assets, they have a private value to the issuer of $\delta E(X)$.¹⁰

In order to raise cash, the issuer creates a limited-liability security backed by the assets. The payoff $F \geq 0$ of the security chosen by the issuer can be made contingent on the outcome of some publicly observable signals $S = (S_1, \dots, S_m)$, the contractible information. Specifically, S is an \mathbb{R}^m -valued random variable,¹¹ for some m , and F is a real-valued random variable measurable with respect to S , so that we may write $F = \varphi(S)$, for some measurable $\varphi: \mathbb{R}^m \rightarrow \mathbb{R}$. For example, if the payoff of the security is to be contingent only on the realized cash flows of the securitized assets, then $S = X$, and we may write $F = \varphi(X)$. Alternatively, the security payment may be contingent on publicly verifiable indices, such as market prices, the sales of the firm, commonly observable actions by management, the actions of credit-card holders (for credit-card backed assets), the mortgage pre-payment behavior of homeowners (for mortgage-backed assets), or any other information that can be used as the basis for a contingent contract. In all cases, we take the distribution of the signal S to be given so as to put aside the distraction and modeling difficulties introduced by moral hazard.

¹⁰Actually, our model does not require $\delta < 1$ with certainty at the design stage. We could instead suppose that the initial security design is chosen in light of the possibility that $\delta < 1$ may occur in the future. We might interpret the security design as an initial “shelf registration” of a security to be used in the event of a future need for cash.

¹¹In our original draft, S was valued in some abstract measurable space \mathcal{S} . Although we take $\mathcal{S} = \mathbb{R}^m$ here for concreteness, precisely the same results apply for abstract \mathcal{S} .

We suppose that the claims of security holders are secured solely by the given assets of the issuer. Thus, the security with stated payoff F has the potential for default in the event that $F > X$. For notational simplicity, however, we assume¹² that the first contractible variable S_1 is X , so that, without loss of generality, we can restrict the issuer to some $F \leq X$.

Given a security design F , the issuer retains the residual cash flow $X - F$. After designing the security, but prior to the sale of the security to outside investors, the issuer or underwriter handling the sale receives information relevant to the payoff of the security. For the moment, we suppose that the issuer is also acting as underwriter, and discuss the role of a third-party underwriter at the end of the section.

The information available to the issuer at sale is represented by a random variable Z valued in¹³ \mathbb{R}^n , for some n . The conditional distribution of S given Z is specified by some transition function μ on \mathbb{R}^n into the space of probability measures on \mathbb{R}^m . For any z in \mathbb{R}^n , we let $\mu(\cdot | z)$ denote the associated conditional distribution of S . As is conventional, we write $E_z[\varphi(S)] = \int_{\mathbb{R}^m} \varphi(s) \mu(ds | z)$ for the conditional expectation of a security payoff $\varphi(S)$ at an outcome z of Z . We assume that μ is continuous,¹⁴ so that for any security design F , whenever $z(n) \rightarrow z$, we have¹⁵ $E_{z(n)}(F) \rightarrow E_z(F)$.

For a security design F , given the information Z , the issuer's conditional valuation of the security is $E(F | Z)$. We assume that the distribution of Z has a compact connected support, implying¹⁶ that, for any security design F , the support of the distribution of $E(F | Z)$ is a compact interval.

For each security design F , the issuer assumes some inverse demand schedule $P_F: [0, 1] \rightarrow \mathbb{R}$. That is, if the issuer sells some fraction q of its holdings, then $P_F(q)$ is the perceived market price of the security F . For now, we take this demand schedule as arbitrarily given, though we might expect it to be downward sloping. In the next section we study how this demand schedule is determined in equilibrium.

Note that, although the issuer may initially sell only a fraction $q < 1$ of the security, we still require that the security design be such that $F \leq X$. This is so since, in almost all instances, holders of F will not have recourse to the residual fraction $1 - q$. This is true, for instance, of CMO's and other asset-backed securities. It is also true in a corporate setting if the residual fraction is held by

¹²In initial drafts of this paper we did not take this shortcut, and reached essentially the same conclusions. In that case the security payoff is given by $\min(F, X)$, and the issuer could issue the entire pool X by taking F to be an upper bound on X .

¹³We do not actually exploit the n -dimensional aspect of this signal space. It would be enough for our results that Z takes values in a metric space, as in previous drafts of this paper.

¹⁴Continuity here is in the usual sense: If $z_n \rightarrow z$, then for any measurable subset $A \subset \mathbb{R}^m$, we have $\mu(A | z_n) \rightarrow \mu(A | z)$.

¹⁵Convergence of $E_{z(n)}[\varphi(S)]$ to $E_z[\varphi(S)]$ follows by Lebesgue dominated convergence, as in Royden (1968, Propositions 18, p. 232).

¹⁶This is a consequence of the continuity of μ and Lebesgue dominated convergence, as in the previous footnote.

an underwriter, or if the residual fraction is eventually liquidated at some later date (with the issuer bearing a retention cost in the interim).

The issuer's liquidation problem is to choose the fraction of the security to sell to investors, given the market demand for the security and the private information Z . The total payoff, for any fraction q sold, is the value of the issuer's ultimate portfolio, which includes the residual cash flow $X - F$, the unsold fraction $1 - q$ of the security F , plus any cash raised from the sale of qF . Because the issuer discounts by δ any retained future cash flows, this portfolio has a value to the issuer of

$$\begin{aligned} & \delta E(X - F | Z) + \delta(1 - q)E(F | Z) + qP_F(q) \\ & = \delta E(X | Z) + q[P_F(q) - \delta E(F | Z)]. \end{aligned}$$

Given a particular outcome f of $E(F | Z)$, the issuer chooses the quantity of the security to sell that maximizes this value, or equivalently, solves

$$(1) \quad \Pi_F(f) = \sup_{q \in [0, 1]} q[P_F(q) - \delta f].$$

We can interpret (1) as a standard monopoly problem in which the issuer faces a marginal cost for the security that is equal to its value to the issuer if retained, δf . The issuer chooses the fraction q of the security to sell in order to maximize profits. Thus Π_F represents the monopoly profits associated with the sale of security F , and

$$V(F) = E[\Pi_F(E(F | Z))]$$

is the ex-ante expected profit associated with the security design F . This profit varies with the liquidity of the security F , which we can think of loosely as the elasticity of the market demand function P_F for the security. Leaving to the next section the determination of P_F , the issuer's problem is thus

$$(2) \quad \sup_F V(F).$$

Figure 1 summarizes the basic timing of the model.

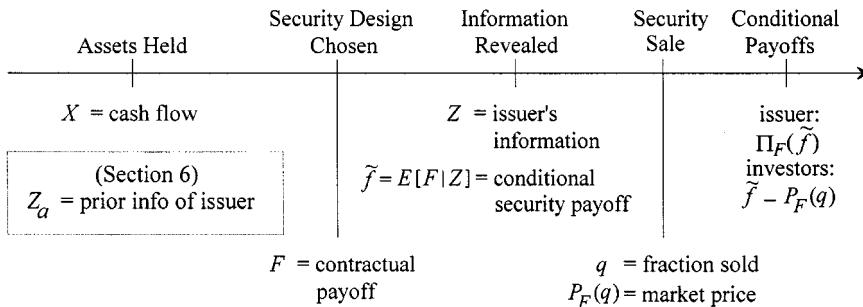


FIGURE 1.—Basic timing of the model.

The identical design problem (2) arises if the sale is handled by a separate underwriter who faces the same retention cost coefficient δ . In that case, the underwriter values the issue at

$$\sup_{q \in [0, 1]} qP_F(q) + \delta(1 - q)E(F | Z).$$

Provided the underwriting market is competitive, this amount (or its expectation, if the underwriter purchases the issue before becoming informed) is paid to the issuer, whose total ex-ante valuation for a design F is once again $V(F)$. This leaves the same problem (2). Thus, in this sense, the design problem is unaffected by the role of an underwriter.

3. MARKET DEMAND FOR SECURITIES

In this section, for a given security design F , we analyze the market demand function P_F and the associated conditional expected payoff function Π_F for the issuer. First, we derive some important general properties of Π_F that hold for an arbitrarily determined demand function P_F . Then we develop an equilibrium model of the market demand function using a signalling model.

General Properties of the Profits of the Public Offering

Suppose $P_F: [0, 1] \rightarrow \mathbb{R}$ is a bounded measurable demand function, taken by the issuer as given. Let the profit function $\Pi_F: \mathbb{R}_+ \rightarrow \mathbb{R}$ be defined by (1). The following conventional properties of a monopoly problem are recorded for later use. For convenience, we henceforth let $[f_0, f_1]$ denote the support of the distribution of $E(F | Z)$.

PROPOSITION 1: *For given $Q: [f_0, f_1] \rightarrow [0, 1]$, suppose that, for each f , $Q(f)$ solves (1). Then:*

- (i) *For any f , $\Pi_F(f) \geq [P_F(1) - \delta f]^+$.*
- (ii) *Π_F is decreasing and convex.*
- (iii) *For any f , $-\delta Q(f)$ is in the subgradient¹⁷ of Π_F at f . In particular, provided Π_F is differentiable at f , we have $\Pi_F'(f) = -\delta Q(f)$.*
- (iv) *Q is weakly decreasing.*

PROOF: For each fixed quantity q to be sold, the function $f \mapsto q[P_F(q) - \delta f]$ is affine and decreasing. As the upper envelope of a family of such functions, Π_F is therefore convex and decreasing, as illustrated in Figure 2.

In order to show (iv), for any g_1 and g_2 in $[f_0, f_1]$, let $q_i = Q(g_i)$ and $p_i = P_F(q_i)$. Then, by revealed preference, $q_i(p_i - \delta g_i) \geq q_j(p_j - \delta g_i)$. Adding these together for $i = 1$ and 2 , and for $j = 1$ and 2 , leaves $(q_1 - q_2)(g_1 - g_2) \leq 0$, so that Q is weakly decreasing. The other properties are by inspection. *Q.E.D.*

¹⁷That is, for any f and f' , we have $\Pi_F(f') - \Pi_F(f) \geq \delta Q(f)(f - f')$.

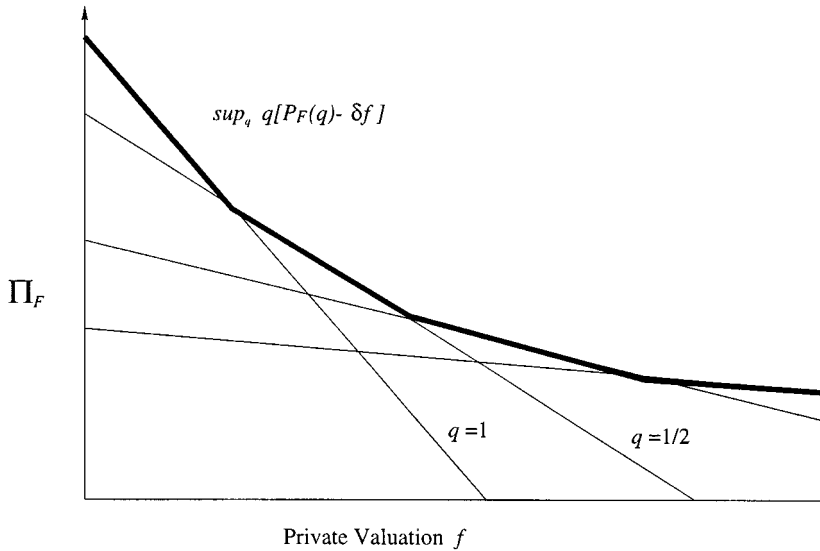


FIGURE 2.—Decreasing and convex profit function.

The fact that any optimal quantity schedule Q is decreasing is consistent with the intuition that the issuer sells less of the security when its conditional expected payoff is higher. The properties of Π_F established in Proposition 1 would hold if the issuer's retention value of the security, which is given here by the linear form $f \mapsto \delta f$, were replaced with a general concave function. For a further generalization to nonconstant returns to scale, see DeMarzo and Duffie (1997). These general properties (i)–(iv) will prove crucial in much of the subsequent analysis.

A Signalling Model of Market Liquidity

Again, we are given a security design F with private valuation $\tilde{f} = E(F|Z)$, which is a nonnegative bounded random variable whose probability distribution has as its support some interval $[f_0, f_1]$. We now develop a model for a market demand function $P: [0, 1] \rightarrow \mathbb{R}_+$ and a quantity schedule $Q: [f_0, f_1] \rightarrow \mathbb{R}$. We focus on a signalling equilibrium for (P, Q) , a model in which investors make inferences regarding the private valuation \tilde{f} of the security based on the liquidation decision of the issuer. We expect to find an endogenous relationship between the fraction $Q(\tilde{f})$ of the security sold and its market price $P(Q(\tilde{f}))$.

We assume that investors observe only the fraction of the security sold to the market.¹⁸ If investors face no uncertainty regarding the issuer's conditional

¹⁸In DeMarzo and Duffie (1997) we also consider a generalization to the case in which investors have independent private information. What is crucial for our analysis is that the issuer has some information that investors do not have, so that the quantity choice of the issuer is a relevant signal to investors.

valuation, that is, $f_0 = f_1$, then we expect that $P(q) = f_0$ for all q . Otherwise, result (iv) of Proposition 1 implies that investors know, whatever the expectations of the issuer regarding demand, that an optimal liquidation decision by the issuer implies a higher quantity sold when the outcome of \tilde{f} is low than when it is high. Thus, rational inference on the part of investors naturally leads to an equilibrium demand function P that is downward sloping. (In fact, we will show P to be strictly decreasing in our setting.)

We propose a signalling game in which uninformed investors compete for purchases of the security in a Walrasian market setting.¹⁹ In this game, the issuer is informed of Z , and computes $\tilde{f} = E(F|Z)$. The issuer then chooses a quantity $Q(\tilde{f})$ to put up for sale to uninformed investors. Investors then bid for the securities. Retention is a credible signal of the security's value because retention is costly to the issuer.²⁰

An outcome of this game is a triple $(f, q, p) \in \Theta \equiv [f_0, f_1] \times [0, 1] \times [f_0, f_1]$, where f is the realization of \tilde{f} known to the issuer, q is the issuer's sales quantity choice, and p is the market price paid by investors. The payoff to the issuer is given by the function $U: \Theta \rightarrow \mathbb{R}$ defined by

$$U(f, q, p) = q(p - \delta f).$$

We use the following standard definition of an equilibrium for the signalling game.

DEFINITION: A Bayes-Nash equilibrium for the liquidation game is a pair (P, Q) of measurable functions satisfying:

(i) $Q(\tilde{f}) \in \arg \max_q U(\tilde{f}, q, P(q))$ almost surely.

(ii) $P(Q(\tilde{f})) = E[\tilde{f} | Q(\tilde{f})]$ almost surely.

An equilibrium (P, Q) is separating if

(iii) $P(Q(\tilde{f})) = \tilde{f}$ almost surely.

Analysis of Bayes-Nash equilibria for signalling games such as ours is standard in the literature. A key necessary condition for the existence of a separating equilibrium, and an important step to establishing the uniqueness of certain types of refined equilibria, is the "single-crossing property." See, for example, Cho and Kreps (1987), Cho and Sobel (1990), Mailath (1987), and Ramey (1996). In our model, the single-crossing property is satisfied, in that, for each fixed $(q, p) \in (0, 1] \times \mathbb{R}_+$,

$$f \mapsto - \frac{U_q(f, q, p)}{U_p(f, q, p)}$$

is strictly monotone (where, as usual, subscripts denote partial derivatives).

¹⁹ For a more general model of Walrasian market equilibrium of this variety, see Gale (1992a).

²⁰ This is much like the signalling model of Leland and Pyle (1977), in which an owner-entrepreneur signals the value of the firm's equity through his retention choice. In their model, the cost of retaining equity is that the risk-averse owner bears additional risk.

PROPOSITION 2: *Let*²¹

$$(3) \quad Q^*(f) = \left(\frac{f_0}{f} \right)^{1/(1-\delta)} \quad \text{and} \quad P^*(q) = \frac{f_0}{q^{1-\delta}}.$$

Then (P^, Q^*) is a separating equilibrium.*

PROOF: Because of the strict monotonicity of Q^* , the rational-expectations condition (ii) and the separating condition (iii) in the definition of a signalling equilibrium are satisfied, as we have

$$P(Q^*(\tilde{f})) = E[\tilde{f} | Q^*(\tilde{f})] = E(\tilde{f} | \tilde{f}) = \tilde{f} \quad \text{almost surely.}$$

With the single-crossing property, it is sufficient for the optimality condition (i) of an equilibrium to check the first-order conditions of the informed agent's optimization problem. (See Mailath (1987).) As with any monopoly problem, optimality implies that marginal revenue equals marginal cost. At f , the first-order condition in (1) for optimal q in a separating equilibrium is

$$P^*(q) + q \frac{d}{dq} P^*(q) = \delta f = \delta P^*(q),$$

where the last equality follows from the fact that, because of separation, $P^*(Q^*(f)) = f$. This yields the differential equation

$$\frac{d}{dq} P^*(q) = - \frac{P^*(q)(1-\delta)}{q},$$

which is uniquely satisfied by (3), with the obvious boundary condition $P^*(1) = f_0$. Finally, Q^* is defined as the inverse of P^* . *Q.E.D.*

If $f_0 > 0$, then (P^*, Q^*) is the unique separating equilibrium, in a sense made precise in DeMarzo and Duffie (1997), using arguments essentially identical to those of Mailath (1987).

While we have verified the existence of an equilibrium, we have not eliminated the possibility of other equilibria. If we were to take a case in which the distribution of \tilde{f} has finite support, we would be able to exploit the uniqueness of the separating equilibrium among all equilibria satisfying the D1 refinement criterion, under fairly general conditions (for example, conditions given by Cho and Sobel (1990)). A proof for the case in which Z has two possible outcomes is given in Appendix B. We have taken the case of a continuum of outcomes for Z in order to obtain a tractable initial security-design stage of our model, where the explicit and natural properties of the equilibrium given by (3) are valuable. The binary-outcome case shown in Appendix B is equally tractable. Even for cases with continuum support for the signaller's private valuation, Ramey (1996)

²¹For the case of $f_0 = 0$, we take the obvious limit to get $Q(f_0) = 1$ and $Q(f) = 0$ for $f > f_0$.

proves, under technical conditions, that the separating equilibrium is the unique D1-stable equilibrium. Ramey's results should extend to our setting.

In general, if one allows for multiple equilibria that are plausible from the point of view of the issuer at the stage of the security design, uncertainty regarding the valuation model that would be applied at the time of the public offering could affect the choice of design. One could imagine, for example, a formalization of this uncertainty through probability assignments to alternative equilibrium valuation models, and maximization with respect to the design choice of the resulting expected valuation. For our purposes, this formalism perhaps causes more trouble than it is worth, and we will assume that the issuer takes as given the unique separating equilibrium valuation model that we have just described.

Simplification of the Design Problem

From this point, for a security design F , we let \tilde{f} denote the issuer's conditional value $E(F|Z)$, and let $[f_0, f_1]$ denote the support of the distribution of \tilde{f} . We fix the equilibrium (P^*, Q^*) given by (3), noting that it depends on the security design F only through the "worst-case" conditional value f_0 . We have

$$(4) \quad \Pi_F(f) = U(f, Q^*(f), f) = \Pi(f, f_0) \equiv (1 - \delta)f_0^{1/(1-\delta)}f^{-\delta/(1-\delta)}.$$

While Π_F is therefore explicit, in order to characterize solutions to the design problem (2), we will rely only on the properties given by the following proposition. These properties are also satisfied for the case, examined in Appendix B, in which Z has two possible outcomes. In the remainder of the paper, we use only these general properties of the issuer's profit function for the public offering. We do not exploit the explicit functional forms derived from the "continuum" or "binary" cases for Z , except for purposes of numerical illustrations.

PROPOSITION 3: Let $\Pi: \mathbb{R}_+^2 \rightarrow \mathbb{R}$ be defined by (4).

(i) Π is homogeneous of degree 1.

(ii) $\Pi(f, 0) = 0$ and $\Pi(f_0, f_0) = (1 - \delta)f_0$.

Whenever $f > f_0 > 0$,

(iii) $(1 - \delta)f > \Pi(f, f_0) > (f_0 - \delta f)^+$.

(iv) $\Pi(\cdot, f_0)$ is strictly decreasing and strictly convex.

(v) Π is differentiable at (f, f_0) , with $-\delta < \Pi_1(f, f_0) < 0$ and $\Pi_1(f_0, f_0) = -\delta$, where Π_1 denotes the partial derivative of Π with respect to its first argument.

PROOF: These properties are immediate from Propositions 1 and 2. *Q.E.D.*

In summary, based on the preceding analysis of the liquidation game, we can restate the security-design problem as (2), where $V(F) = E[\Pi(\tilde{f}, f_0)]$, Π has the properties given in Proposition 3, $\tilde{f} = E(F|Z)$, and $f_0 = \min_z E_z(F)$.

For the case of $\delta = 98\%$, Figure 3 shows the public-offering profit function, normalizing to the case of $f_0 = 1$. Case C depicts the continuous case derived in (4); Case B depicts the binary example studied in Appendix B. We illustrate

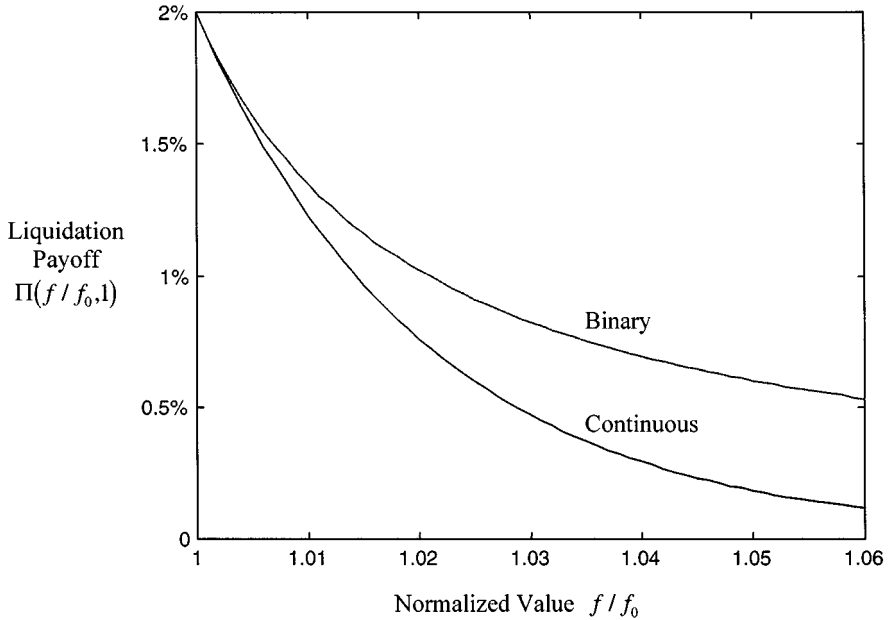


FIGURE 3.—Profit of public offering.

both cases in order to emphasize the fact that our results do not rely on the explicit functional form in (4).

In interpreting Figure 3, one notes that $\Pi(f_0, f_0) = (1 - \delta)f_0 = .02f_0$, which reflects the fact that in the worst case the issuer sells the entire issue and so saves the full holding costs. If $f/f_0 > 1$, the issuer retains a fraction of the issue to “prove” its value to the market, and so the full holding costs are not recovered.

We assume from this point that $\min_z E_z(X) > 0$, so that there exist securities that have positive conditional value given any outcome for the seller’s information.²²

If the support of the distribution of S is countable, there is always²³ an optimal design. This case is not restrictive on economic grounds, and, for any F ,

²²The design problem is trivial if $\min_z E_z(X) = 0$, for in this case $\Pi(\tilde{f}, f_0) = 0$ for all F .

²³In order to see this, let F^1, F^2, \dots be a sequence of designs, with $V(F^n) \rightarrow v^*$, the supremum value in (2). We can write $F^n = \varphi_n(S)$, for some $\varphi_n: \mathbb{R}^m \rightarrow \mathbb{R}$. Assuming that the support of the distribution of S is countable, and because X is bounded, we can always take a subsequence of $\{\varphi_n\}$, also denoted $\{\varphi_n\}$, that converges pointwise on the support of the distribution of S to some $\varphi^*: \mathbb{R}^m \rightarrow \mathbb{R}$. We let $F^* = \varphi^*(S)$ and $\tilde{f}^* = E(F^* | Z)$. We can take a further subsequence, again denoted $\{\varphi_n\}$, for which \tilde{f}_0^n converges to some number y . We have, by continuity of Π and dominated convergence, $v^* = \lim_n V(F^n) = \lim_n E[\Pi(\tilde{f}_0^n, f_0^n)] = E[\Pi(\tilde{f}^*, y)]$. The result is shown if $\tilde{f}_0^* = y$. If $\tilde{f}_0^* > y$, then by strict monotonicity of $\Pi(f, \cdot)$ for each $f > 0$ (which follows from (i) and (iv) of Proposition 3), we would have $V(F^*) = E[\Pi(\tilde{f}^*, \tilde{f}_0^*)] > E[\Pi(\tilde{f}^*, y)] = v^*$, a contradiction of definition of v^* . If $\tilde{f}_0^* < y$, then we would have, for some $\epsilon < y - \tilde{f}_0^*$, $\mathcal{P}(\tilde{f}^* \in [f_0^*, \tilde{f}_0^* + \epsilon]) > 0$, by the definition of \tilde{f}_0^* . This, however, contradicts the fact that f^n converges to f^* almost surely, and therefore in distribution. (Note that this convergence-in-distribution argument does not work for $\tilde{f}_0^* > y$.) Thus, indeed, $\tilde{f}_0^* = y$, and we have the optimality of F^* .

the support of the distribution of $\tilde{f} = E(F | Z)$ is an interval even if the support of the distribution of S is discrete.

4. PROPERTIES OF OPTIMAL SECURITY DESIGN

This section provides some characterization of optimal security designs.

First-Order Conditions

We begin with a first-order necessary²⁴ condition for optimality of a security design, namely that, given a security design F , there is at most zero marginal benefit to the issuer of adding any incremental cash flows to the design.

Specifically, given a candidate security F , we can calculate the marginal benefit of adding to the design some bounded S -measurable random variable G , with appropriate truncation above at X and below at 0. While one can compute the derivative quite generally, we will only have occasion to apply it for the case of nonnegative G , which we assume for simplicity of notation. This directional derivative $\nabla V(F; G)$ is the right derivative of $V(\min(F + kG, X))$ with respect to $k \geq 0$, evaluated at $k = 0$, and can be computed as follows. We will show that the marginal private valuation,²⁵

$$\gamma(F, G) = \frac{d}{dk} E(\min(F + kG, X) | Z) |_{k=0^+},$$

and that the marginal minimum private valuation,

$$\gamma_0(F, G) = \frac{d}{dk} \min_z E_z[\min(F + kG, X)] |_{k=0^+},$$

both exist. We then compute $\nabla V(F, G)$ in terms of these derivatives.

PROPOSITION 4: *For any F and $G \geq 0$, let A be the event that $X - F > 0$. Then $\gamma(F, G) = E(G1_A | Z)$ and*

$$(5) \quad \gamma_0(F, G) = \min_{z \in \mathcal{Z}^*} E_z(G1_A) \quad \text{where} \quad \mathcal{Z}^* = \{z: E_z(F) = f_0\}.$$

If²⁶ $f_0 > 0$ and $\gamma_0(F, G) > 0$, then

$$(6) \quad \nabla V(F; G) = \gamma_0(F, G) E \left[\Pi \left(\frac{\tilde{f}}{f_0}, 1 \right) + \Pi_1 \left(\frac{\tilde{f}}{f_0}, 1 \right) \left(\frac{\gamma(F, G)}{\gamma_0(F, G)} - \frac{\tilde{f}}{f_0} \right) \right].$$

²⁴The first-order necessary conditions, while useful in characterizing solutions, are not generally sufficient, as the objective function is not generally concave. We therefore restrict our attention to a limited set of useful necessary conditions, rather than providing a fully elaborated treatment of first-order conditions.

²⁵We note that $\gamma(F, G)$ is a random variable; the derivative is state by state.

²⁶These strict positivity restrictions are purely to give a convenient expression. The derivative exists, and one can easily calculate it, for the general case.

PROOF: Let $C(k, z) = E_z[\min(F + kG, X)]$. By convexity, the right partial derivative $C_k(k, z)$ exists. We have $\gamma(F, G) = C_k(0, Z)$, which gives the result by passing the derivative through the expectation, using dominated convergence. In Appendix A, we generalize the envelope theorem to prove (5). We can use the homogeneity of Π to write

$$\Pi(\tilde{f}, f_0) = f_0 \Pi\left(\frac{\tilde{f}}{f_0}, 1\right).$$

Differentiation and dominated convergence yield (6).

Q.E.D.

The result is illustrated in Figure 4. The bracketed quantity in (6) is, at a particular outcome of Z , equal to the height of the line tangent to $\Pi(\cdot, 1)$ at \tilde{f}/f_0 , evaluated at the outcome of $\gamma(F, G)/\gamma_0(F, G)$. Roughly speaking, at a particular outcome of Z , we can interpret \tilde{f}/f_0 as a measure of the “information sensitivity” of the security F , as it represents the quotient of the realized private valuation by the lowest possible private valuation by the issuer. Similarly, we can interpret $\gamma(F, G)/\gamma_0(F, G)$ as the marginal information sensitivity of adding the new cash flows G to F . One obvious consequence of Figure 4 is that if this marginal sensitivity is everywhere less than the information sensitivity of F , then adding G to F increases the issuer’s payoff.

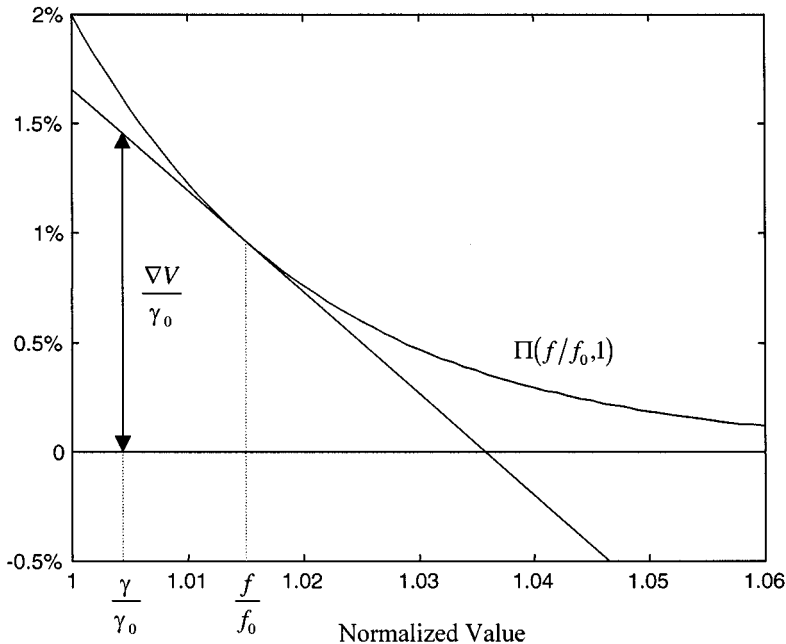


FIGURE 4.—Marginal gain of adding G to F .

This intuition suggests that it cannot be optimal to issue a security that never exhausts all available cash flows, that is, with $F < X$ almost surely. Because risk-free cash flows are insensitive to information, such a security could be improved upon by a risk-free increase in its payoff.

PROPOSITION 5: *If F is an optimal design, then the event that $F = X$ has strictly positive probability.*

PROOF: Without loss of generality, $f_0 > 0$. Suppose, for purposes of contradiction, that $F < X$ almost surely. For $G = 1$, we can compute that $\gamma(F, G) = \gamma_0(F, G) = 1$. From (6),

$$\nabla V(F; G) = \frac{V(F)}{f_0} + a,$$

where

$$a = E \left[\Pi_1(\tilde{f}, f_0) \left(1 - \frac{\tilde{f}}{f_0} \right) \right].$$

We have $a \geq 0$ because $\Pi_1 \leq 0$ and $\tilde{f} \geq f_0$. Thus $\nabla V(F; G) > 0$, and F cannot be optimal. *Q.E.D.*

Proposition 5 tells us that adding risk-free cash flows always improves a security design. A similar argument can also be used to show that adding risky cash flows can also improve the design so long as those cash flows are not too informationally sensitive. For instance, even if the initial security F has no information sensitivity ($\tilde{f}/f_0 = 1$), Figure 5 demonstrates that adding a contingent cash flow G improves the issuer's payoff so long as $\gamma(F, G)/\gamma_0(F, G) < 1/\delta$, or equivalently $\gamma_0(F, G) > \delta\gamma(F, G)$. In this case, we know from (6) that the lemons cost that stems from adding these cash flows is less than the retention cost of holding them. This would mean, for example, that it may be optimal to issue the entire asset pool X if the degree of asymmetric information is not too severe.

We can formalize this idea as follows. The informational sensitivity of S to Z is

$$\epsilon_S \equiv \sup_{(B, z_1, z_2)} \frac{\mu(B | z_1)}{\mu(B | z_2)} - 1.$$

(The supremum is taken with respect to B in the collection of measurable subsets of outcomes of S , and with respect to any outcomes z_1 and z_2 of Z . The ratio of two probabilities is defined to be 1 if both probabilities are zero.) If ϵ_S is close to 0, then conditioning on Z has relatively little impact on the distribution of S . For example, if the conditional density functions of S given z_1 and z_2 , respectively, exist and are bounded and continuous, then their largest ratio is no larger than $1 + \epsilon_S$. For sufficiently small sensitivity ϵ_S , the lemons

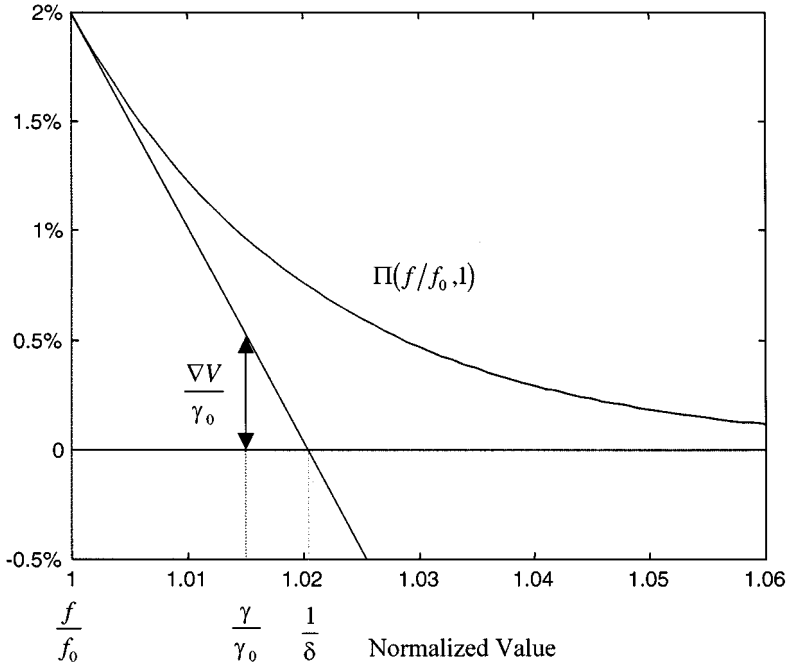


FIGURE 5.—Adding risky cash flows to a risk-free security.

premium is small enough that the issuer has an incentive to sell an equity claim in the underlying assets. In fact, the next result shows that pure equity is optimal unless the informational sensitivity exceeds the excess rate of return r on cash over retained assets, defined by $1/(1+r) = \delta$.

PROPOSITION 6: *Suppose the informational sensitivity of S to Z is strictly less than r . Then F is optimal if and only if $F = X$ (pure equity) almost surely.*

PROOF: Suppose not, and $F < X$ with positive probability. Without loss of generality, $f_0 > 0$. We let $G = 1$ and note that by the informational sensitivity assumption, $\gamma(F, G)/\gamma_0(F, G) < \delta^{-1}$. If $\gamma(F, G)/\gamma_0(F, G) > \tilde{f}/f_0$, then, from Proposition 3,

$$\begin{aligned}
 & \Pi\left(\frac{\tilde{f}}{f_0}, 1\right) + \Pi_1\left(\frac{\tilde{f}}{f_0}, 1\right) \left(\frac{\gamma(F, G)}{\gamma_0(F, G)} - \frac{\tilde{f}}{f_0} \right) \\
 & \geq \Pi\left(\frac{\tilde{f}}{f_0}, 1\right) - \delta \left(\frac{\gamma(F, G)}{\gamma_0(F, G)} - \frac{\tilde{f}}{f_0} \right) \\
 & > \left(1 - \delta \frac{\tilde{f}}{f_0} \right)^+ - \left(1 - \delta \frac{\tilde{f}}{f_0} \right) \\
 & \geq 0.
 \end{aligned}$$

On the complimentary event, $\gamma(F, G)/\gamma_0(F, G) \leq \tilde{f}/f_0$, we have

$$\begin{aligned} & \Pi\left(\frac{\tilde{f}}{f_0}, 1\right) + \Pi_1\left(\frac{\tilde{f}}{f_0}, 1\right)\left(\frac{\gamma(F, G)}{\gamma_0(F, G)} - \frac{\tilde{f}}{f_0}\right) \\ & \geq \Pi\left(\frac{\tilde{f}}{f_0}, 1\right) \geq \left(1 - \delta\frac{\tilde{f}}{f_0}\right)^+ \geq 0. \end{aligned}$$

Because $\delta\tilde{f} < f_0$ with strictly positive probability, taking expectations and using (6) shows that $\nabla V(F; G) > 0$. This contradicts the optimality of F . *Q.E.D.*

This result can be extended as follows. The informational sensitivity of a measurable set B of outcomes of S is defined to be

$$\epsilon_B = \sup_{(z_1, z_2)} \frac{\mu(B|z_1)}{\mu(B|z_2)} - 1.$$

The next result states: If it is possible for the issuer to strictly increase the payoff of the security on a set of contractible outcomes whose informational sensitivity is less than the excess rate of return r of cash over retained assets, then the issuer should do so.

PROPOSITION 7: Suppose F is an optimal security design and B is a measurable subset of the outcomes of S at which $F < X$. Then the informational sensitivity of B is more than r .

The proof follows that of Proposition 6, taking $G = \min(1_{S \in B}, X - F)$.

For example, suppose $S = X$ and $F = \varphi(X)$ is an optimal security. Letting $p(x|z)$ denote the conditional likelihood of X given Z at x and z (supposing that this density exists), the last result implies that $\varphi(x) = x$ at any outcome x of X satisfying

$$\inf_z p(x|z) \geq \delta \sup_z p(x|z).$$

That is, all cash flows are paid out if the z -conditional density of these cash flows is not too sensitive to the particular outcome z for private information.

Contractible Signals

If the issuer's signal Z can be used in the contractual payoff of the security, then it is optimal to design a security that does not involve any lemons premium, as shown in the following result. In effect, the security design can act as a warranty against adverse private information that may have been held by the issuer at the time of sale.

PROPOSITION 9: *Suppose Z is S -measurable. Then there exists an optimal security F whose private valuation does not depend on Z , in particular satisfying $\tilde{f} = f_0 = \min_z E_z(X)$. This security is not retained by the issuer (that is, $Q^*(\tilde{f}) = 1$).*

PROOF: Recall that X is bounded above by some constant, say M . For each outcome z of Z , let

$$K(z) = \left\{ y \in [0, M] : E_z[\min(y, X)] = \min_z E_z(X) \right\}.$$

Because $E[\min(0, X) | Z] = 0 \leq \min_z E_z(X) \leq E(X | Z) = E[\min(M, X) | Z]$, and because $E_z[\min(y, X)]$ is continuous in y by the dominated convergence theorem, we know that $K(z)$ is nonempty and compact. There exists, by continuity of $(z, y) \mapsto E_z[\min(y, X)]$, a measurable selection $h: \mathbb{R}^n \rightarrow [0, M]$ from K . Letting $F = \min(h(Z), X)$, we have $\tilde{f} = f_0 = \min_z E_z(X)$.

It remains to show that F is optimal. In order to see this, we note from Proposition 3, part (ii), that $\Pi(f_0, f_0) = (1 - \delta) \min_z E_z(X)$. For a security G , let $\tilde{g} = E(G | Z)$. Since $\Pi(\cdot, g_0)$ is decreasing

$$\begin{aligned} \Pi(\tilde{g}, g_0) &\leq \Pi(g_0, g_0) = (1 - \delta) g_0 \leq (1 - \delta) \min_z E_z(X) \\ &= \Pi(\tilde{f}, f_0) = V(F), \end{aligned}$$

so $V(G) = E[\Pi(\tilde{g}, g_0)] \leq V(F)$, and we are done. Q.E.D.

For a simple example, suppose $Z = X$, so that the issuer has precise advance knowledge of the asset cash flows. Then the optimal design is to issue a claim F equal to the minimum outcome of X .

In general, however, this proposition does not imply that if the underwriter's private information is contractible, then the optimal security is riskless. Rather, in this case we know only that the private valuation of the issuer is constant, and therefore that there need be no lemon's premium for an optimal security. For example, suppose that the conditional distribution of X given Z is uniform on $[0, Z]$, where the distribution of Z has support $[z_0, z_1]$, with $z_1 > z_0 > 0$. Then the security $F = \min(Z(1 - (1 - z_0/Z)^{1/2}), X)$ would be optimal.²⁷ This security is obviously risky but its conditional expected payoff \tilde{f} is always $z_0/2$.

We remark that the contractability of Z is an extremely strong assumption that cannot be expected in general. Consider the following typical example. Suppose that Z corresponds to a noisy estimate of some factor Y that affects the distribution of X . For example, Y might correspond to the future sales of a firm. If Y is itself verifiable ex post, and hence contractible, one might be tempted to conclude that an optimal security design would eliminate the sensitivity to the information Z by conditioning on Y . While this is feasible, it is not generally optimal. In order to see this, we let $Z = Y + e$, where e is

²⁷In order to check this, one needs to show that $E(F | Z) = z_0/2$, which is straightforward to verify given that $E_z[\min(k, X)] = k - k^2/(2z)$ for $k < z$.

independent of $\{Y, X\}$. Then

$$E(X|Z) = E[E(X|Y, Z)|Z] = E[E(X|Y)|Z].$$

Hence, in general, $E(X|Y)$ will take values below $\min_z E_z(X)$ with strictly positive probability. Thus, a security whose payoff is independent of Y would generally be too conservative due to the cost of nonissuance. In general, it is advantageous to add a marginal amount of information sensitivity in order to save on holding costs.

5. STANDARD DEBT

In order to illustrate cases in which standard debt may be optimal, in this section we consider the case in which the contractible information S is X itself, the cash flows of the underlying assets. This is a natural setting for many standard asset-backed securities in which the cash flows from the underlying assets are easily monitored and verified.

We also restrict attention to a security design whose payoff is increasing in the outcome of X , a natural and commonplace form of security. The fact that we consider only security designs that are increasing in the asset cash flows implies some loss of generality that is not without precedent in the security-design literature. See, for example, Innes (1990) or Nachman and Noe (1991, 1994), for the use of, and potential justification for, such a restriction. We note that if the security design is not monotone in X , then the issuer could benefit by contributing additional funds to the assets. If such “charitable contributions” cannot be prevented, then only monotone payoffs would be observed and our assumption is without loss of generality.²⁸

In short, we now try to characterize solutions to the security-design problem when optimizing within the set of securities of the form $F = \varphi(X)$, for some nondecreasing φ . A simple example of such a security is standard debt, defined at some constant face value d by $F = \min(X, d)$. If $d \geq X$, then the standard-debt security $\min(X, d)$ is equivalent to X , or pure equity.

A strict interpretation of $\min(X, d)$ as standard debt depends, however, on an assumption that the unsold fraction $1 - q$ of the security is not held on the balance sheet of the issuer at the time of default. As we have discussed earlier, this may be the case contractually, as with CMOs and other asset-backed securities which are not direct obligations of the issuer, but rather of a legally distinct pool of assets, or it may be the case because the entire issue was sold (at time of issuance) to an underwriter, with the same incentives as the issuer, who

²⁸In order to see this, let $F = \varphi(X)$ and suppose that $\varphi(x_1) > \varphi(x_2)$ for some $x_1 < x_2$. At the outcome x_1 of X , suppose the issuer can donate $x_2 - x_1$ and get a net payoff of $x_2 - \varphi(x_2) - (x_2 - x_1) = x_1 - \varphi(x_2)$. This exceeds the payoff of $x_1 - \varphi(x_1)$. Thus the payoff $\varphi(x_1)$ will not be realized.

subsequently sells it to public investors.²⁹ Alternatively, if the unsold fraction $1 - q$ does remain “comingled” with X , a true standard debt contract would be defined $F(X) = \min(X + (1 - q)F(X), d)$, or equivalently $F(X) = \min(X/q, d)$. This case is not consistent with our model.

We introduce the following notion of a worst-case outcome for Z :

DEFINITION: An outcome z_0 of Z is a *uniform worst case* if, for any other outcome z and any interval $I \subset \mathbb{R}_+$ of outcomes of X ,

- (i) if $\mu(X \in I | z) > 0$, then $\mu(X \in I | z_0) > 0$;
- (ii) the conditional of $\mu(\cdot | z)$ given $X \in I$ has first-order stochastic dominance over the conditional of $\mu(\cdot | z_0)$ given $X \in I$.

The assumption of a uniform worst case is equivalent to the property that, for any z , the Radon-Nikodym derivative $\pi_z: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ of $\mu(\cdot | z)$ with respect to $\mu(\cdot | z_0)$ exists (from (i)), and can be chosen to be an increasing³⁰ function (from (ii)). We can actually drop the technical condition (i), losing only the convenience of being able to work with the Radon-Nikodym derivative π_z .

The existence of a uniform worst case is weaker than the monotone likelihood ratio property (MLRP).³¹ For example, there is a uniform worst case if $X = h(Z) + Y$ or if $X = h(Z)e^Y$ for some continuous function h and some random variable Y whose density is log-concave. (For instance, the normal, uniform, and exponential densities are log-concave.) For these examples, the MLRP would require, in addition, that h be monotone.

PROPOSITION 10: *If there is a uniform worst case, then a standard debt contract is optimal.*

PROOF: Let $G = \varphi(X)$ be any monotone security and $\tilde{g} = E(G | Z)$. By the monotonicity of φ and the fact that there is a uniform worst case z_0 , we know that $g_0 = E_{z_0}(G)$. Consider a standard-debt security $F = \min(X, d)$. By dominated convergence, $E_{z_0}[\min(X, d)]$ is continuous in d , so we may choose d so that $f_0 = g_0$. Letting $H = G - F$ and $h(Z) = E(H | Z)$, we have $h(z_0) = 0$. Because $\varphi(x) \leq x$ and φ is increasing, there exists some x^* such that $H = \varphi(X) - \min(X, d) > 0$ if and only if $X > x^*$. Then, for any z ,

$$h(z) = E_z(H) = E_{z_0}[H\pi_z(X)] \geq E_{z_0}[H\pi_z(x^*)] = \pi_z(x^*)h(z_0) = 0.$$

²⁹Alternatively, $\min(X, d)$ can be interpreted as standard debt if the fraction retained by the issuer is intended for sale at a later date, but before X is realized. In this case, the retention discount δ is relative to the time of subsequent sale.

³⁰If not, then there are sufficiently small adjacent intervals $(a, a + b)$ and $[a + b, a + 2b)$ such that $\int_a^{a+b} \pi_z(x) d\mu(x | z_0) < \int_{a+b}^{a+2b} \pi_z(x) d\mu(x | z_0)$, which leads to a contradiction of stochastic dominance of $\mu(\cdot | z)$ over $\mu(\cdot | z_0)$ conditional on $X \in (a, a + 2b)$. For details, see a related result by Whitt (1980).

³¹The MLRP assumes the existence of a conditional density of X given Z , and assumes in addition a complete order \succsim on the outcomes of Z under which $\mu(\cdot | z)$ uniformly dominates $\mu(\cdot | z')$ (in the above sense) whenever $z \succsim z'$.

Thus $\tilde{g} = \tilde{f} + h \geq \tilde{f}$. Because $\Pi(\cdot, g_0)$ is decreasing (see Proposition 3),

$$\Pi(\tilde{g}, g_0) = \Pi(\tilde{f} + h, g_0) \leq \Pi(\tilde{f}, f_0).$$

Taking expectations, we see that $V(F) \geq V(G)$. Because G is arbitrary, an optimal security is standard debt at the face value d^* achieving $\max_d V(\min(d, X))$. *Q.E.D.*

The intuition for this result can be explained roughly as follows. Illiquidity costs increase with the informational sensitivity of the security, measured in terms of the private valuation \tilde{f} relative to the worst-case private valuation f_0 . In order to efficiently trade off illiquidity costs for retention costs, the issuer minimizes the informational sensitivity of the security, among those with the same worst-case private valuation. The worst-case private valuation for any monotone security occurs at the same outcome z_0 of Z . Fixing the worst-case private valuation f_0 , regardless of the outcome z , standard debt has the lowest ratio of private valuation at z to private valuation at z_0 . To put it another way, changing the outcome of the issuer's information from z_0 to z (weakly) increases the private valuation of any monotone security more than it increases the private valuation of standard debt.

High Retention Costs Imply Junk Debt or Equity

As we know from Proposition 5, riskless debt is not optimal. In fact, if d is an optimal level of standard debt, then, by Proposition 7,

$$(7) \quad \min_z \mu(X > d | z) \leq \delta.$$

Using (7), we can study how the optimal face value of the debt varies with the retention-value coefficient δ . If δ is relatively small, the issuer faces a high cost of holding the assets (equivalently, the issuer has highly profitable investment opportunities) and so the issuer's demand for funds is high. In this case we might expect the debt to be relatively large, perhaps to the point of issuing equity (in the form of standard debt at some face value that is an upper bound for X). On the other hand, for δ close to 1, the issuer's holding cost is small, and we might expect the issuance of a small amount of liquid, low-risk debt. We may roughly interpret this intuition as the Myers-Majluf pecking-order hypothesis. Specifically, we can vary the coefficient δ and let d_δ denote the optimal face value of standard debt in the model with coefficient δ . We assume a uniform worst case z_0 . From (7), $\mu(X > d_\delta | z_0) \leq \delta$. Then $\liminf_{\delta \downarrow 0} \mu(X \leq d_\delta | z_0) = 1$. Assuming that the support of $\mu(\cdot | z)$ does not depend on z , the limiting contract, in this sense, simply pays X , or pure equity.

Variation of Debt Level with Assets

We next relate the size of the optimal debt issue to parameters affecting the characteristics of the underlying assets. In particular, we show that rescaling the assets simply rescales the optimal quantity of debt, while increasing the amount

of risk-free cash flows increases “debt capacity” by less than one for one. The idea is that the marginal benefit of increasing the debt load, while a nonnegative function of cash inflows, is also a declining function. Thus, as cash flows are added, the face value of debt grows, but not as fast as cash grows.³²

To this end, taking a comparative-statics approach, we consider a family $\{X(b): b \geq 0\}$ of asset cash flows, defined in the next proposition, and we let $D(b)$ be the optimal face value of debt given the asset cash flow $X(b)$.

PROPOSITION 11: *Let Y be a fixed nonnegative random variable.*

(i) *For the case $X(b) = bY$, we have $D(b) = bD(1)$.*

(ii) *For the case $X(b) = b + Y$, suppose there is a uniform worst case for $X(0)$. For any $b \geq 0$, we have $D(b) - D(0) \leq b$.*

(iii) *For case (ii), suppose moreover that neither riskless debt nor pure equity are optimal security designs for asset cash flow $X(0)$. Then, for any b , we have $D(b) - D(0) < b$.*

The first part of the result follows simply from the homogeneity of Π . In general, standard debt or not, re-scaling the asset cash flows causes a like re-scaling of the optimal security design. As for part (iii), the assumption that riskless debt is not optimal follows from Proposition 5 provided that the distribution of X has a density function, or under weaker conditions.

PROOF: For $X(b) = bY$, the result follows from homogeneity of degree 1 of both Π and $F(b) = \min(X(b), bd) = b \min(Y, d)$.

For the case of $X(b) = b + Y$, we let $F(d) = \min(Y, d)$ and $\tilde{f}(d) = E[F(d) | Z]$. Note that $E[\min(d + b, X(b)) | Z] = b + \tilde{f}(d)$. We define $G(d, b) = E[\Pi(b + \tilde{f}(d), b + f(d)_0)]$ and let $\Delta(b)$ solve $\max_d G(d, b)$. (A solution exists by continuity.) Thus $D(b) = b + \Delta(b)$, and it remains to show that Δ is decreasing, and strictly decreasing under the additional condition that neither riskless debt nor pure equity are optimal for Y .

We define $p = \mu(Y > d | Z) / \mu(Y > d | z_0)$, where z_0 is a uniform worst case. (We take $\mu(Y > d | z_0) > 0$ without loss of generality.) We also let $\theta = (b + \tilde{f}(d)) / (b + f(d)_0)$. Then, from Proposition 4,

$$G_d(d, b) = \mu(Y > d | z_0) E[\Pi(\theta, 1) + \Pi_1(\theta, 1)(p - \theta)].$$

Because z_0 is a uniform worst case, for any z we know that $\mu(Y > d | z) / \mu(Y > d | z_0)$ is increasing in d . This implies that

$$\theta = \frac{b + \tilde{f}(d)}{b + f(d)_0} \leq \frac{\tilde{f}(d)}{f(d)_0} = \frac{\int_0^d \mu(Y > x | Z) dx}{\int_0^d \mu(Y > x | z_0) dx} \leq \frac{\mu(Y > d | Z)}{\mu(Y > d | z_0)} = p.$$

³²This is related to the fact that, as we show in the concluding remarks, if the issuer could “strip out” the risk-free cash flows as a separate riskless security, it would pay to do so, since the issuer would avoid the lemon’s cost altogether on that component of the cash flows.

Now, since Π is strictly convex in its first argument, $\Pi(\theta, 1) + \Pi_1(\theta, 1)(p - \theta)$, is strictly increasing, state by state, in the outcome of θ . Thus, because θ is weakly decreasing in b , state by state, and strictly decreasing in b on the event $A(d)$ that $\tilde{f}(d) > f(d)_0$, we know that $G_a(d, b)$ is weakly decreasing in b , and strictly decreasing in b if $A(d)$ has nonzero probability. This implies that G satisfies the single-crossing property in $(d, -b)$ defined by Milgrom and Shannon (1994). (See, in particular, Edlin and Shannon (1996).) Hence, by their Theorem 4, Δ is decreasing. The strict version (iii) of the result follows similarly, using the fact that $A(D(0))$ has nonzero probability by assumption, and by the fact that the optimal level of debt is interior. *Q.E.D.*

Numerical Example

We compute an example showing how the optimal level of debt varies with the parameters governing the distribution of (X, Z) . We suppose that $X = (1 + Z)Y$, where Y is lognormally distributed with volatility σ ; that is, $Y = \exp(\alpha - \sigma^2/2 + \sigma e)$, where α is a constant and e is standard normal. We take Z to be uniformly distributed on $[-m, m]$, for some coefficient $m \in (0, 1)$. For this parameterization, $z_0 = -m$ is a uniform worst case, so the previous propositions apply. Although, as defined, X is not bounded, the example could be modified by truncating the distribution of Y . Boundedness was assumed primarily for technical convenience and does not critically affect the examples.

First we consider the case in which $e^\alpha = 100$, $\sigma = 50$ percent, and $\delta = 98$ percent. Figure 6 shows a plot of the securitization profit $V(F(d)) = E[\Pi(\tilde{f}(d), f(d)_0)]$ against the face value d of debt, for several levels of informational sensitivity: $m = 1, 3, 7$, and 10 percent.

We note that the securitization profit $V(F(d))$ is between 0 and 2 percent of $E(X) = 100$, because the maximum securitization profit is the holding cost $1 - \delta$. Moreover, the profit is strictly less than 2 percent since there is a lemons problem ($m > 0$). For any given m , the issuer must trade off the increased lemons cost associated with increasing the face value (and riskiness) of the debt, with the increased holding cost associated with decreasing the face value (and market value) of the debt. This leads to an interior solution for an optimal level of debt. Note also that as m increases, the lemons problem becomes more severe. This lowers the securitization profit associated with any level of debt, and also reduces the optimal debt level.

Figure 7 is a plot of the securitization profit associated with different levels of volatility for the underlying asset. We fix m at 7 percent and plot $V(F(d))$ against d for $\sigma = 10, 25, 50$, and 75 percent. For relatively low levels of volatility, we see that the issuer's securitization profit is relatively high.³³ The intuition for this is that a larger quantity of debt can be issued that is relatively default-free,

³³In a more complicated model in which the underlying assets are themselves endogenous, this suggests that the security-design problem induces risk aversion by the issuer regarding the asset payoff X . See DeMarzo (1998).

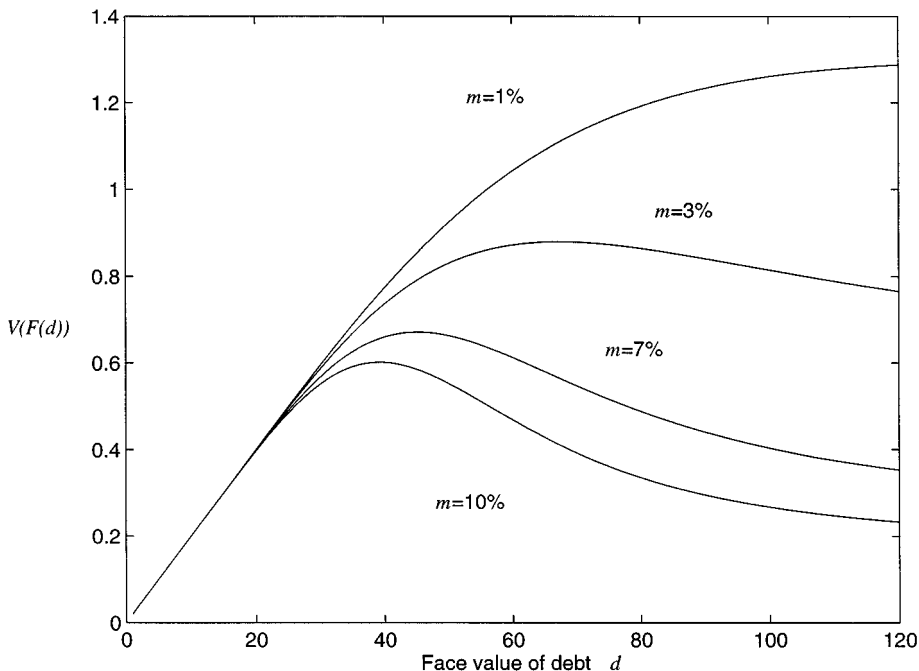


FIGURE 6.—Securitization profit $V(F(d))$ at debt level d .

and thus not subject to a lemons problem. The optimal face value of the debt is decreasing in the volatility σ . This suggests an alternative to bankruptcy costs as an explanation³⁴ for why riskier firms issue less debt. The explanation³⁵ suggested by our model is that relatively riskier firms, other things equal, are more exposed to a lemon's premium on the sale price of their debt.

6. PRE-DESIGN INFORMATION

Thus far, we have assumed that the private information held by the issuer is learned after the security is designed. We briefly consider the possibility that, even before the design stage, the issuer has some private information bearing on the distribution of the cash flows available. In particular, we suppose that $Z = (Z_a, Z_b)$, where Z_a is learned prior to the issuer's choice of the design of the security, while Z_b is learned after the security design is chosen, but prior to the liquidation stage. The ex-ante information Z_a affects the security design problem. First, when the issuer chooses F to maximize the expected securitization profit, that expectation is taken conditional on Z_a . Moreover, the chosen

³⁴ Haugen and Senbet (1978) have already criticized bankruptcy costs as an explanation for capital structure.

³⁵ We are grateful to a referee for suggesting that we emphasize this point.

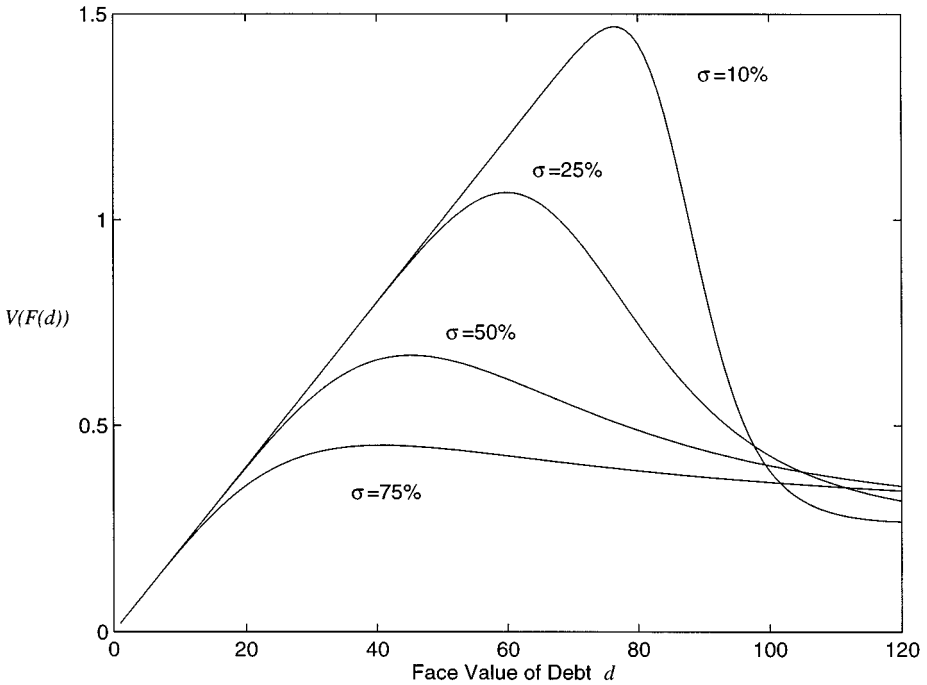


FIGURE 7.—Effect of asset volatility.

design F serves as a signal to investors about Z_a . The incentive for the issuer to use F itself as a signal changes the nature of the security-design problem.

Although this complicates the problem in general, we show that, under certain reasonable assumptions regarding the information structure, our prior analysis is still valid. We can assume, for example, that for some k and continuous $h: \mathbb{R}^n \rightarrow \mathbb{R}^k$,

(i) $h(Z)$ is a sufficient statistic for Z , in that S and Z are independent given $h(Z)$;

(ii) the support of the conditional distribution of $h(Z)$ given Z_a is constant (that is, does not vary with the outcome of Z_a).

These conditions imply that, although the issuer may learn nontrivial information about cash flows and contractible variables at the initial stage, none of this early information bears on the support of the private valuation \tilde{f} , regardless of the security F chosen. In this case, we claim that an optimal security design is given by the solution to

$$(8) \quad V(F | Z_a) = \sup_F E \left[\Pi(\tilde{f}, f_0) | Z_a \right],$$

where, using (i),

$$\tilde{f} = E(F | Z) = E[F | h(Z)],$$

and f_0 is the lower boundary of the support of the distribution of \tilde{f} , which by (ii) is the same whether or not one first conditions on Z_a . In the liquidation game, we propose the separating equilibrium in which the issuer signals \tilde{f} by selling $Q^*(\tilde{f})$. Because the equilibrium is separating, and because the worst case f_0 is unaffected by first conditioning \tilde{f} on Z_a , the signalling game is unaffected by whether investors know the outcome of Z_a . The issuer therefore gains nothing by attempting to manipulate investors' inference of Z_a by announcing a choice of F . This confirms that (8) is the relevant design problem, and that all of our earlier results apply, once the issuer conditions on Z_a .

As a specific example, suppose the assets correspond to the revenues associated with a pharmaceutical entering clinical trials. Our assumptions hold if, at the design stage, the issuer has private information regarding the likelihood of success of these trials, but learns the outcome of the trials prior to issuance. In this case, Z_a corresponds to the likelihood of success, whereas $h(Z) = Z_b$ is the actual outcome. For a parametric example, suppose we take univariate Z_a and Z_b , and let $S = X = \phi(Z_a, Z_b, Y)$, where $Z_a \geq 0$, Z_b has support $[0, \infty)$, and Z_a, Z_b , and Y are independent. Then the sufficient statistic $h(Z)$ is equal to $Z_a Z_b$, and condition (ii) is satisfied. Thus, the issuer can apply our original model after conditioning on Z_a .

If, with either of the above variations in our information structure, the assumptions of Section 5 hold (that is, $S = X$ and there is a uniform worst case), then the optimal monotone security design remains standard debt. The face value of the debt, however, depends in principle on Z_a .

For example, if the issuer's private information at the pre-design stage is unexpectedly favorable, then the face value of debt is reduced because, at the time of sale the lemon's premium is expected to be more severe. (The issuer anticipates the need to retain more of the issue in order to signal the anticipated higher-than-normal private valuation of the issue.)

7. CONCLUDING REMARKS

This paper presents a model of security design based on the liquidity of the created security. We model liquidity by demonstrating that private information on the part of the issuer leads naturally to a downward-sloping demand curve for securities. We consider several information structures and show that this lemons problem implies that the issuer's profit function for the public offering is homogeneous of degree one and convex in the issuer's private valuation of the security. These and several other general properties allow us to characterize the optimal security design in several cases. Most importantly, we show that if securities are restricted to be monotone in the available cash flows, then (under distributional assumptions) risky standard debt is optimal.

Convexity of the issuer's profit function for the public offering also suggests that there may be a potential gain to the issuer from issuing separate securities. We have only considered in this paper the single security-design problem. In principle, however, the issuer prefers to decompose this security into heteroge-

neous pieces. For example, suppose the assumptions of Proposition 10 hold, so that the optimal security is standard debt with some face value d . If $X \geq d_0$ for some constant $d_0 > 0$, then the issuer is strictly better off issuing risk-free senior debt with face value d_0 , *as well as* junior debt with face value $d_1 = d - d_0$. That is, letting $F = \min(d_1, X - d_0)$, $G = \min(d, X)$, and $\tilde{g} = E(G | Z)$, we have

$$\Pi(d_0, d_0) + E[\Pi(\tilde{f}, f_0)] > E[\Pi(\tilde{g}, g_0)].$$

This follows from the fact that $\tilde{f} = \tilde{g} - d_0$, which implies that

$$\begin{aligned} & \Pi(d_0, d_0) + \Pi(\tilde{g} - d_0, g_0 - d_0) - \Pi(\tilde{g}, g_0) \\ &= g_0 \left[\frac{d_0}{g_0} \Pi(1, 1) + \left(\frac{g_0 - d_0}{g_0} \right) \Pi\left(\frac{\tilde{g} - d_0}{g_0 - d_0}, 1 \right) - \Pi\left(\frac{\tilde{g}}{g_0}, 1 \right) \right] \\ &> g_0 \left[\Pi\left(\frac{\tilde{g}}{g_0}, 1 \right) - \Pi\left(\frac{\tilde{g}}{g_0}, 1 \right) \right] = 0, \end{aligned}$$

using the homogeneity of Π and the strict convexity of $\Pi(\cdot, 1)$.

This reasoning suggests that an issuer might prefer to issue prioritized debt structures when multiple securities can be issued. Examples include the various tranches of a structure of collateralized mortgage obligations backed by a given pool of mortgages, or the subordination structure for the bonds of a corporation. Modeling this formally requires several modifications to the basic model described in this paper, since in general the liquidation game becomes a multi-dimensional signalling problem. DeMarzo and Duffie (1993) develop a more general model of this sort. The motivation for “splitting” securities in this setting is independent of motivations due to spanning or clientele effects that have been the focus of much of the prior literature (see Allen and Gale (1994)).

Finally, in DeMarzo and Duffie (1997), we also consider the following extensions:

- a. We endogenize the degree of private information by allowing the issuer to acquire information at a cost.
- b. We allow for the possibility that investors also have private information not known to the issuer.
- c. We remarked in Section 2 that the retention costs represented by the parameter δ might naturally correspond to profitable investment opportunities. That is, if the cash generated by securitizing the assets can be invested in projects earning a return r that exceeds the market discount rate (which we have normalized to zero), then $\delta = 1/(1+r)$. This interpretation of the model requires, however, that the return r be independent of the scale of the new investment. In general, one might expect nonconstant returns to scale. In DeMarzo and Duffie (1997), we investigate this generalization of the model and show that the conditions for standard debt to be optimal are unchanged.

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APPENDIX A: TECHNICAL RESULTS

In order to verify (5), we need only deal with a continuous $C: \mathbb{R}_+ \times \mathcal{Z} \rightarrow \mathbb{R}$, for compact $\mathcal{Z} \subset \mathbb{R}^n$, satisfying the property that, for each z , $C(\cdot, z)$ is concave. We let $C^*(\epsilon) = \min_z C(\epsilon, z)$ and let $C_\epsilon^*(0)$ and $C_\epsilon(0, z)$ denote the right derivatives at 0 of C^* and $C(\cdot, z)$, respectively. (These derivatives exist by concavity of $C(k, z)$ in k , and by the fact that the minimum of a family of concave functions is concave.) We assume that $C_\epsilon(0, z)$ is continuous in z . This is justified in our application by dominated convergence. Finally, we let $\mathcal{Z}^*(\epsilon) = \operatorname{argmin}_z C(\epsilon, z)$.

PROPOSITION A: $C_\epsilon^*(0) = \min_{z \in \mathcal{Z}^*(0)} C_\epsilon(0, z)$.

PROOF: By definition, $C^*(\epsilon) \leq C(\epsilon, z)$ for all z . Thus,

$$C^*(\epsilon) - C^*(0) \leq C(\epsilon, z) - C(0, z), \quad z \in \mathcal{Z}^*(0).$$

Then,

$$C_\epsilon^*(0) \leq C_\epsilon(0, z), \quad z \in \mathcal{Z}^*(0),$$

and it follows that

$$(A.1) \quad C_\epsilon^*(0) \leq \inf_{z \in \mathcal{Z}^*(0)} C_\epsilon(0, z).$$

We let $a = C_\epsilon^*(0)$. For any positive integer n and for $k \geq 1/n$ and z_n in $\mathcal{Z}^*(1/n)$, the concavity of $C(\cdot, z_n)$ implies that

$$\begin{aligned} \frac{1}{k} [C(k, z_n) - C(0, z_n)] &\leq n \left[C\left(\frac{1}{n}, z_n\right) - C(0, z_n) \right] \\ &\leq n \left[C^*\left(\frac{1}{n}\right) - C^*(0) \right] \\ &\leq a. \end{aligned}$$

Thus, $C(k, z_n) - C(0, z_n) \leq ka$. Taking a subsequence if necessary, we have z_n converging to some z^* . Continuity of C implies that $C(k, z^*) - C(0, z^*) \leq ka$, for all $k \geq 0$. Thus, $C_\epsilon(0, z^*) \leq a$.

Finally, $C(1/n, z_n) - C^*(0) \leq a/n$, and again by continuity we have $C(0, z^*) \leq C^*(0)$. This implies that $C(0, z^*) = C^*(0)$, so z^* is in $\mathcal{Z}^*(0)$. By (A.1), we have

$$C_\epsilon(0, z^*) \leq a = C_\epsilon^*(0) \leq \inf_{z \in \mathcal{Z}^*(0)} C_\epsilon(0, z).$$

The result follows. *Q.E.D.*

APPENDIX B: BINARY INFORMATION

In this appendix we extend by treating the case in which the outcomes of Z are 0 and 1, and show that the natural equilibrium properties that we exploit in the body of the paper are robust, in the sense of equilibrium uniqueness subject to a standard equilibrium refinement for signalling games.

PROPOSITION B1: *With binary information, let*

$$(B.1) \quad \hat{Q}(f) = (1 - \delta) \frac{f_0}{f - \delta f_0},$$

and

$$\hat{P}(q) = \left[\frac{1}{q} + \frac{\delta}{1 - \delta} \right] (1 - \delta) f_0.$$

Then (\hat{P}, \hat{Q}) is a separating equilibrium for the signalling game:³⁶

PROOF: With this definition of \hat{Q} ,

$$U(f_0, 1, f_0) = (1 - \delta) f_0 = U(f_0, \hat{Q}(f), f),$$

so that the low type has no incentive to mimic the high type. We can also check that $U(f, \hat{Q}(f), f) > f_0 - \delta f = U(f, 1, f_0)$, so that the high type does not mimic the low type. *Q.E.D.*

While \hat{Q} defines an equilibrium, this is not the unique equilibrium. There exist other separating equilibria, as well as pooling equilibria. This equilibrium is the “natural” one, however, in the following sense. First, it is Pareto-dominant among all separating equilibria. Second, we show below that it is the unique equilibrium satisfying the weak D1 refinement of Cho and Kreps (1987) and Banks and Sobel (1987). In our context, this refinement essentially states that if the issuer sells a quantity that would make only the high type better off, then the market should believe that security’s value is high. This is a weak refinement, and one supported by much of the signalling literature.

PROPOSITION B2: *The equilibrium of Proposition B1 is the unique equilibrium that satisfies the D1 refinement.*

PROOF: First consider a pooling equilibrium. A pooling equilibrium must have $q > 0$, since otherwise the low type would be better off choosing $q' = 1$. Also, the equilibrium price p must be less than f_1 , the high outcome of $E(F|Z)$. Then one can check that for any deviation

$$q' \in \left[\frac{q(p - \delta f_1)}{f_1 - \delta f_1}, \frac{q(p - \delta f_0)}{f_1 - \delta f_0} \right],$$

only the high type would gain if the market responds with a price of f_1 . Hence such a deviation by the high type is “credible,” and no pooling equilibria satisfy D1.

Next, suppose we have a separating equilibrium (P, Q) . If the low type deviates, the market price cannot decline (the price cannot go below f_0 in a sequential equilibrium). Thus, we must have $Q(f_0) = 1$, since otherwise the low type pays a retention cost. Any other separating equilibrium Q must have $Q(f_1) < \hat{Q}(f_1)$, for otherwise the low type would mimic the high type. Hence, if we choose any deviation $q' \in (Q(f_1), \hat{Q}(f_1))$, only the high type could gain with q' , and this deviation shows that the equilibrium cannot satisfy D1. *Q.E.D.*

For the binary case, the equilibrium price function \hat{P} , defined by $\hat{P}(q) = f_0[\delta + (1 - \delta)/q]$, is only applicable at the equilibrium quantities $\hat{Q}(f_0)$ and $\hat{Q}(f_1)$. The demand “curve” therefore should be interpreted as a comparative static, as we vary F and the corresponding definition of $Q(f_1)$. The resulting profit function is $\hat{\Pi}: \mathbb{R}_+^2 \rightarrow \mathbb{R}$, defined by $\hat{\Pi}(f, f_0) = U(f, \hat{Q}(f), f)$, where \hat{Q} is defined by (c.1). The profit function satisfies the properties for Π described in Proposition 4.

³⁶In a case with $f_0 = 0$, we define $\hat{Q}(f_0) = 1$ (though, since the price is zero, this is arbitrary and payoff irrelevant).

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